

Antennas & wave propagation

Introduction:- Antennas are electronic eyes & ears on the world.
An antenna is defined by as a metallic device (in the form of wire (or) rod) used for radiating (or) receiving radio waves.

An Antenna is a means of transporting signal from one end to other.

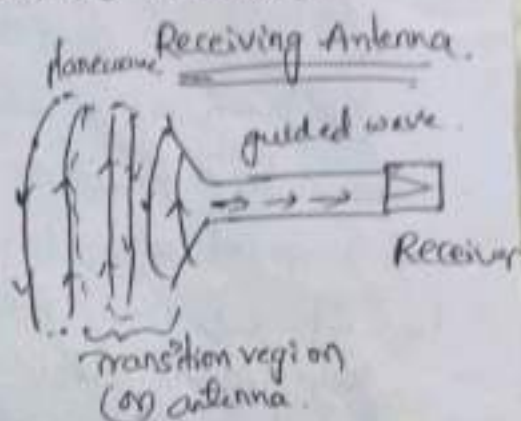
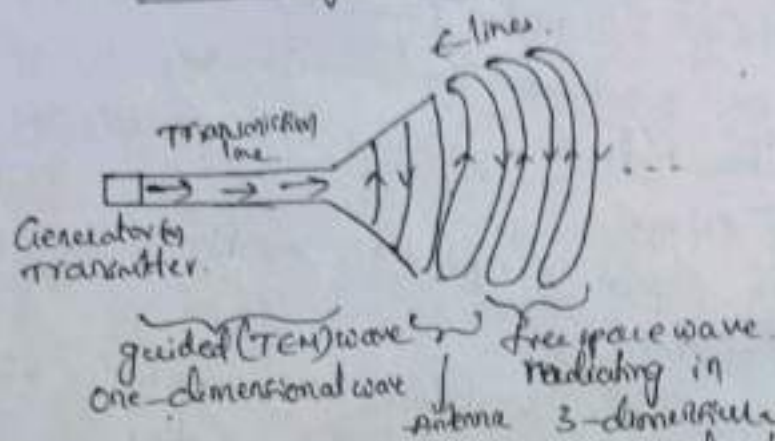
- An Antenna produces an electromagnetic field consisting electric field and magnetic field. together with these fields only it is possible to transmit & receive EM energy.
- when radio frequency signal is applied to an antenna, the electric & magnetic fields are produced.
- we find everywhere at home, workplaces, on cars, vehicles, ships - all operate with the basic principles of electromagnetics.
- First Radio antenna was assembled in 1886 by Heinrich Hertz.
- Now-a-days a antennas are the most essential Communication link for aircrafts and ships.

Definition According to IEEE standard definitions of antenna (or) aerial is a means for radiating (or) receiving radio waves.

- Antenna produces electromagnetic fields and these fields constitute an electromagnetic wave. It can be defined as source or radiator of electromagnetic waves.
- Antenna can also be defined as a transducer which converts radio frequency electrical current into an electromagnetic wave of the same frequency as that of an electrical current.
- An Antenna acts as a coupling device b/w a generator (or) transmitter and free space.
- It is an impedance matching device b/w free space & transmission line.

- Antenna is the transitional structure b/w free space and a guiding device. The guiding device or transmission line may take the form of a coaxial line (or) a hollow pipe (waveguide).
- It is used to transport electromagnetic energy from the transmitting source to the antenna (or) from the receiver.

Transmitting Antenna



- 1) An antenna may be a piece of conducting material in the form of a wire, rod (or) any other shape with excitation.
- 2) An antenna is a source (or) radiator of electromagnetic waves.
- 3) An " is a sensor of electromagnetic waves.
- 4) " is a transducer. (Converts electrical energy into an electromagnetic wave or converts electromagnetic wave into electrical energy)
- 5) " is an impedance matching device b/w the free space & transmitting/receiving.
- 6) " is a coupler b/w a generator & space (or) vice-versa.

Important properties of antenna

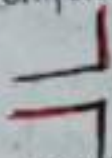
- 1) An Antenna has identical impedance used for Tx (or) Rx purpose. This property is called equality of impedances.
- 2) ~~exists~~ identical directional characteristics and patterns whether it is used for Tx (or) Rx purposes. This property is known as equality of directional pattern.
- 3) Antenna has same effective length irrespective of being used for Tx (or) Rx purpose. This property is called equality of effective lengths.

Basic Antenna Elements

- 1) alternating current element (Hertzian dipole) It is the basic short linear antenna. It is assumed that the current along the length of linear antenna is constant.
- 2) short dipole It is a linear antenna with a length less than $\lambda/4$. The current distribution of short dipole is assumed to be a triangular.
- 3) short monopole It is also a linear antenna with a length less than $\lambda/4$ with a current distribution assumed to be triangular.
- 4) Half wave dipole It is a linear antenna with length equal to $\lambda/2$. This antenna is generally a centred and its current distribution is sinusoidal.
- 5) Quarter wave monopole length equal to $\lambda/4$. This is also sinusoidal current distribution.

Types of Antenna

Antenna can be used as Tx antenna (or) receiving antenna. It has directional properties. It is the important component of a wireless communication system.

- a)  The most commonly used antenna is the dipole. It is made up of two straight wires (or) conductors laying along the same axis. The exciting source at the centre symmetrical dipole is produced. Dipole antenna is excited by voltage obtained from a transmission line, wave guide or directly from generator.

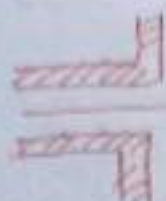
b) Loop Antenna

It consists of single (or) many turns of wire forming loop. It is generally excited by a generator directly. Field produced by a loop antenna is very much similar to the dipole.



loop

c) Helical antenna



Antenna with a wire in the form of a helix backed by a ground plane is called helical.

These 2 antennas are wire antennas which are used in aircrafts, ships, automobiles etc.

→ Travelling wave antenna: This antenna travelling wave in one direction is obtained. The velocity of this wave equals the velocity of light & it excites the waves in the space in the same direction strongly. maximum directivity can be achieved.

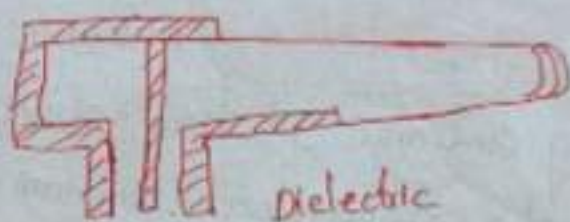
→ Travelling wave antenna in which the travelling wave is guided by a dielectric is called dielectric antenna.

→ Fields produced extend outside a dielectric guide.

→ Useful in Broadband signals.



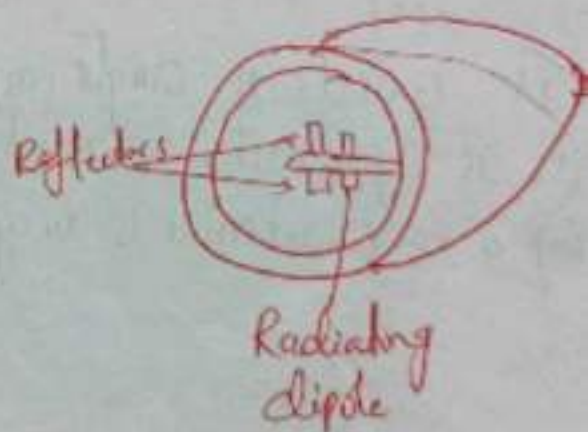
Rhombic



Dielectric

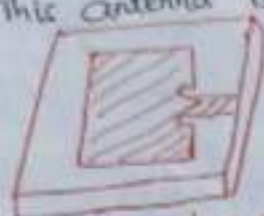
→ Field across an aperture excites radiation in space. If an aperture is ^{small} must be resonant to excite large amount of power. If aperture is large, need not be resonant. Ex: aperture antenna is horn (or) pyramidal.

It is useful in Broadband signals.



②

Parabolic reflectors are most common used.
 Electromagnetic waves are reflected by a conducting sheet.
 The dish of the parabolic reflector acts as a mirror and it
 reflects the radiation from a dipole (or) horn placed at the focus.
 This antenna is most suitable in radars, communications, astronomy etc.



(iv) microstrip patch antenna.



(v) coplanar strip horn.

These are an integrated circuit may be placed on a dielectric substrate. (iv) (v) show also called microstrip antenna.
 Integrated type antenna is also called as patch antenna.

Types of Antennas

- 1) Wire antennas
 - * dipole, monopole, loop antenna, helix
 - * Usually used in personal applications, automobiles, building, ships, aircrafts and spacecrafts.
- 2) Aperture antennas
 - * Horn antennas, waveguide opening.
 - * Usually used in aircrafts & space crafts, because these antennas can be flush mounted.
- 3) Reflector antennas
 - * Parabolic reflectors, corner reflectors.
 - * These are high gain antennas usually used in radio astronomy, microwave communication & satellite tracking.

4) Lens antennas

→ Convex-plane, Convex-Convex, Convex-concave and Concave-plane lenses.

* These antennas are usually used for very high freq applications

5) Microstrip antennas

* rectangular, circular shaped metallic patch above a ground plane

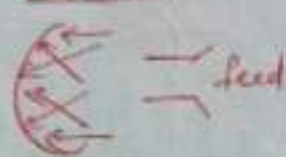
* used in aircraft, space crafts, satellite, missiles, cars, mobile phones

6) Array antennas

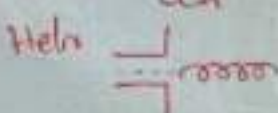
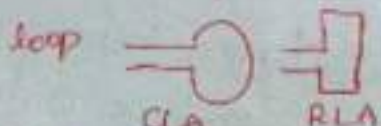
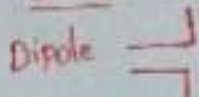
* Yagi-Uda antenna, microstrip patch array, aperture array, slotted waveguide array.

* used for very high gain applications with added advantage, such as, controllable radiation pattern.

Reflector



Wire



Convex plane Convex-convex plane



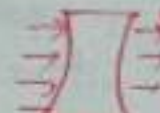
Convex-Concave plane



Concave plane



Concave-Concave plane



Concave-Convex



Aperture antennas

Pyramidal



Conical



Rectangular



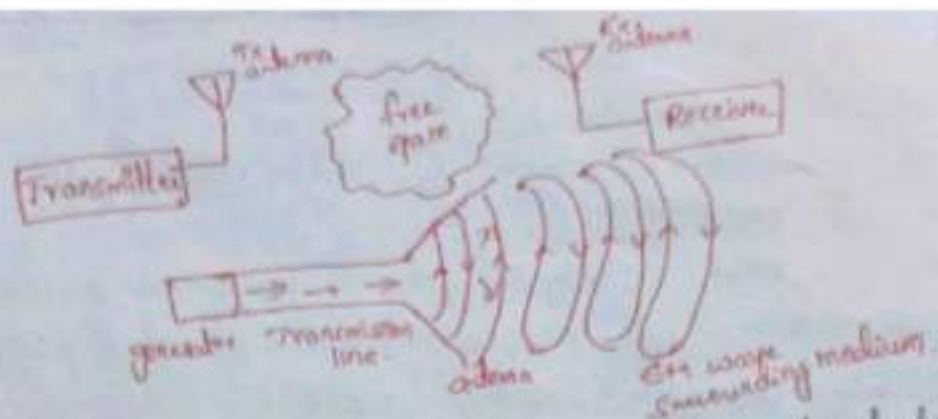
Microstrip



Rectangular Patch



Circular patch



- An antenna is an electrical device which converts electrical power into radio waves and vice-versa.
- It is usually used with radio transmitter or radio receiver.
- In transmission the transmitter supplies the high frequency alternating current to the antenna terminals and the antenna radiates the energy from the current to EM waves (radio waves)
- In reception an antenna intercepts some of the power of an EM wave in order to produce a tiny voltage at its terminals and then applied to a receiver and after that the signal will be amplified.
- typically an antenna consist of an arrangement of a metallic elements (conductors) connected to transmitter or receiver.
- Transducer It converts radio waves into electrical signals & vice-versa is a form of transducer.
- According to the applications & technology available the antennas classified into two categories.
 - 1) omni directional
 - 2) Directional.

Omni directional These are the antennas which receive or radiate in all the directions.

→ These are required when the relative position of the other station is unknown.

→ Omni directional usually refers to all horizontal directions typically with reduced performance in the direction of sky or ground.

Ex. monopole (vertical antenna) or dipole

Directional Beam antennas which are intended to preferentially radiate (or) receive in a particular direction.

→ These are more complex than the monopole (or) dipole.

→ These are intended to directivity and consequently the gain of the antenna.

Ex. parabolic reflector, horn antenna, reflector antenna.

Single wire

Radiation Mechanism The radiation from the antenna takes place when the electromagnetic field generated by the source is transmitted to the antenna system through the transmission line and separated from the antenna into free space.



Radiation from single wire

Conducting wires are characterized by the motion of electric charges and the creation of current flow. Assume that an electric volume charge density ρ_v (coulombs/m³) is distributed uniformly in a circular wire of cross-sectional area A & volume v .

→ Current density in a volume with charge density ρ_v (C/m³)

$$J_z = \rho_v v_z \text{ (A/m}^2\text{)}$$

→ Surface current density in a section with a surface charge density ρ_s (C/m²)

$$J_s = \rho_s v_z \text{ (A/m)}$$

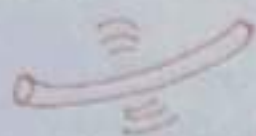
→ Current in a thin wire with a linear charge density ρ_l (C/m)

$$I_z = \rho_l v_z \text{ (A)}$$

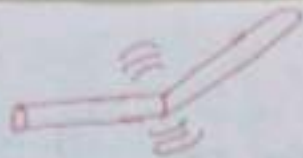
→ To Accelerate/decelerate charges, one needs source of electromotive force and/or discontinuities of the medium in which the charge move.

→ Fundamental principle of radiation There must be some time varying current.

- 1) If charge is not moving current is not created then is no radiation.
- 2) If charge is moving with a uniform velocity
- 3) There is no radiation if there is wire is straight, as
- 4) There is radiation if the wire is curved, discontinuous, terminated or truncated
- 5) If charge is oscillating in a time-motion it radiates. Even if the wire is straight.



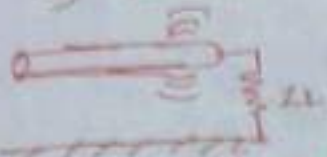
a) Curved



b) Bent



(c) Discontinuous

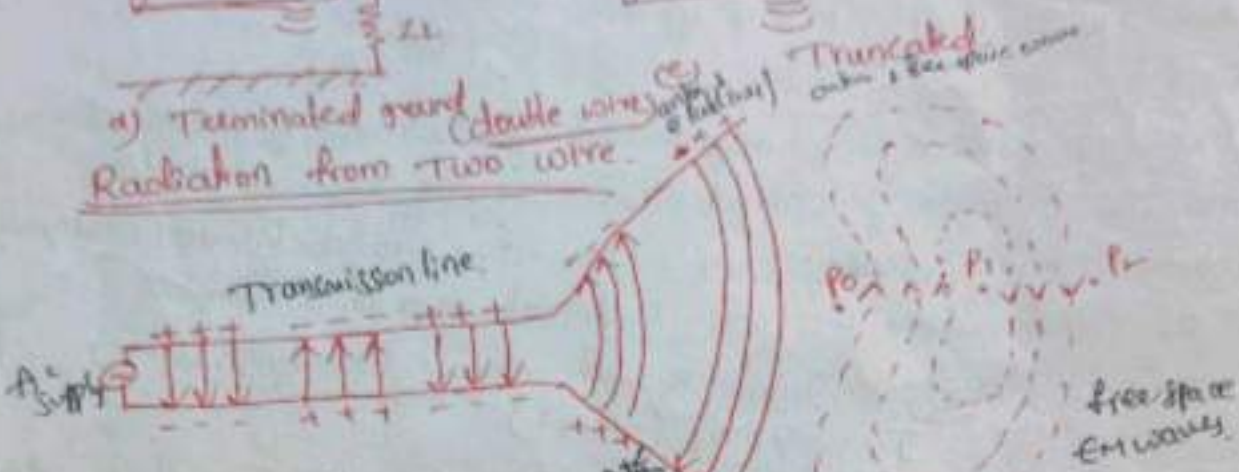


d) Terminated ground



(e) Truncated

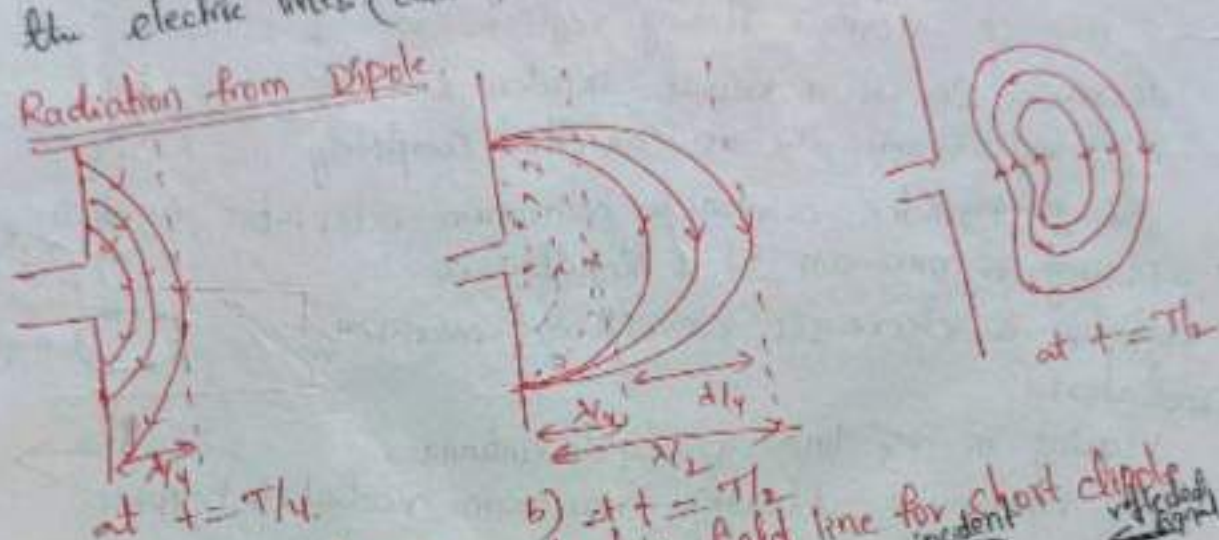
Radiation from Two wire



- Apply the voltage source across the two conductors. This creates the E field b/w the conductors.
- The E field has associated with it electric line of force which are tangent to the electric field at each point and their strength is proportional to the E field intensity.
- electric lines of force have a tendency to act on the free electrons (easily detachable from atoms)
- The movement of the charges creates a current that in turn creates magnetic field intensity.
- Associated magnetic field intensity are magnetic lines of force which are tangent to the magnetic field.
- electric field lines start on +ve charges to -ve charges. it forms a start +ve charge end at ∞ , start at ∞ end on -ve charges. it forms closed loop encircling current-carrying conductors because Physically there are no magnetic charges.

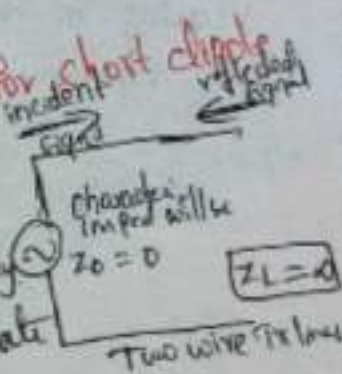
- Volts source is sinusoidal, we expect electric field b/w the conductors to also be sinusoidal with a period equal to that of applied source.
- the relative magnitude of the electric field intensity is indicated by the density (bundling) of the lines of force with arrows showing direction.
- creation of time varying electric & magnetic fields b/w conductors forms electromagnetic waves which travel along the transmission line.
- EM waves enter the antenna and have associated with them electric charges & corresponding currents. If we remove part of antenna structure.
- free space wave can be formed by connecting the open ends of the electric lines (dashed).

Radiation from Dipole

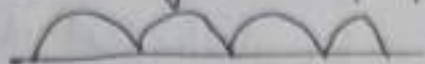


Formation of electric field line for short dipole

- How dipole an
- Two wire transmission lines. Consider AC
- we give AC supply signal to this circuit supply.
- (→) and signal character impedance Z_0 . wave propagate in this direction (→) at the end. the impedance -
- $Z_L = \infty$ Because of impedance. signal will reflected (←) direction because of open loop. (no connection)



→ Because of incident & Reflected signal there will be generation of standing wave propagation pattern on Transmission line.

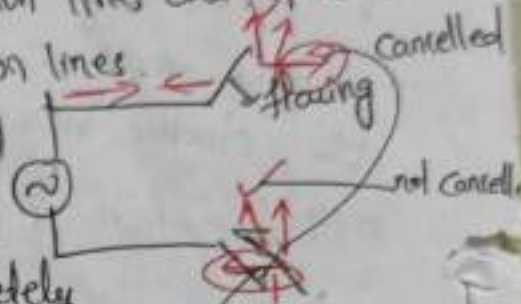


→ If don't connect anything impedance at load $Z_L = \infty$ very high
 reflection coefficient $= \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{1 - Z_0/Z_L}{1 + Z_0/Z_L} \left[\begin{matrix} Z_0 \neq 0, \text{ if } \infty \\ Z_L = \infty \end{matrix} \right]$
 $= \frac{1 - 0}{1 + 0} = 1$

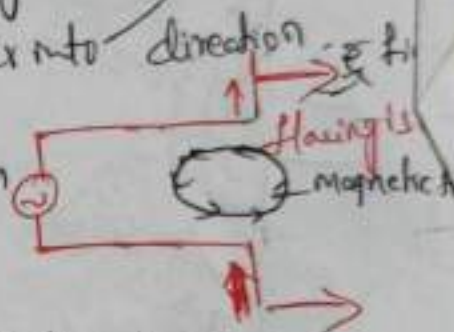
→ If reflection coefft is ~~max~~ ^{maximum} ± 1 we will get minimum radiation. This is happen at Transmission lines are opened.

→ Now, Flaring is added to Transmission lines.

→ As we increase flaring reflection will decrease. So as a Result incident & Reflected signals are not cancelled completely.



→ If I consider current is maximum enter into direction of flaring is maximum It is Resulting is Reflection is decreases. we will get maximum radiation.



→ Flaring in Tx line is dipole antenna.
 → At maximum flaring, maximum radiation happens.

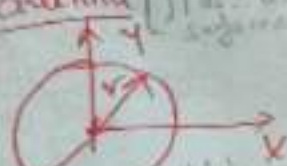
Basic Antenna parameters

- Communication systems are mainly dependent on the characteristics of the antennas used in the system.
- Antennas used in different systems are of diff types.
- Applications of antennas are diff in diff systems.
- All the antennas possess certain fundamental properties

- | | |
|-----------------------------------|--------------------------|
| 1) Radiation pattern | 7) Antenna s/p impedance |
| 2) Radiation intensity | 8) Effective length |
| 3) Directive gain and directivity | 9) Effective aperture |
| 4) power gain | 10) Antenna temperature |
| 5) Antenna beamwidth | 11) Antenna polarization |
| 6) Antenna bandwidth | |

Radiation pattern

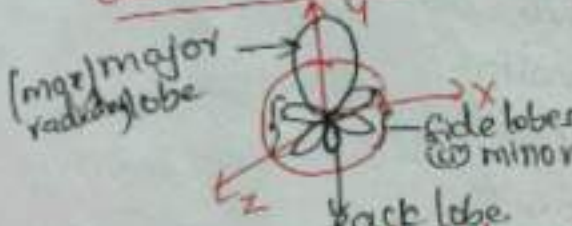
Isotropic antenna



" Isotropic antenna radiates equally in all the direction.

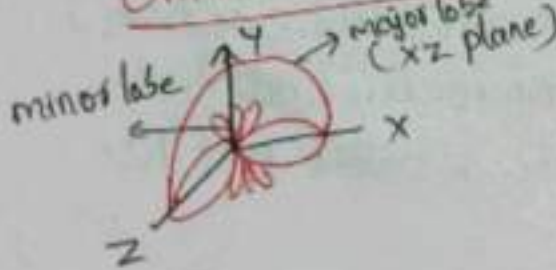
→ It's radiation pattern will be sphere.

Unidirectional Antenna



" Directional antenna radiates in particular direction.

Omnidirectional antenna



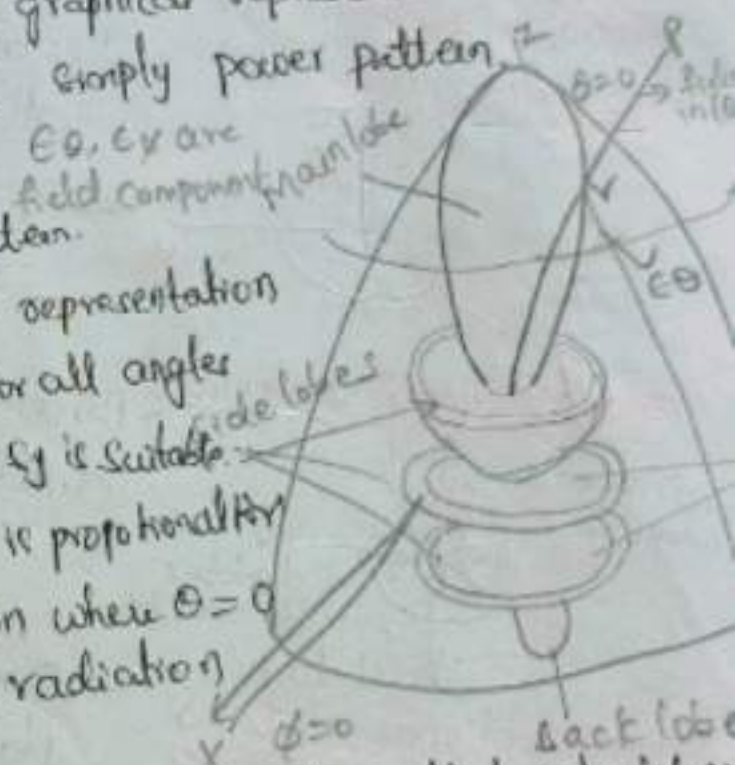
- 1) Omnidirectional antenna radiates in plane
- 2) major lobe of omnidirectional antenna is in plane.
- 3) In omnidirectional antenna we don't have back lobe.

Radiation pattern

- In general the radiation pattern is nothing but a graph which shows the variation of actual field strength of electromagnetic field at all the points equidistant from the antenna.
- Hence it is a three dimensional graph.
- There are two basic radiation patterns.
- If the radiation of the antenna is represented graphically as a function of direction it is called radiation pattern.
- But if the radiation of the antenna is expressed in terms of the field strength E (in volt/metre) then the graphical representation is called Field strength pattern (or) Field Radiation pattern.
- If the radiation of the antenna is expressed in terms of the power P (in watt) per unit solid angle, then the graphical representation is called Power Radiation pattern (or) Power pattern.

Field Radiation pattern

- This is a 3-dimensional pattern.
- It requires 2 dimensional representation to represent the radiation for all angles θ & ϕ . spherical co-ordinate system is suitable.
- P field intensity point 'p' is proportional to E^2 .
- Main lobe in z -direction where $\theta = 0$ (which represents maximum radiation in that direction).
- minor lobes on the sides (which are also called side lobes).
- nulls b/w lobes indicating minimum or zero radiation.
- Exactly opposite to the main lobe is called back lobe.



1) Field radiation pattern can be described completely w.r.t θ + the field intensity and polarization using 2 imp factors

→ 1) $E_\theta(\theta, \phi)$ → The θ -component of the $\vec{E}$ -field as function of angle θ & ϕ (Expressed in V/m)

② $E_\phi(\theta, \phi)$ → The ϕ -component of the $\vec{E}$ -field as function of angles θ & ϕ (Expressed in V/m)

③ $\delta_\theta(\theta, \phi)$ → The phase angle of both the field components (Expressed in degree (or) radian)

→ Field pattern is defined as the ratio of the field component to its maximum value. This is dimensionless quantity with maximum value equal to (unity). given as

$$E_{\theta n}(\theta, \phi) = \frac{E_\theta(\theta, \phi)}{E_\theta(\theta, \phi)_{\max}} \quad \text{by} \quad E_{\phi n}(\theta, \phi) = \frac{E_\phi(\theta, \phi)}{E_\phi(\theta, \phi)_{\max}}$$

→ 3-dimensional pattern can't be plotted in a plane. It is avoided. Instead of this polar plots of the relative magnitude of the field in any cleared plane are sketched.

→ polar plots are plotted in two planes namely one containing the antenna & other normal to it. These planes are called principle planes and two plots (or) patterns are called principle plane patterns.

→ These patterns are obtained by plotting the magnitude of the normalized field strengths.

→ when magnitude the normalized field strength is plotted versus θ with const ϕ , the pattern is called E -plane (or) vertical pattern.

→

1) Field radiation pattern can be described completely w.r.t ② + the field intensity and polarization using 3 imp factors.

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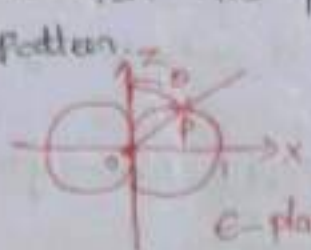
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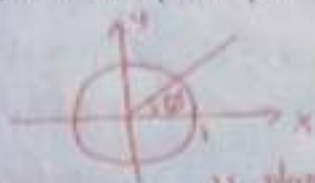
→ when magnitude the normalized field strength is plotted versus θ with const ϕ , the pattern is called θ -plane or vertical pattern.

→

→ When the normalized field strength is plotted versus θ for $\theta = \pi/2$. The pattern is called H-plane pattern (or Horizontal pattern).



E-plane pattern



H-plane pattern

(Note: along the line in which antenna is cut)

(a) E-plane pattern $\perp$ H-plane pattern for the horizontal dipole.

→ Radiation pattern for a one wavelength long vertical dipole



(b) Field radiation pattern for vertical dipoles of half & one wavelength.



(c) $1\frac{1}{2}$ wavelength

Two wavelength

3 Power Radiation pattern

Radiation in a given direction is expressed in terms of power per unit solid angle, the pattern is called Power radiation pattern.

→ Power density $P_d(\theta, \phi)$ is defined as power flow per unit area and is a function of the direction (θ, ϕ) . The power density can be expressed in terms of the magnitude of $\vec{E}$ field intensity as,

$$P_d$$

$$P_d(\theta, \phi) = \frac{1}{2} \frac{|E(\theta, \phi)|^2}{\eta_0} = \frac{1}{2} \frac{|E(\theta, \phi)|^2}{120\pi} \text{ watt/m}^2 \quad (2)$$

where $|E(\theta, \phi)| = \sqrt{E_\theta^2(\theta, \phi) + E_\phi^2(\theta, \phi)}$

η_0 = intrinsic impedance of free space = $120\pi \Omega$

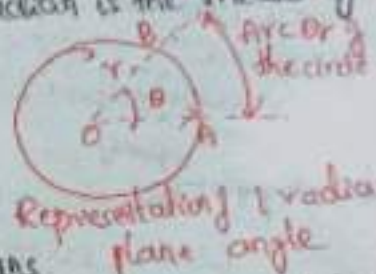
$$P_{dn}(\theta, \phi) = \frac{P_d(\theta, \phi)}{P_d(\theta, \phi)_{\max}} = \frac{|E(\theta, \phi)|^2}{|E(\theta, \phi)_{\max}|^2} = f_n^2(\theta, \phi)$$

$$P_d(\theta, \phi) = I^2(\theta, \phi) = \sin^2 \theta$$

Radian, steradian and beam solid angle (sr)

→ Basic diff b/w radian & steradian is that the radian is the measure of a plane angle, while the steradian is the measure of a solid angle.

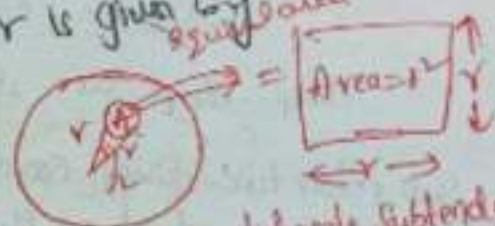
→ Total circumference C of a circle with radius r is given by $C = 2\pi r$



over a complete circle there are 2π radians

→ By measuring a solid angle steradian defined as a solid angle with vertex at the centre of the sphere with radius r that is subtended by a spherical surface area equal to that of a square with each side equal to r . The angle in steradian is expr in sr

→ Area of complete sphere with radius r is given by $A = 4\pi r^2$



→ Over a closed sphere with radius r , solid angle subtended by it is 4π steradian

1 steradian = $1 sr = \frac{\text{Solid angle of sphere}}{4\pi}$

Representation of 1 steradian solid angle

Light from a torch in form of solid angle
Vegetable cutting falling water

$1 \text{ sr} = 1 \text{ rad}^2 = \left(\frac{180}{\pi}\right)^2 (\text{deg})^2 = 3282.8 \text{ square degree}$
 (Solid angle)
 The infinitesimal area ds on the surface of a sphere with radius r is given by $ds = r^2 \sin\theta d\theta d\phi \text{ m}^2$

Hence the element of solid angle $d\Omega$ of a sphere is given by

$$d\Omega = \frac{ds}{r^2} = \sin\theta d\theta d\phi \text{ steradian}$$

Beam solid angle (or Beam Area) Ω_A

→ This is expressed in steradian. It is defined as the Integral of normalized power pattern over a sphere. It is

denoted by Ω_A

$$\Omega_A = \int_0^{2\pi} \int_0^\pi P_n(\theta, \phi) \sin\theta d\theta d\phi$$

$\left(\frac{ds}{r^2} = \frac{\text{area of the sphere}}{r^2} \right)$
 $= \frac{4\pi r^2}{r^2} = 4\pi$

solid angle of sphere $d\Omega = \sin\theta d\theta d\phi$

$$\text{beam area } \Omega_A = \int_0^{2\pi} \int_0^\pi P_n(\theta, \phi) d\Omega \text{ steradian}$$

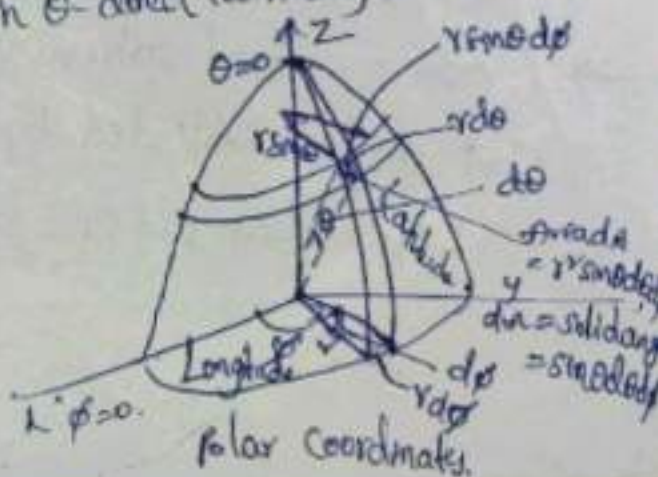
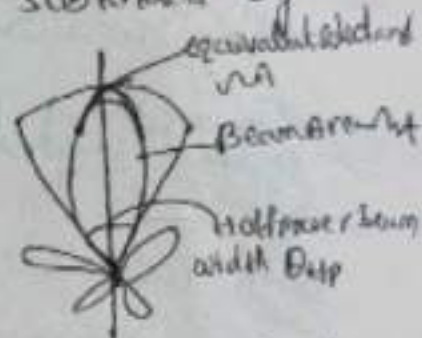
→ Ω_A is described in terms of the angles subtended by half power points of the main lobe.

Ω_A can be written as.

$$\Omega_A = \theta_{HP} \phi_{HP} \text{ steradian}$$

θ_{HP} , ϕ_{HP} are half power beamwidths

In polar two-dim coordinates an incremental area A on the surface of sphere is the product of the length $r d\theta$ in θ -direction (latitude) & $r \sin\theta d\phi$ in the ϕ -direction (longitude)



Radiation Intensity $U(\theta, \phi)$

→ The radiation intensity is defined as power per unit solid angle. It is denoted by U . It doesn't depend on the distance from the radiator (or antenna). It is expressed in W/sr (watts/steradian).

→ Radiation intensity of an antenna is defined as

$$U(\theta, \phi) = r^2 P_d(\theta, \phi)$$

→ Total power radiated can be expressed in terms of the radiation intensity

$$P_{\text{rad}} = \oint P_d(\theta, \phi) ds = \oint P_d(\theta, \phi) [r^2 \sin\theta d\theta d\phi]$$
$$= \int_0^{2\pi} \int_0^{\pi} [P_d(\theta, \phi) r^2] \sin\theta d\theta d\phi$$

Let $du = \sin\theta d\theta d\phi$ be the differential solid angle in steradian

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) du$$

→ Average value of the radiation intensity is $U_{\text{avg}} = \frac{P_{\text{rad}}}{4\pi}$

→ Using radiation intensity $U(\theta, \phi)$ we can also calculate normalized power pattern as the ratio of radiation intensity $U(\theta, \phi)$ to its maximum value $U(\theta, \phi)_{\text{max}}$.

$$P_{\text{dn}}(\theta, \phi) = \frac{U(\theta, \phi)}{U(\theta, \phi)_{\text{max}}}$$

Directive Gain $G_D(\theta, \phi)$ = Directivity D

If the antenna were isotropic i.e. if it were to radiate uniformly in all directions, then the power density at all the points on the surface of a sphere will be same. The average power can be expressed in terms of the radiated power as

$$P_{avg} = \frac{P_{rad}}{4\pi r^2} \text{ w/m}^2$$

The directive gain is defined as the ratio of the power density $P_d(\theta, \phi)$ to the average power radiated. For isotropic antenna the value of the directive gain is unity.

$$G_D(\theta, \phi) = \frac{P_d(\theta, \phi)}{P_{avg}} = \frac{P_d(\theta, \phi)}{\frac{P_{rad}}{4\pi r^2}}$$

$$G_D(\theta, \phi) = \frac{P_d(\theta, \phi) r^2}{\left(\frac{P_{rad}}{4\pi}\right)}$$

The numerator in the above ratio is radiation intensity. denominator is average value of the radiation intensity.

directive gain can be written as $G_D(\theta, \phi) = \frac{U(\theta, \phi)}{U_{avg}} = \frac{4\pi U(\theta, \phi)}{P_{rad}}$

→ Directive gain can be defined as a measure of the concentration of the radiated power in a particular direction (θ, ϕ)

→ The ratio of the maximum power density to the average power radiated is called maximum directive gain (or directivity) of the antenna. It is denoted by $G_{Dmax}(\infty) D$.

$$D = G_{Dmax} = \frac{P_{dmax}}{\frac{P_{rad}}{4\pi r^2}} \quad \text{--- (1)}$$

$$D = G_{Dmax} = \frac{U_{max}}{U_{avg}} = \frac{4\pi U_{max}}{P_{rad}}$$

→ Directivity of an antenna is dimensionless quantity. The directivity can also be expressed in terms of the E field intensity as

$$D = G_{Dmax} = \frac{4\pi |E_{max}|^2}{\int_0^{2\pi} \int_0^\pi |E(\theta, \phi)|^2 \sin\theta d\theta d\phi}$$

We know that average power density P_{avg} can be given as

$$P_{avg} = \frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi P(\theta, \phi) \sin\theta d\theta d\phi$$

We know that $du = \sin\theta d\theta d\phi$

$$P_{avg} = \frac{1}{4\pi} \iint_{4\pi} P(\theta, \phi) du \text{ watt/sr} \quad \text{--- (2)}$$

Subst eq (2) in (1)

$$D = \frac{P_{dmax}}{P_{avg}} = \frac{P(\theta, \phi)_{max}}{\frac{1}{4\pi} \iint_{4\pi} P(\theta, \phi) du}$$

$$D = \frac{1}{\frac{1}{4\pi} \iint_{4\pi} \left[\frac{P(\theta, \phi)}{P(\theta, \phi)_{max}} \right] du}$$

But $\frac{P(\theta, \phi)}{P(\theta, \phi)_{max}} = P_n(\theta, \phi) = \text{normalized power pattern}$

$$D = \frac{4\pi}{\iint_{4\pi} P_n(\theta, \phi) du} = \frac{4\pi}{\Omega_A} \quad \text{--- (3)}$$

(3) represents the directivity intensity beam area (or beam solid angle)

Ω_A → If beam area of an antenna is smaller its directivity is greater.
An isotropic radiator has lowest possible directivity of value $D=1$

All practical antennas have directivity value $D > 1$

→ If the half power beamwidths of an antenna are known

then we can express $D = \frac{41253}{\theta_{HP}^2 \phi_{HP}^2}$

where 41253 = no of ~~sphere~~ ^{square} degrees in sphere = $4\pi \left(\frac{180}{\pi}\right)^2$

θ_{HP} = HPBW in one principle plane

ϕ_{HP} = HPBW in other "

→ Above eqn is obtained by neglecting minor lobes. If we consider minor lobes too, $D = \frac{4\pi \cos^2 \theta}{\sin^2 \theta}$

Directivity & Resolution

Resolution of an antenna is defined as half of the beamwidth b/w first nulls. $\text{Resolution} = \frac{\text{FNBW}}{2}$

But half the beamwidth b/w first nulls is approximately equal to the half power beamwidth (HPBW) of an antenna.

$$\text{HPBW} \approx \frac{\text{FNBW}}{2}$$

→ In practice for an antenna HPBW is slightly less than $\frac{\text{FNBW}}{2}$

→ Antenna beam area is given by the product of two half power beamwidths in two principle planes. we can write

$$\Omega_A = \Omega_{HP} \approx \left(\frac{\text{FNBW}}{2}\right) \theta \left(\frac{\text{FNBW}}{2}\right) \theta$$

→ If there are N no of point sources of radiations distributed uniformly, then antenna resolves those and its expression is

$$N = \frac{4\pi}{\Omega_A}, \text{ where } \Omega_A = \text{Beam area Express in sr.}$$

$$D = \frac{4\pi}{\Omega_A} \Rightarrow \boxed{D = N}$$

→ Ideally no of point sources resolved by an antenna is equal to directivity of an antenna.

Power Gain $[G_p(\theta, \phi)]$ & Radiation Efficiency or Antenna Efficiency

The practical antenna is made up of a conductor having finite conductivity. Hence we must consider the ohmic power loss of the antenna. If the practical antenna has ohmic losses ($I^2 R$) represented by P_{loss} . Then the power radiated P_{rad} is less than the P/P power P_{in} . Then we can express the P_{rad} in terms of P_{in} as

$$P_{rad} = \eta_r P_{in}$$

where η_r is called radiation efficiency of an antenna.
Thus radiation efficiency of an antenna can be written as

$$\eta_r = \frac{P_{rad}}{P_{in}}$$

→ Total i/p power to the antenna can be written as

$$P_{in} = P_{rad} + P_{loss}$$

Hence the radiation efficiency can be written as

$$\eta_r = \frac{P_{rad}}{P_{rad} + P_{loss}}$$

Power radiated and ohmic power loss can be expressed in terms of r.m.s. current as:

$$P_{rad} = I_{rms}^2 R_{rad}$$

$$P_{loss} = I_{rms}^2 R_{loss}$$

Radiation efficiency is given by

$$\eta_r = \frac{R_{rad}}{R_{rad} + R_{loss}}$$

→ Ratio of the power radiated in a particular direction (θ, ϕ) to the actual power i/p to the antenna is called power gain of antenna. The power gain of the antenna is denoted by $G_p(\theta, \phi)$ and it is given by

$$G_p(\theta, \phi) = \frac{P_d(\theta, \phi)}{P_{in}}$$

→ Maximum power gain can be defined as the ratio of the maximum radiation intensity to the radiation intensity due to isotropic lossless antenna.

$$G_{pmax} = \frac{\text{Max radiation intensity}}{\text{Radiation intensity due to isotropic lossless antenna}}$$

$$G_{pmax} = \frac{U_{max}}{\left(\frac{P_{in}}{4\pi}\right)}$$

→ Above eqn is obtained by neglecting minor lobes. If we consider maximum radiation intensity is given by

$$U_{\max} = \frac{P_{\text{rad}}}{4\pi} G_{\max} = \left(\frac{\eta_r P_{\text{in}}}{4\pi} \right) G_{\max} = \left(\frac{\eta_r P_{\text{in}}}{4\pi} \right) D$$

Substituting value $U_{\max}$ in the expression for max power gain

$$G_{\max} = \frac{\eta_r \left(\frac{P_{\text{in}}}{4\pi} \right) G_{\max}}{\left(\frac{P_{\text{in}}}{4\pi} \right)} = \frac{\eta_r \left(\frac{P_{\text{in}}}{4\pi} \right) D}{\left(\frac{P_{\text{in}}}{4\pi} \right)}$$

$$G_{\max} = \eta_r G_{\max} = \eta_r D$$

- ⇒ For many practical antennas, Radiation efficiency η_r is 100%.
- ⇒ maximum power gain is approximately same as the directivity (or) maximum directional gain of the antenna.
- ⇒ power gain & directional gain are expressed in (dB).

Front to Back Ratio (FBR)

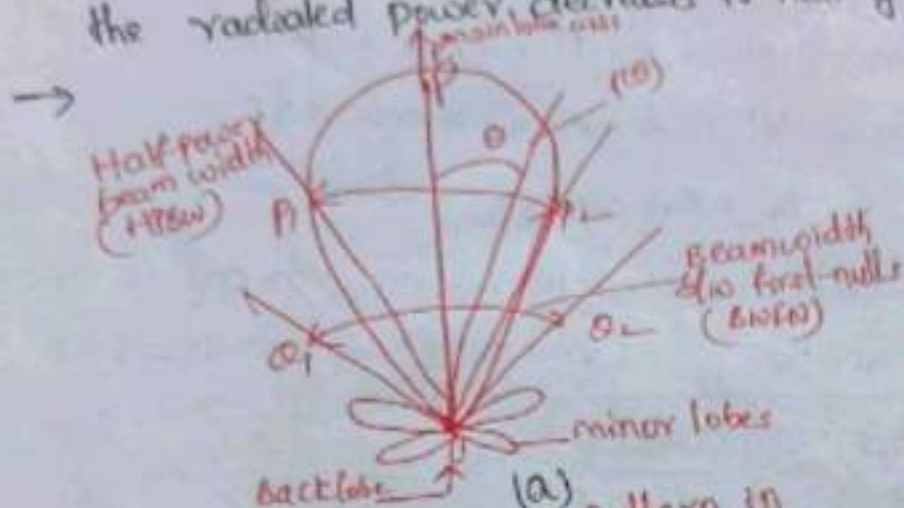
- It is the ratio of the power radiated in the desired direction to the power radiated in the opposite direction.

$$FBR = \frac{\text{power radiated in desired direction}}{\text{power radiated in opposite "}}$$

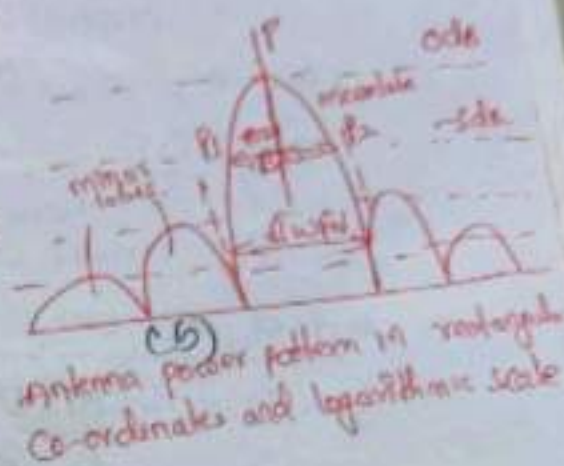
- FBR ratio value desired is very high as it is expected to have large radiation in the front (or) desired direction rather than that in the back (or) opposite direction.
- It depends on freq of operation. Freq of antenna changes FBR also changes.
- FBR depends on spacing b/w the antenna elements.
- If the spacing b/w antenna elements increases the FBR decreases.
- It depends on electrical length of the parasitic elements.
- practically the FBR is important in case of the receiving antenna rather than transmitting antennas.

Antenna Beamwidth

- Antenna Beamwidth is the measure of the directivity of the antenna. The antenna beamwidth is an angular width in degrees.
- It is measured on a radiation pattern on major lobe.
- Antenna beamwidth is defined as the angular width in degrees b/w the two points on a major lobe of a radiation pattern where the radiated power decreases to half of its maximum value.



(a) Antenna power pattern in Polar Co-ordinates



(b) Antenna power pattern in rectangular Co-ordinates and logarithmic scale

- Beamwidth is also called Half power beamwidth (HPBW) because it is measured b/w two points on the major lobe where the power is half of its maximum power from fig(a). Power is maximum at point P , while it is half at pts P_1 & P_2 both. Hence the angular width b/w pts P_1 & P_2 is nothing but antenna beamwidth or (HPBW). Beamwidth is also called 3-dB beamwidth as reduction of power to half of its maximum value corresponds to the reduction of power (Exp in dB) by 3 dB. From fig(b)

→ Radiation pattern is described in terms of the angular width b/w first nulls (or first side lobes). Then such an angular beamwidth is called beamwidth b/w first nulls (BWFN).

→ Directivity (D) of the antenna is related with beam solid angle Ω_A (or) beam area B , $D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{B}$.

B = Beam Area

$\approx$ HPBW in horizontal plane $\times$ (HPBW) in vertical plane

$\approx$ (HPBW) in E-plane $\times$ (HPBW) in H-plane.

(or) $B \approx \theta_E \times \theta_H$ when θ_E & θ_H in radians.

$$D = \frac{4\pi}{\theta_E \theta_H} \quad \text{if } \theta_E \text{ \& } \theta_H \text{ are in radians.}$$

we can convert angles expressed in radians into angles in degrees by using relation, $1 \text{ rad} = \frac{180^\circ}{\pi} = 57.295^\circ = 57.3^\circ$

$$D = \frac{4\pi (57.3)^2}{\theta_E^\circ \theta_H^\circ} = \frac{41257}{\theta_E^\circ \theta_H^\circ}$$

→ This formula is approximate formula and it is applicable only to the antennas with narrow beamwidth (about 20°) with no minor lobes in the radiation pattern.

→ Beamwidth of the antenna is affected by the shape of the radiation pattern, wavelength & dimensions.

Antenna beam efficiency (ϵ_b)

To examine the quality of the transmitting & receiving antennas, the antenna beam efficiency parameter is important. For antenna with major lobe coincident with z-axis, the beam efficiency is defined as

$$BE = \frac{\text{power transmitted (or received) within the cone angle } \theta_1}{\text{power transmitted (or received) by antenna}}$$

where θ_1 is the half angle of the cone within which the percentage of total power is found.

$$BE = \frac{\int_0^{2\pi} \int_0^{\theta_1} U(\theta, \phi) \sin\theta d\theta d\phi}{\int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin\theta d\theta d\phi}$$

→ Beam efficiency can be expressed in terms of the main beam area (Ω_M) and total beam area (Ω_A).

→ Beam efficiency can also be defined as the ratio of the main beam area to the total beam area.

$$BE = \epsilon_M = \frac{\Omega_M}{\Omega_A}$$

→ total beam area (Ω_A) is the combination of the main beam area (Ω_M) & minor lobe area (Ω_m)

$$\Omega_A = \Omega_M + \Omega_m \quad \text{--- (1)}$$

dividing eq (1) by Ω_A

$$1 = \frac{\Omega_M}{\Omega_A} + \frac{\Omega_m}{\Omega_A}$$

$$\frac{\Omega_M}{\Omega_A} = \epsilon_M = \text{beam efficiency}$$

$$\frac{\Omega_m}{\Omega_A} = \epsilon_m = \text{stray factor}$$

$$\boxed{\epsilon_M + \epsilon_m = 1}$$

Antenna Bandwidth ($\Delta\omega$)

$$\text{Bandwidth} = (BW) = \Delta\omega = \omega_2 - \omega_1 = \frac{\omega_0}{Q}$$

$$\Delta f = f_2 - f_1 = \frac{f_0}{Q} \text{ Hz}$$

where f_0 is the centre freq (or design freq or resonant freq), while Q factor of antenna is given by $Q = 2\pi \frac{\text{Total energy stored by antenna}}{\text{Energy radiated per cycle}}$.
For lower Q antennas, the antenna bandwidth is very high average.

Radiation Resistance

→ In general an antenna radiates power into free space in the form of electromagnetic waves. So the power dissipated is given by

$$P_{\text{rad}} = I^2 R$$

→ Assuming all the power dissipated in the form of electromagnetic waves then write $R = \frac{P_{\text{rad}}}{I^2}$

→ Resistance which relates power radiated by radiating antenna and the current flowing through the antenna is a fictitious resistance. Such resistance is called radiation resistance of antenna and it is denoted by

$$R_{\text{rad}}, \text{ or } R_a \text{ or } R_0$$

→ But practically antenna is not completely radiated in the form of electromagnetic waves, but there are certain radiation losses due to the loss resistance denoted by R_{loss} . Total power is given by

$$P = P_{\text{rad}} + P_{\text{loss}} = \text{radiation loss} + \text{ohmic loss}$$

$$P = I^2 R_{\text{rad}} + I^2 R_{\text{loss}}$$

→ Radiation Resistance depends on Antenna Configuration, ratio of length, diameter, conductor used, location of antenna w.r.t ground.

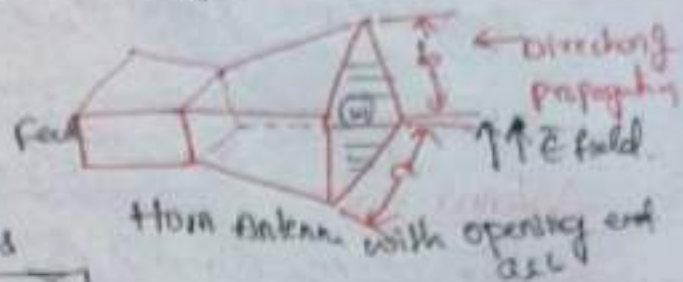
Antenna Apertures

→ with each antenna we can associate a number of a equivalent areas. These are used to describe the power capturing character of antenna. There are various Apertures (ie areas)

→ Different apertures are
1) physical ap 2) effective ap 3) scattering ap 4) loss ap 5) collecting

Physical Aperture

→ Rectangular Horn antenna with dimensions 'a' & 'b' is given
→ The Area of opening called as Physical Aperture. ie $A_p = a \times b$



→ This value diff for diff Antennas.
→ When incident wave has power density w then Received power that has to be $P = w A_p$ (watts)
→ AP based on dimension & depends on power w density received
→ practically it is not happen. b/c that we need to calculate effective aperture

Effective Aperture

→ when we (calculate) receive power, it is less than we calculate. [Actual power is less than we get & it occurred due to various reasons]
due to following reasons:
a) Horn Antenna is not having uniform over opening.
b) E field at wall must be zero but practically not.
c) due to tapering loss.
d) Because of conduction loss that happens.
→ so to understand effective aperture we need to calculate aperture efficiency.

$$\epsilon_{ap} = \frac{A_e}{A_p} \text{ th. ap}$$

$$A_{em} = (G_{max}) \frac{\lambda^2}{4\pi}$$

Antenna Apertures

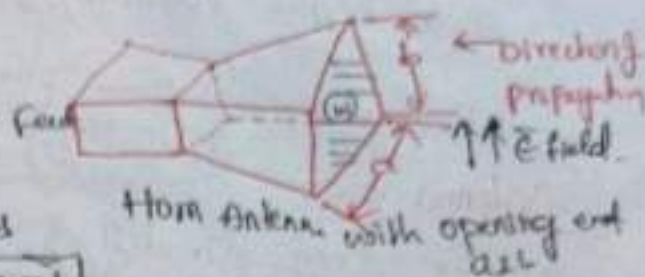
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b) E field at wall must be zero but practically not.

c) due to tapering loss.

d) Because of conduction loss that happens.

→ So to understand effective aperture we need to calculate aperture efficiency.

$$\epsilon_{ap} = \frac{A_e}{A_p} \text{ (s. ap phys. ap)}$$

$$f_{em} = (G_{max}) \frac{\lambda^2}{4\pi}$$

Scattering

That happens at border of antenna. E.g. These are edges that having what happens is scattering on this border. Electromagnetic waves get scattered. Edges are not proper, from border we get scattering.



→ loss aperture may be happening with conduction, (turning) could happen conducting, dielectric material & even the wall.

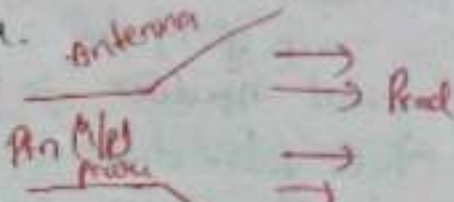
→ collecting opening. This opening receiving EM waves, which is aperture. These from antenna so this aperture is related to this antenna. That is collecting aperture.

Antenna gain

We have seen directivity of antenna.

$$D = \frac{U_{\text{max}}}{P_{\text{rad}}} \quad \text{--- (1)}$$

→ calculation of gain of antenna we need to use P_{in} instead of P_{rad} in directivity.



$$G = \frac{U_{\text{max}}}{P_{\text{in}}} \quad \text{--- (2)}$$

→ efficiency of an antenna i.e. $k = \frac{P_{\text{rad}}}{P_{\text{in}}}$

$$\frac{(2)/(1)}{D} \Rightarrow \frac{G}{D} = \frac{U_{\text{max}}/P_{\text{in}}}{U_{\text{max}}/P_{\text{rad}}} \Rightarrow \frac{P_{\text{rad}}}{P_{\text{in}}} = k$$

$$\boxed{G = kD}$$

→ Gain of antenna can be measured by Antenna under test (AUT) with respect to reference antenna.

$$\boxed{G = \frac{U_{\text{AUT}}}{U_{\text{Ref}}}} \quad \text{--- practically}$$

→ Radiation intensity of Antenna i.e. $U_{\text{Ref}} = \frac{P_{\text{in}}}{4\pi} \Rightarrow G = \frac{U_{\text{AUT}}}{P_{\text{in}}/4\pi}$

$$G = \frac{U_{\text{AUT}}}{P_{\text{in}}}$$

→

Ex The radiation resistance of an antenna is 72Ω & the loss resistance is 8Ω what is the directivity if antenna power gain is 16

Ans $R_r = 72\Omega$, $R_L = 8\Omega$, $G = 16$

$$k = \frac{R_r}{R_r + R_L} = \frac{72}{72 + 8} = \frac{72}{80} = 0.9$$

$$\text{gain } G = kD \Rightarrow D = \frac{G}{k} = \frac{16}{0.9} = 17.77$$

$$D = 10 \log(17.77) \Rightarrow \boxed{12.498 \text{ dB} = D}$$

Ex An Antenna loss resistance of 10Ω , power gain 20 & directivity 22 calculate the radiation resistance.

Ans $R_L = 10\Omega$, $G = 20$, $D = 22$

$$G = kD \Rightarrow k = G/D = 20/22 = 10/11 = 0.909$$

$$\text{Antenna eff. } k = \frac{R_r}{R_r + R_L} \Rightarrow \frac{10}{11} = \frac{R_r}{R_r + 10} \Rightarrow 10R_r + 100 = 11R_r$$

$$\boxed{R_r = 100\Omega}$$

Radiation Resistance (R_r)

→ The antenna is a radiating device, which radiates EM wave in the space.

→ If we supply I current to antenna then power dissipated by antenna. that will be $\boxed{P = I^2 R}$

→ The energy supplied to antenna is dissipated in two ways.

1) Radiated power P_{rad} 2) due to ohmic loss P_{loss}
 $P_{\text{rad}} = I^2 R_{\text{rad}}$
 $P_{\text{loss}} = I^2 R_{\text{loss}}$

→ So total power $P = P_{\text{rad}} + P_{\text{loss}}$
 $= I^2 R_{\text{rad}} + I^2 R_{\text{loss}}$

$$\boxed{P = I^2 (R_{\text{rad}} + R_{\text{loss}})}$$

→ Radiation efficiency $\epsilon_{rad} = \frac{R_r}{R_r + R_L}$

1) $R_r = 50\Omega$, $R_L = 10\Omega$, $\epsilon_{rad} = \frac{50}{50+10} = \frac{50}{60} = \frac{5}{6}$

2) $R_r = 10\Omega$, $R_L = 10\Omega$, $\epsilon_{rad} = \frac{10}{20} = \frac{1}{2}$

→ R_{rad} is high Radiation effien is high
" " " " " " low

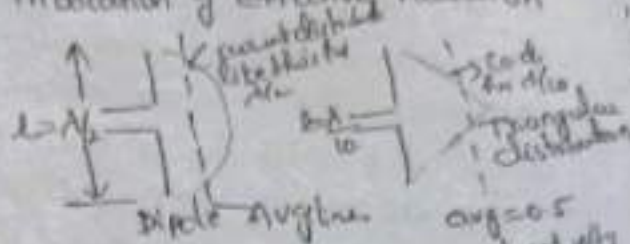
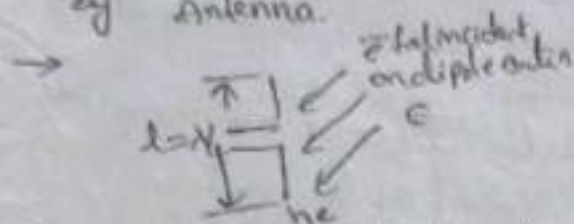
→ Radiation Resistance depends upon

- 1) Configuration antenna
- 2) Ratio of length to the diameter of conductor.
Used in Antenna.
- 3) Depends upon point where radiation resistance is considered
- 4) Location of antenna w.r.t. ground and other objects.

Effective length (or Effective Height)

(12) @

→ Effective height of antenna gives indication of effective radiation by antenna.



→ If E say height h_e

→ Induced voltage $V = h_e \times \text{Incident field } (E)$
 $= h_e E \Rightarrow h_e = \frac{V}{E}$

→ Based on current distribution we can calculate height.
 → we need to calculate avg of current distribution (avg line)
 that avg it will be $\frac{2}{\pi} = 0.64$

→ From that effective length achieve $h_e = 0.69 l$ (Hertz)
 → effective height $h_e = 0.5 l$ (Hertz) $[h_e = \text{avg} \times \text{length}]$

→ Radiated power with radiation resistance $P = \frac{V^2}{4R_r} = \frac{h_e^2 E^2}{4R_r} = \frac{P_{\text{inc}}}{Z_0} A_e$
 $h_e = \sqrt{\frac{4R_r A_e}{Z_0}}$ (m) $Z_0 = 120\pi$
 $A_e = \frac{h_e^2 Z_0}{4R_r}$ (m²) effective aperture.
 → power radiated based on impedance intensity of the effective aperture power is given

→ Effective height and effective aperture are related via radiation resistance and the intrinsic impedance of space.

Antenna Temperature It is a parameter that describes how much noise the antenna produces in a given temperature. This temp is not the physical temp of the antenna.

→ Moreover the antenna doesn't have an intrinsic antenna temp associated with it rather the temp.

→ depends on its gain pattern and the thermal environment that is placed in it. And it's also called as the antenna noise temperature.

→ For an antenna with a radiation pattern given by $R(\theta, \phi)$ the noise temp is given by

$$T_A = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{2\pi} R(\theta, \phi) T(\theta, \phi) \sin\theta d\theta d\phi$$

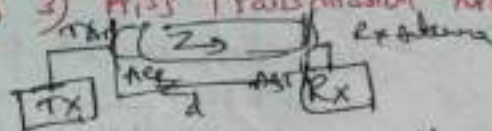
where $R(\theta, \phi)$ is Radiation pattern of antenna.

$T(\theta, \phi)$ is temp distribution.

1) Antenna e/p impedance 2) Bandwidth 3) Fris Transmission Formula

Fris Transmission Formula

$$P_R = P_T (G_T G_R) \left(\frac{\lambda}{4\pi d} \right)^2$$



P_T → Transmitted power in watt

P_R → Receiving "

G_T → Gain of Tx antenna

G_R → gain of Rx antenna

Ex Examples on HPBW & FNBW

① An antenna is having a field pattern given by

$$E(\theta) = \cos \theta, \text{ for } 0 \leq \theta \leq 90^\circ \text{ find Half power beamwidth}$$

Ans Power $\propto E^2 \Rightarrow E \propto \sqrt{P}$

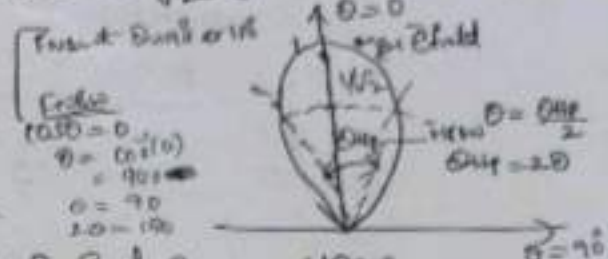
at half power beam width power is half, and if the same time electric field should be $\frac{1}{\sqrt{2}}$ of max. field

$$E\text{-field } E(\theta) = \cos \theta = \frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ$$

$$\rightarrow \text{HPBW } \theta_{\text{HPBW}} = 2\theta = 90^\circ$$

$$\rightarrow \theta_{\text{FNBW}} = 2\theta' = 2 \times 90^\circ = 180^\circ$$

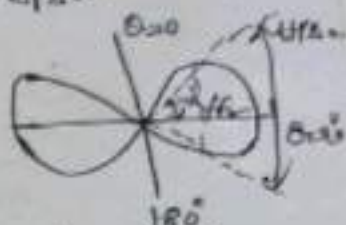


② Ex For an antenna $E(\theta) = \sin \theta$ find HPBW & FNBW

$$E(\theta) = \sin \theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$$

$$\theta_{\text{HPBW}} = 2\theta = 90^\circ$$

$$\text{For FNBW is } \theta_{\text{FNBW}} = 180 - 0 = 180^\circ$$



③ Find the beam area if the antenna has field pattern given by $E(\theta) = \cos^2 \theta$, $0 \leq \theta \leq 90^\circ$ [if solid angle]

$$\Omega = \int_0^{2\pi} \int_0^{\pi/2} P(\theta) \sin \theta d\theta d\phi$$

Normalize E-field $E_n(\theta) = \frac{E(\theta)}{E_{\text{max}}} = \frac{\cos^2 \theta}{1} = \cos^2 \theta$

$$\Omega = \int_0^{2\pi} \int_0^{\pi/2} E_n^2(\theta) \sin \theta d\theta d\phi = \int_0^{2\pi} \int_0^{\pi/2} \cos^4 \theta \sin \theta d\theta d\phi$$

$$= \int_0^{2\pi} d\phi \int_0^{\pi/2} \cos^4 \theta \sin \theta d\theta$$

$$= (2\pi) \int_0^{\pi/2} \cos^4 \theta \sin \theta d\theta$$

$$= 2\pi \int_0^{\pi/2} -x^4 dx \Rightarrow -2\pi \int_1^0 x^4 dx$$

$$= -2\pi \left(\frac{x^5}{5} \right)_1^0 \Rightarrow -2\pi \left(0 - \frac{1}{5} \right) = \frac{2\pi}{5}$$

$$\begin{aligned} \cos \theta &= x, \Rightarrow \\ -\sin \theta d\theta &= dx \\ \theta=0 \Rightarrow x=1 \\ \theta=\pi/2 \Rightarrow x=0 \end{aligned}$$

Q₂ If an antenna has a main lobe with $\theta_{HP} = 2^\circ$, $\theta_{EP} = 2^\circ$, calculate approximate directivity

Ans $D = \frac{41253 (\text{deg})^2}{\theta_{HP} \theta_{EP}} = \frac{41253 (\text{deg})^2}{(2^\circ)(2^\circ)} = 10313.25$

$$D \text{ in dB} = 10 \log_{10} \frac{10313.25}{1} = 40.2561 \text{ dB}$$

Q₃ Calculate FBR of an antenna expressed in dB if the antenna radiates 3 kW in its optimum direction while 300 W in opposite dir.

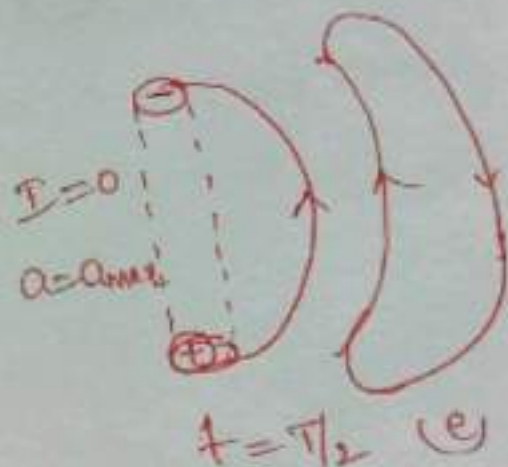
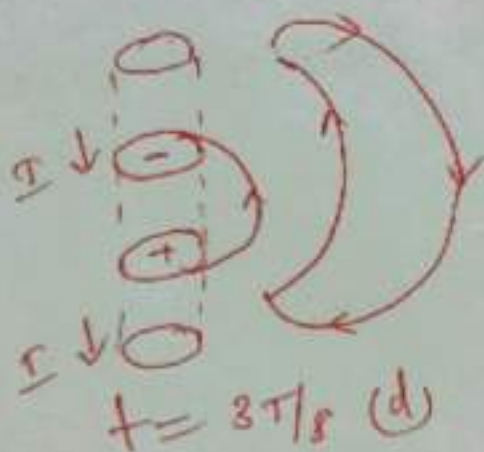
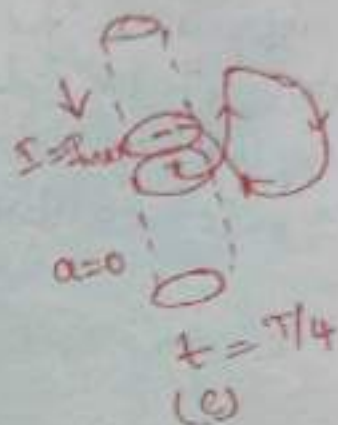
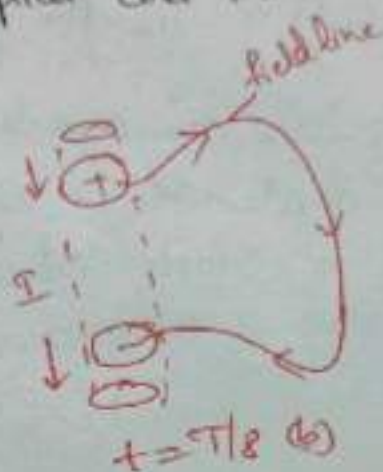
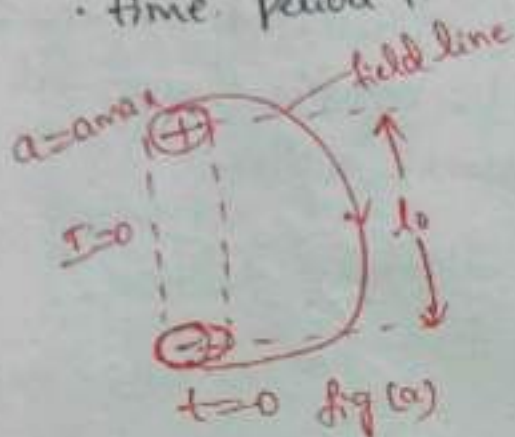
$$\text{FBR} = \frac{\text{Power radiated in desired direction}}{\text{Power radiated in opposite direction}}$$

$$\text{FBR} = \frac{3000}{300} = 10 \Rightarrow 10 \log_{10}(10) = 10 \text{ dB}$$

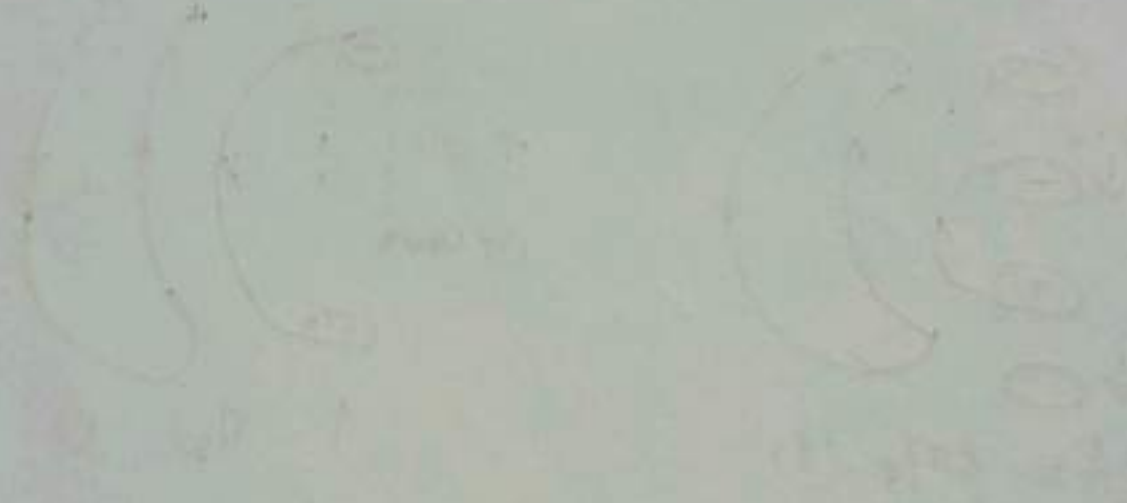
Fields From oscillating Dipole

(19)

- In this we discuss the radiation mechanism of an oscillating charge which moves back and forth in a simple harmonic motion along a dipole.
- Let us consider a dipole antenna with two equal and opposite charges oscillating up & down with simple harmonic motion.
- Consider ' l_0 ' to be the maximum separation b/w two equal and opposite charges while ' l ' be an instantaneous separation b/w charges.
- When an osc field is applied and their separation varies with time period τ .



- fig (a) equal & opposite charges are at max separation $t=0$. Hence acceleration of charges is max i.e. a_{max} as they reverse direction. Then current $I=0$. The field line shown in fig.
- At $t=T/8$, charges move towards each other and field line which is different that as shown in fig (b)
- At $t=T/4$ charges reach at mid-point of a axis of the dipole. The acceleration of charges become zero ($a=0$) with current of max value $I=I_{max}$. In this case field lines separate to new field lines of opp. charge shown in fig (c)
- At $T=3T/8$, i.e. field $T=T/2$ the field lines keep moving outward in fig d i.e. rec.
- Electric field lines are separated (radiates) into the free space by an oscillating dipole with 2 equal & opposite charges at 2 ends oscillating in simple and harmonic motion.

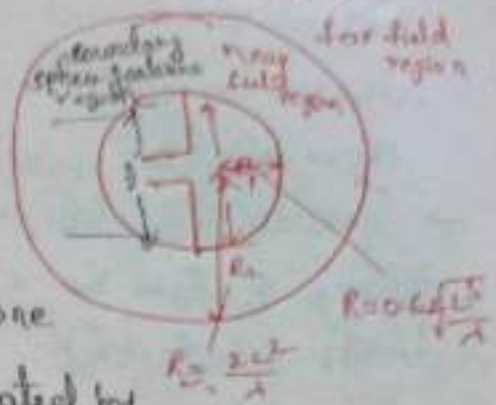


Field zones of Antenna

There are two types of field zones. Fields radiated by an antenna can be divided into two principle regions.

- 1) The near field (or) Fresnel zone
- 2) The far field (or) Fraunhofer zone

Two regions can be arbitrarily separated by a boundary whose radius can be given by $R = \frac{2L^2}{\lambda}$



→ In the far field the shape of the field pattern is independent of the distance.

→ In the far region, the measurable field components are transverse to the radial from the field direction of the (radiated) antenna. Power flow is directed radially outwards.

→ The near field the shape of field pattern depends on the distance.

→ In the near region the longitudinal component of electric field may be significant and power flow is not entirely radial.

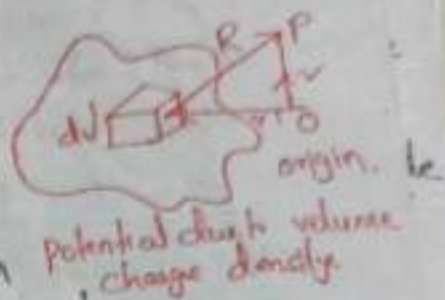
Heuristic Approach

(obtaining the potentials for the $\vec{E}$ and $\vec{B}$ fields and defining in this way the corresponding scalar potentials satisfy the independence of $\vec{E}$ and $\vec{B}$)

In this approach first we have to obtain the potentials for $\vec{E}$ & $\vec{B}$ fields. Consider a uniform volume charge density ρ_v over the given volume.

shown in fig 1

→ Consider the differential volume dv at a point distance r' from the origin where the charge density is $\rho_v(r')$.



→ The scalar electric potential V at point P can be expressed in terms of a static charge distribution as

$$V(r) = \int_V \frac{\rho_v(r') dv'}{4\pi\epsilon_0 R} \quad \text{--- (1)}$$

→ Then the fundamental electric field can be obtained by finding the gradient of a scalar potential V as $\vec{E} = -\nabla V$ --- (2)



→ If for a steady magnetic field, in a homogeneous medium, the vector magnetic potential $\vec{A}$ can be expressed in terms of a current distribution which is constant with time, as

$$\vec{A}(r) = \int_V \frac{\mu_0 \vec{J}(r') dv'}{4\pi R} \quad \text{--- (3)}$$

→ Then the fundamental magnetic field can be obtained by finding the curl of the vector magnetic potential $\vec{A}$ as

$$\vec{B} = \nabla \times \vec{A} \quad \text{--- (4)}$$

→ The potentials given by eqns (1) & (3) represent the potentials for the static electric & magnetic fields respectively, where the charge & current distributions don't vary with time.

→ Charges & current distributions producing the E-field vary with respect to time.

→ Time varying fields must modify the potentials represented by eqs ① & ② for the time variations as

$$V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon} \int_V \frac{\rho(\mathbf{r}', t)}{R} dV' \quad \text{--- ⑤}$$

$$\bar{A}(\mathbf{r}, t) = \frac{\mu}{4\pi} \int_V \frac{\mathbf{J}(\mathbf{r}', t)}{R} dV' \quad \text{--- ⑥}$$

where $R = |\mathbf{r} - \mathbf{r}'|$

eqs ⑤ & ⑥ are modified by introducing a time delay R/v as

$$V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon} \int_V \frac{\rho(\mathbf{r}', t - R/v)}{R} dV' \quad \text{--- ⑦}$$

$$\bar{A}(\mathbf{r}, t) = \frac{\mu}{4\pi} \int_V \frac{\mathbf{J}(\mathbf{r}', t - R/v)}{R} dV' \quad \text{--- ⑧}$$

eqs ⑦ & ⑧ the potentials are delayed (or) retarded by the time R/v . Hence the potentials are called Retarded potentials.

Helmholtz theorem

Maxwell's eqns approach

→ In this approach starting with Maxwell's eqns the differential eqns are derived.

→ For time varying fields, Maxwell's eq in the point form are given by

$$\nabla \times \bar{E} = -\epsilon \frac{\partial \bar{H}}{\partial t} \quad \text{--- ⑨}$$

$$\nabla \times \bar{H} = \bar{J} + \epsilon \frac{\partial \bar{E}}{\partial t} \quad \text{--- ⑩}$$

$$\nabla \cdot \bar{E} = \frac{\rho_v}{\epsilon} \quad \text{--- ⑪}$$

$$\nabla \cdot \bar{H} = 0 \quad \text{--- ⑫}$$

From eq (1) it is clear that the divergence of $\vec{H}$ is zero but from the vector identity 'the divergence of a curl of a vector is zero'.
 → This clearly indicates that to satisfy eq (1), $\vec{H}$ must be expressed as a curl of some vector. so defining vector potential $\vec{A}$ as follows. $\vec{H} = \nabla \times \vec{A}$ — (13)

Putting the value of $\vec{H}$ in eq (9)

$$\nabla \times \vec{E} = -\mu \frac{\partial}{\partial t} (\nabla \times \vec{A})$$

Interchanging the operations on R.H.S of above eqn

$$\nabla \times \vec{E} = - \left(\nabla \times \frac{\partial \vec{A}}{\partial t} \right)$$

$$\therefore \nabla \times \vec{E} + \nabla \times \frac{\partial \vec{A}}{\partial t} = 0$$

$$\nabla \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = 0 \text{ — (14)}$$

→ According to vector identity 'curl of a gradient of a scalar is always zero. so the eq (14) only satisfied only if the term $\left(\vec{E} + \frac{\partial \vec{A}}{\partial t} \right)$ is defined as a gradient of a scalar.

→ Let us introduce a scalar potential V such that

$$\vec{E} + \frac{\partial \vec{A}}{\partial t} = -\nabla V \text{ — (15)}$$

Then the Electric field strength is given by

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t} \text{ — (16)}$$

→ From eqs (13) & (16) it is clear that the $\vec{E}$ & $\vec{H}$ fields can be expressed in terms of a scalar potential V and a vector potential $\vec{A}$.

→ The fields eqs (13) & (16) satisfy the two of Maxwell's eqs namely (9) & (12).

→ The Differential eqs for the potentials are obtained from the remaining two of Maxwell's eqs.

→ Subst the values of $\vec{E}$ & $\vec{A}$ from the eqs (16) & (17) respect in eq (10)

$$\nabla \times \left[\frac{1}{\mu} \nabla \times \vec{A} \right] = \vec{J} + \epsilon \frac{\partial}{\partial t} \left[-\nabla V - \frac{\partial \vec{A}}{\partial t} \right]$$

Interchanging the operators.

$$\frac{1}{\mu} [\nabla \times \nabla \times \vec{A}] = \vec{J} + \epsilon \left[-\nabla \frac{\partial V}{\partial t} - \frac{\partial^2 \vec{A}}{\partial t^2} \right]$$

$$\nabla \times \nabla \times \vec{A} = \mu \vec{J} - \epsilon \mu \nabla \frac{\partial V}{\partial t} - \epsilon \mu \frac{\partial^2 \vec{A}}{\partial t^2} \quad (17)$$

From the vector identity

$$\nabla \times \nabla \times \vec{A} = \nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$

Resulting eq (17) using above identity.

$$\nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu \vec{J} - \epsilon \mu \nabla \frac{\partial V}{\partial t} - \epsilon \mu \frac{\partial^2 \vec{A}}{\partial t^2}$$

$$\nabla^2 \vec{A} - \nabla (\nabla \cdot \vec{A}) = -\mu \vec{J} + \epsilon \mu \nabla \frac{\partial V}{\partial t} + \epsilon \mu \frac{\partial^2 \vec{A}}{\partial t^2} \quad (18)$$

Subst value of $\vec{E}$ from eq (16) in eq (11) we get

$$\nabla \cdot \left(-\nabla V - \frac{\partial \vec{A}}{\partial t} \right) = \frac{\rho_V}{\epsilon}$$

$$\nabla^2 V + \nabla \cdot \frac{\partial \vec{A}}{\partial t} = -\frac{\rho_V}{\epsilon} \quad (19)$$

From eq (18) & (19) are not sufficient enough to define $\vec{A}$ and V comp

→ In other words we can not get a unique solution of these eqs. eqs give necessary but not sufficient conditions.

→ Using Helmholtz theorem we can find the unique solution

→ The H. theorem states that any vector field can be defined uniquely if the curl and divergence of the field both are known at any point.

The curl of $\vec{A}$ is already specified in eq (1) choose divergence of $\vec{A}$ from eq (1) as

$$\nabla \cdot \vec{A} = -\mu \epsilon \frac{\partial v}{\partial t} \quad (20)$$

eq (20) substituted in (18) & (19)

$$\nabla^2 \vec{A} - \nabla(-\mu \epsilon \frac{\partial v}{\partial t}) = -\mu \vec{J} + \mu \epsilon \nabla \frac{\partial v}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{A}}{\partial t^2}$$

$$\nabla^2 \vec{A} + \mu \epsilon \nabla \frac{\partial v}{\partial t} = -\mu \vec{J} + \mu \epsilon \nabla \frac{\partial v}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{A}}{\partial t^2}$$

$$\nabla^2 \vec{A} - \mu \epsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu \vec{J} \quad (21)$$

$$\text{in (19)} \quad \nabla^2 v + \frac{\partial}{\partial t}(\nabla \cdot \vec{A}) = -\frac{\rho v}{\epsilon}$$

$$\nabla^2 v + \frac{\partial}{\partial t}(-\mu \epsilon \frac{\partial v}{\partial t}) = -\frac{\rho v}{\epsilon}$$

$$\nabla^2 v - \mu \epsilon \frac{\partial^2 v}{\partial t^2} = -\frac{\rho v}{\epsilon} \quad (22)$$

eq (21) & (22) are standard wave eqs. solution of this eqs. are retarded potential eqs.

$$A(\vec{r}, t) = \frac{\mu}{4\pi} \int_V \frac{\vec{J}(\vec{r}', t - R/v)}{R} dv' \quad (23)$$

$$v(\vec{r}, t) = \frac{1}{4\pi \epsilon} \int_V \frac{\rho(\vec{r}', t - R/v)}{R} dv' \quad (24)$$

Antenna theorems

(25)

The validity of these theorems is based upon the linearity (or) the bilaterality of the n/w. In em field theory the solution of any antenna problem can be obtained by application of Maxwell's eqs and appropriate boundary conditions. The field eqs themselves are linear, as long as the conditions ϵ , μ , σ of the media involved are truly const. i.e. don't vary with magnitude of the signal. nor with the direction. The same theorems can be applied.

→ A few theorems that are most commonly used in antenna problems are as follows.

1) Super position theorem

In a n/w of generators and linear impedances the current flowing at any point is the sum of the currents that ^{would} flow if each generator were considered separately, all other generators being replaced at the time by impedances equal to their internal impedances.

2) Thevenin's Theorem

If an impedance Z_R be connected b/w any two terminals of a linear n/w containing one or more generators the current which flows through Z_R will be the same as it would be if Z_R were connected to a simple generator whose generated voltage is the open ckt voltage that appeared at the terminals and whose impedance is the impedance of the n/w looking back from the terminals. i.e. all generators replaced by impedances equal to the internal impedances of these generators.

③ maximum power transfer theorem

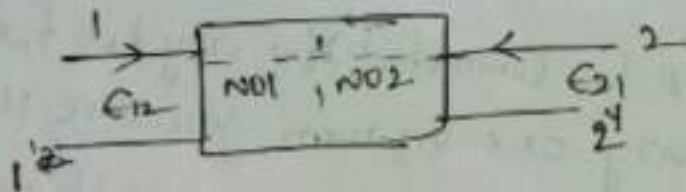
An impedance connected to two terminals of a n/w will absorb maximum power from the n/w when the impedance is equal to the conjugate of the impedance looking back from the output terminals.

④ Compensation theorem

Any impedance in a n/w may be replaced by a generator of zero internal impedance whose generated voltage at every instant is equal to the instantaneous potential difference that existed across the impedance because of the current flowing through it.

⑤ Reciprocity theorem

It states that if an emf is applied to the terminals of an antenna no. 1 and the current measured at the terminals of another antenna no. 2 then the equal current both in amplitude and phase will be obtained at the terminals of another antenna no. 1 if the same emf is applied to the terminals of antenna no. 2.



UNIT-IV

Antenna Arrays

In some wireless communication applications, we need to have narrow beam for long distance communication. It is possible by two ways.

- 1) Increasing the size of antenna
 - 2) Using antenna array
- TO increase the gain of antenna
TO have narrow beam

→ TO increase the field strength in the desired direction by using group of antennas is called array of antennas (or) Antenna array

→ Antenna array can be defined as the system of similar antennas directed to get required high directivity in the desired direction.

There are 4 types of Antenna arrays. These are

- 1) Broadside Array
- 2) Endfire array
- 3) Collinear Array
- 4) Parasitic Array

1) Broadside Array.

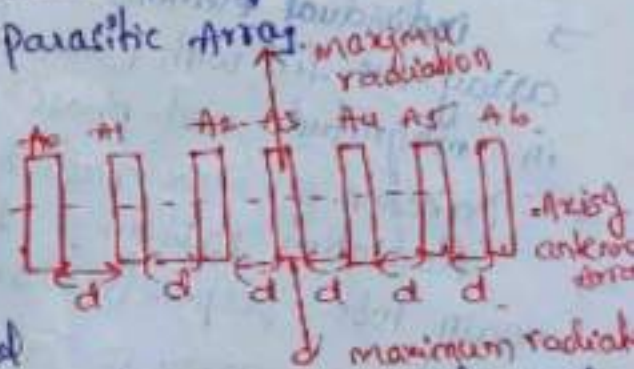
→ Broadside array is the array of antennas in which all the elements are placed parallel to each other and the direction of maximum radiation is always perpendicular to the plane containing elements.

→ All the individual antennas are spaced equally along the axis of antenna array.

→ The spacing b/w any two elements is denoted by 'd'.

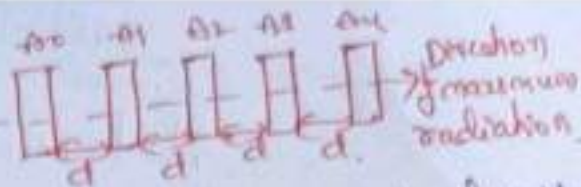
→ All the elements are fed with currents with equal magnitude same phase.

→ radiation pattern for the broadside array is bidirectional i.e. arrangement of antenna in which maximum radiation is in the direction perpendicular to the axis of array and plane containing the elements of array.



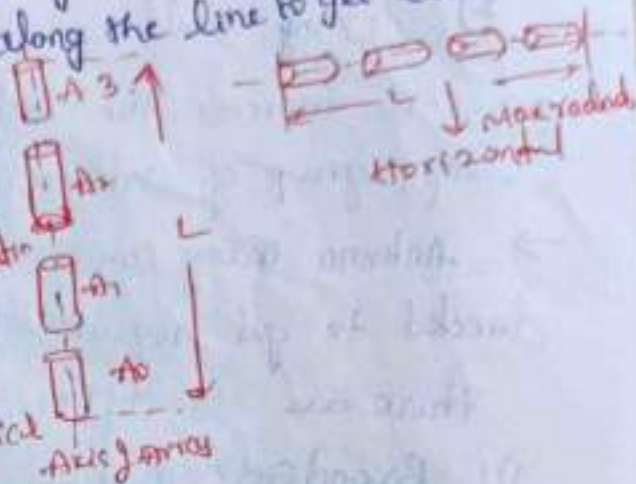
② End fire Array

- End fire Array is very similar to the broadside array from the point of view of arrangement.
- Difference is in the direction of maximum radiation (radiation along the axis of array).
- direction of the maximum radiation is along the axis of array.
- Antennas are spaced equally along a line.
- All the antennas are fed individually with currents of equal magnitude but their phases vary progressively along the line to get entire arrangement unidirectional finally.



③ Collinear Array

- Antennas are arranged co-axially i.e. the antennas are arranged end to end along a single line.
- individual elements in the collinear array are fed with currents equal in magnitude and phase. Similar to broadside array.
- Direction of max radiation is perpendicular to the axis of array.
- Radiation pattern of this array has circular symmetry with main lobe perpendicular everywhere to the principle axis.
- Collinear array is also called omnidirectional array (as broadside array).
- gain of the collinear array is maximum if the spacing b/w the elements is of the order of 0.5λ to 0.7λ . Small spacing gives feeding problems.
- To overcome this the elements of the array are operated with their ends very close to each other by connecting ends by an insulator.
- power gain of the collinear array doesn't increase in proportion with no. of elements. E.g. 2-element array power gain is 1.9 dB but for 4-element array it is not equal to twice of 2-element but it is equal to 4.2 dB.
- Collinear array with more than 4-element is not practically used as power gain is not sufficient. Practically use 2-element array.



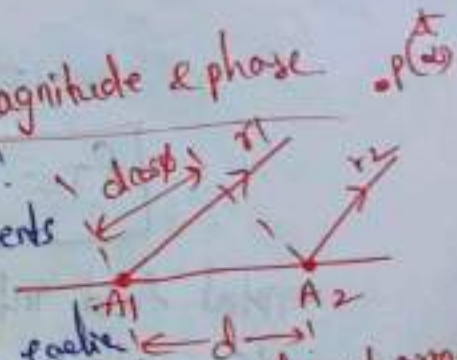
- ④ Parasitic Array.
- Parasitic array consists one driven element and one parasitic element
 - In multielement parasitic array, there may be one or more driving elements and also one or more parasitic elements.
 - So in general multielement parasitic array is the array with at least one driven element and one or more parasitic elements.
 - Ex: 2 parasitic array with half wave dipoles as elements
 - Array is $\lambda/4$ or $\lambda/2$
 - Amplitude and phase of the current induced in the parasitic element depends on the spacing b/w the driven element and parasitic element.
 - Radiation pattern Unidirectional
 - phase of the currents are changed by adjusting the spacing b/w elements.

Array of Point Sources

- Point Source in this an antenna is regarded as point source or volume radiator.
- Depending upon the magnitude of and phase currents given to two point sources we obtain three cases.
 - 1) Equal amplitude and phase
 - 2) " " and opposite phase
 - 3) Unequal amplitude and " " and opposite phase

① Two point sources with currents equal in magnitude & phase

- Consider two pt sources A_1 & A_2 separated by 'd'.
- both the point sources are supplied with currents equal in magnitude & phase $I_1 = I_2 = I$
- radiation from the pt source A_2 will reach earlier at pt P than that from pt source A_1 because of path difference.
- pt P far away from the array. distance b/w pt P & point sources A_1 & A_2 be r_1 & r_2 respectively.



Hence path difference is given by.

$$\text{path difference} = d \cos \phi \quad \text{--- (1)}$$

path difference can be expressed in terms of wavelength as

$$\text{path diff} = \frac{d \cos \phi}{\lambda} \quad \text{--- (2)}$$

Hence the phase angle ψ is given by.

$$\text{phase angle} = \psi = 2\pi (\text{path difference}) \\ = 2\pi \left(\frac{d \cos \phi}{\lambda} \right)$$

$$\boxed{\psi = \frac{2\pi}{\lambda} d \cos \phi \text{ rad}} \quad \text{--- (3)}$$

But phase shift $= \beta = \frac{2\pi}{\lambda}$, then eq (3) becomes.

$$\psi = \frac{2\pi}{\lambda} d \cos \phi \Rightarrow \boxed{\beta d \cos \phi \text{ rad} = \psi} \quad \text{--- (4)}$$

$E_1 \rightarrow$ the field at a distance point P due to point source A_1 .

$$\text{By } E_2 = E_0 e^{j\psi/2}$$

$$E_1 = E_0 e^{-j\psi/2}, \quad E_2 = E_0 e^{j\psi/2}$$

Amplitude of both the field components is E_0 as currents are same & the 2 sources are identical.

$\rightarrow$ Total field at Pt P is given by.

$$E_T = E_1 + E_2 \Rightarrow E_0 e^{-j\psi/2} + E_0 e^{j\psi/2}$$

$$E_T = E_0 (e^{-j\psi/2} + e^{j\psi/2})$$

$$= 2E_0 \left(\frac{e^{-j\psi/2} + e^{j\psi/2}}{2} \right)$$

$$= 2E_0 \cos(\psi/2) \quad (\text{sub } \psi \text{ value})$$

$$\boxed{E_T = 2E_0 \cos \left(\frac{\beta d \cos \phi}{2} \right)}$$

Total field intensity at Pt P. due to two point sources having currents same amp & phase. Total amplitude is $2E_0 \cos \left(\frac{\beta d \cos \phi}{2} \right)$

Maxima direction.

Total field is max when $\cos\left(\frac{\beta d \cos\phi}{2}\right)$ is maximum.

Condition for max. is $\cos\left(\frac{\beta d \cos\phi}{2}\right) = \pm 1$

$$\cos\left(\frac{\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cos\phi}{2}\right) = \pm 1 \Rightarrow \cos\left(\frac{\pi}{2} \cos\phi\right) = \pm 1$$

$$\frac{\pi}{2} \cos\phi = \cos^{-1}(\pm 1)$$

$$\frac{\pi}{2} \cos\phi_{\max} = \pm n\pi \quad \text{where } n = 0, 1, 2, \dots$$

If $n=0$, then $\frac{\pi}{2} \cos\phi_{\max} = 0 \Rightarrow \cos\phi_{\max} = 0$.

$n=1$, then $\frac{\pi}{2} \cos\phi_{\max} = \pm \pi \Rightarrow \cos\phi_{\max} = \pm 1$

$$\phi_{\max} = 90^\circ \text{ (or) } 270^\circ$$

minima direction.

$$\cos\left(\frac{\beta d \cos\phi}{2}\right) = 0$$

$$\frac{\pi}{2} \cos\phi_{\min} = \cos^{-1}(0) \Rightarrow \frac{\pi}{2} \cos\phi_{\min} = \pm (2n+1)\frac{\pi}{2}$$

If $n=0$, $\frac{\pi}{2} \cos\phi_{\min} = \pm \frac{\pi}{2}$

$$\cos\phi_{\min} = \pm 1 \Rightarrow \phi_{\min} = \cos^{-1}(\pm 1)$$

$$\phi_{\min} = 0^\circ \text{ (or) } 180^\circ$$

2) Half power point directions.

$$\cos\left(\frac{\beta d \cos\phi}{2}\right) = \pm \frac{1}{\sqrt{2}}$$

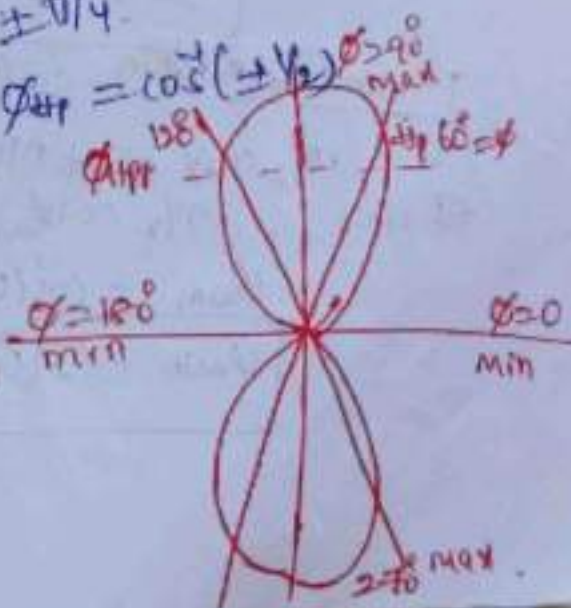
$$\frac{\pi}{2} \cos\phi = \cos^{-1}\left(\pm \frac{1}{\sqrt{2}}\right) = \pm (2n+1)\frac{\pi}{4}$$

If $n=0$, then $\frac{\pi}{2} \cos\phi = \pm \frac{\pi}{4}$

$$\cos\phi_{\text{HPP}} = \pm \frac{1}{\sqrt{2}} \Rightarrow \phi_{\text{HPP}} = \cos^{-1}\left(\pm \frac{1}{\sqrt{2}}\right)$$

$$\phi_{\text{HPP}} = 60^\circ \text{ (or) } 120^\circ$$

Field pattern for two point source with spacing $d = \lambda/2$ & fed with currents equal in mag & phase.



Two point sources with currents equal in magnitude but opposite in phase.

All the conditions are exactly same except the phase of the currents is opposite i.e. 180° . Total field at P is given by -

$$E_T = -E_1 + E_2$$

$$E_1 = E_0 e^{j\psi/2}, \quad E_2 = E_0 e^{j\psi/2}$$

$$E_T = -E_0 e^{-j\psi/2} + E_0 e^{j\psi/2}$$

$$E_T = E_0 \left(-e^{-j\psi/2} + e^{j\psi/2} \right)$$

$$= 2jE_0 \left(\frac{-e^{-j\psi/2} + e^{j\psi/2}}{2j} \right)$$

$$\frac{e^{j0} - e^{-j0}}{2j} = \sin 0$$

$$E_T = 2jE_0 \sin \psi/2$$

phase angle $\psi = \beta d \cos \phi$

$$E_T = j(2E_0) \sin \left(\frac{\beta d \cos \phi}{2} \right)$$

Maxima direction

$$\sin \left(\frac{\beta d \cos \phi}{2} \right) = \pm 1$$

$$\sin \left(\frac{\pi}{2} \cos \phi \right) = \pm 1 \Rightarrow \sin \left(\pi/2 \cos \phi \right) = \pm 1$$

$$\pi/2 \cos \phi = \sin^{-1}(\pm 1) \Rightarrow \pi/2 \cos \phi = \pm (2n+1)\pi/2$$

If $n=0$, then $\pi/2 \cos \phi_{\max} = \pm \pi/2$

$$\cos \phi_{\max} = \pm 1 \Rightarrow \phi_{\max} = \cos^{-1}(\pm 1)$$

$$\phi_{\max} = 0^\circ \text{ or } 180^\circ$$

Minima direction

$$\sin(\pi/2 \cos \phi) = 0 \Rightarrow \pi/2 \cos \phi = \sin^{-1}(0)$$

$$\pi/2 \cos \phi = \pm n\pi \Rightarrow$$

If $n=0$, $\pi/2 \cos \phi_{\min} = 0 \Rightarrow \cos \phi_{\min} = 0$

$$\phi_{\min} = \cos^{-1}(0) \Rightarrow$$

$$\phi_{\min} = 90^\circ \text{ (or) } -90^\circ$$

Half power point Direction (HPD)

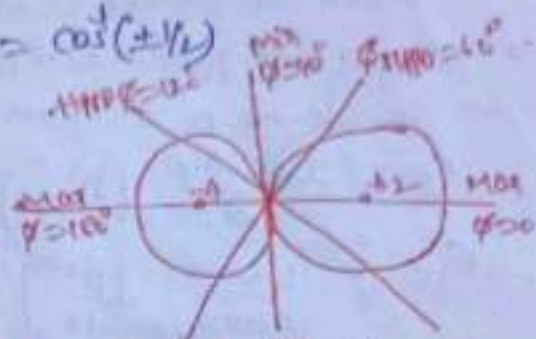
$$\sin(\pi/2 \cos \phi) = \pm \frac{1}{\sqrt{2}}$$

$$\pi/2 \cos \phi = \sin^{-1}(\pm 1/\sqrt{2}) = \pm (2n+1)\pi/4$$

$$\text{Let } n=0, \quad \pi/2 \cos \phi = \pm \pi/4$$

$$\cos \phi = \pm 1/2 \Rightarrow \phi = \cos^{-1}(\pm 1/2)$$

$$\boxed{\phi_{HPD} = 60^\circ \text{ (or) } 120^\circ}$$



(2) Two point sources with unequal magnitude in Magnitude and with any phase

→ Here we consider a general condition, where amplitude of two point sources are not equal and they have phase diff say α

$$\psi = \frac{2\pi}{\lambda} d \cos \theta + \alpha$$

→ let us also assume that source 1 is taken as reference point

$$\text{then } E_r = E_1 e^{j\psi} + E_2 e^{j\psi}$$

$$\text{where } \frac{E_2}{E_1} = k \quad \text{Here } E_1 > E_2 \\ k < 1 \\ 0 \leq k \leq 1$$

$$E_r = E_1 \left[1 + \frac{E_2}{E_1} e^{j\psi} \right]$$

$$E_r = E_1 [1 + k e^{j\psi}]$$

$$= E_1 [1 + k(\cos \psi + j \sin \psi)]$$

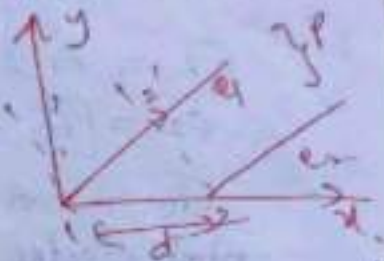
$$|E_r| = E_1 \sqrt{(1 + k \cos \psi)^2 + (k \sin \psi)^2}$$

$$\angle E_r = \tan^{-1} \left[\frac{k \sin \psi}{1 + k \cos \psi} \right]$$

phase angle b/w two fields at the far point p is

given by

$$\boxed{\theta = \tan^{-1} \frac{k \sin \psi}{1 + k \cos \psi}}$$



N Element Uniform Linear array with equal spacing and currents equal in magnitude & phase - Broadside array
 Max radiation occurs in the directions normal to the line of array. Hence such an array is known as uniform broadside array.

→ Electric field produced at pt P due to an element A_0 is given by

$$E_0 = \frac{\rho dL \sin\theta}{4\pi r_0^2 \epsilon_0} \left[\frac{\rho}{r_0} \right] e^{-j\beta r_0} \quad \text{--- (1)} \quad E = \frac{\rho dL \sin\theta}{4\pi r_0^2 \epsilon_0} \left[\frac{\rho}{r_0} \right] e^{-j\beta r_0}$$

$$E_1 = \frac{\rho dL \sin\theta}{4\pi r_0^2 \epsilon_0} \left[\frac{\rho}{r_0} \right] e^{-j\beta(r_0 - d \cos\theta)}$$

$$E_1 = E_0 e^{j\beta d \cos\theta} \quad \text{--- (2)}$$

$$E_2 = E_1 e^{j\beta d \cos\theta} = E_0 e^{j2\beta d \cos\theta} \quad \text{--- (3)}$$

$$E_{n-1} = E_0 e^{j(n-1)\beta d \cos\theta} \quad \text{--- (4)}$$

total electric field at pt P is given by.

$$E_T = E_0 + E_1 + E_2 + \dots + E_{n-1} = E_0 + E_0 e^{j\beta d \cos\theta} + E_0 e^{j2\beta d \cos\theta} + \dots + E_0 e^{j(n-1)\beta d \cos\theta}$$

Let $\beta d \cos\theta = \psi$, then rewriting above eqn

$$E_T = E_0 + E_0 e^{j\psi} + E_0 e^{j2\psi} + \dots + E_0 e^{j(n-1)\psi} \quad \text{--- (5)}$$

$$= E_0 [1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(n-1)\psi}] \quad \text{--- (6)}$$

consider a series given by $S = 1 + r + r^2 + r^3 + \dots + r^{n-1}$ --- (7)

$r = e^{j\psi}$
 multiplying with r to eq (7)

$$S \cdot r = r + r^2 + r^3 + \dots + r^n \quad \text{--- (8)}$$

$$\text{Subtracting (8) from (7)} \quad S - S \cdot r = 1 - r^n$$

$$S(1 - r) = 1 - r^n \Rightarrow S = \frac{1 - r^n}{1 - r} \quad \text{--- (9)}$$

Using eq (9) & eq (6) can be modified as

$$E_T = E_0 \left[\frac{1 - e^{jn\psi}}{1 - e^{j\psi}} \right] \Rightarrow \frac{E_T}{E_0} = \frac{e^{jn\psi/2} [e^{jn\psi/2} - e^{-jn\psi/2}]}{e^{j\psi/2} [e^{j\psi/2} - e^{-j\psi/2}]}$$

From the trigonometric identities

$$e^{j\theta} = \cos\theta - j\sin\theta, \quad e^{-j\theta} = \cos\theta + j\sin\theta$$

$$e^{j\theta} - e^{-j\theta} = -j2\sin\theta$$

Using above trigonometric identities eq (8) can be written as

$$\frac{E_T}{E_0} = \frac{e^{jn\psi/2} [-j2\sin(n\psi/2)]}{e^{j\psi/2} [-j2\sin(\psi/2)]}$$

$$\frac{E_T}{E_0} = e^{j(n-1)\psi/2} \left[\frac{\sin(n\psi/2)}{\sin(\psi/2)} \right] \quad \text{--- (9)}$$

exponential term in eq (9) is the phase shift. Now considering magnitudes of the electric fields, we can write

$$\left| \frac{E_T}{E_0} \right| = \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} \quad \text{--- (10)}$$

properties of Broadside Array.

- (1) Major lobe. Field is max in the direction normal to the axis of array. condition for max field at P is given by.

$$\psi = 0, \text{ i.e. } \cos\phi = 0. \text{ i.e. } \cos\phi = 0 \Rightarrow \phi = \cos^{-1}(0)$$

$\phi = 90^\circ (\text{or}) 270^\circ$ are called directions of principle max.

- (2) Magnitude of major lobe. Max radiation occurs when $\psi = 0$,

$$|\text{Major lobe}| = \left| \frac{E_T}{E_0} \right| = \lim_{\psi \rightarrow 0} \left\{ \frac{\frac{d}{d\psi} (\sin n\psi/2)}{\frac{d}{d\psi} (\sin \psi/2)} \right\}$$

$$= \lim_{\psi \rightarrow 0} \left\{ \frac{\cos(\psi/2)(n\psi/2)}{\cos(\psi/2)(\psi/2)} \right\} = n.$$

$$|\text{Major lobe}| = n$$

③ Nulls direction of minima, equating ratio of magnitude is zero.

$$\left| \frac{E}{E_0} \right| = \frac{\sin n\psi/2}{\sin \psi/2} = 0$$

Condition for minima is given by

$$\sin n\psi/2 = 0, \text{ but } \sin \psi/2 \neq 0$$

Hence we can write $\sin n\psi/2 = 0$.

$$\text{i.e. } n\psi/2 = \sin^{-1}(0) = \pm m\pi, \text{ where } m = 1, 2, 3, \dots$$

$$\text{Now } \psi = \beta d \cos \phi = \frac{2\pi}{\lambda} d \cos \phi$$

$$\frac{n}{2} \left(\frac{2\pi}{\lambda} d \cos \phi \right) = \pm m\pi$$

$$\frac{nd}{\lambda} \cos \phi_{\min} = \pm m \Rightarrow \boxed{\phi_{\min} = \cos^{-1} \pm \left(\frac{m\lambda}{nd} \right)} \quad \text{--- (1)}$$

where n = no. of elements in array, d = spacing b/w elements in array

$$\lambda = w.v., m = \text{const}$$

Eq. (1) gives the direction of nulls.

④ Subsidiary maxima (or side lobes)

Side lobes can be obtained $\sin(n\psi/2) = \pm 1$

$$n\psi/2 = \pm 3\pi/2, \pm 5\pi/2, \pm 7\pi/2, \dots$$

Hence $\sin(n\psi/2) = \pm 1$ is not considered. Bcz if $n\psi/2 = \pi/2$ then

$\sin n\psi/2 = 1$, which is the direction of principle maxima.

So we can skip $\sin n\psi/2 = \pm \pi/2$ value.

Thus we get $\psi = \pm 2\pi/n, \pm 4\pi/n, \pm 6\pi/n, \dots$

$$\psi = \beta d \cos \phi = \left(\frac{2\pi}{\lambda} \right) d \cos \phi = \pm 2\pi/n, \pm 4\pi/n, \pm 6\pi/n, \dots$$

$$\cos \phi = \frac{\lambda}{2\pi d} \left[\pm \frac{(2m+1)\pi}{n} \right] \text{ where } m = 1, 2, 3, \dots$$

$$\phi = \cos^{-1} \left[\pm \frac{\lambda(2m+1)}{2nd} \right] \quad \text{--- (2)}$$

Eq. (2) represents directions of side lobes.

⑤ Beamwidth of major lobe.

⑥

Beamwidth between first nulls is given by

$$\text{BWFN} = 2\gamma, \text{ where } \gamma = 90 - \phi$$

$$\phi_{\min} = \cos^{-1}\left(\pm \frac{m\lambda}{nd}\right), \text{ where } m=1, 2, 3$$

$$\text{Also } 90 - \phi_{\min} = \gamma, \text{ i.e. } 90 - \gamma = 0 \text{ min}$$

$$\text{Hence } 90 - \gamma = \cos^{-1}\left(\pm \frac{m\lambda}{nd}\right)$$

Taking cosine of angle on both the sides.

$$\cos(90 - \gamma) = \cos\left[\cos^{-1}\left(\pm \frac{m\lambda}{nd}\right)\right], \sin \gamma = \pm \frac{m\lambda}{nd}$$

If γ is very small, $\sin \gamma \approx \gamma$, Sub in eq. $\gamma = \pm \frac{m\lambda}{nd}$

$$\text{At } m=1, \gamma = \pm \frac{\lambda}{nd}$$

$$\text{BWFN} = 2\gamma = \frac{2\lambda}{nd}$$

$$[nd=L]$$

$$\text{BWFN} = \frac{2\lambda}{L} \text{ rad} = \frac{2}{(L/\lambda)} \text{ rad. mdeg BWFN} = \frac{114.6}{(L/\lambda)} \text{ deg}$$

$$\text{HPBW} = \frac{\text{BWFN}}{2} = \frac{1}{(L/\lambda)} \text{ rad. HPBW} = \frac{57.3}{(L/\lambda)} \text{ deg}$$

Directivity.

$$G_{\text{d max}} = \frac{\text{Maximum radiation intensity}}{\text{Average}} = \frac{U_{\text{max}}}{U_{\text{avg}}} = \frac{U_{\text{max}}}{U_0}$$

$$G_{\text{d max}} = 2\left(\frac{L}{\lambda}\right)$$

2) n elements with equal spacing and currents equal in Magnitude but with opposite phase — End Fire Array.
 Consider that the current supplied to first element A_0 be I_0 . Then the current supplied to A_n is given by.

$$I_1 = I_0 e^{-j\beta d}$$

$$\text{Ily } I_2 = I_1 e^{-j\beta d}$$

$$= [I_0 e^{-j\beta d}] e^{-j\beta d} = I_0 e^{-j2\beta d}$$

$$I_{n-1} = I_0 e^{-j(n-1)\beta d}$$

Electric field produced at pt P , due to A_0 is given by.

$$E_0 = \frac{I_0 L \sin\theta}{4\pi r_0 \epsilon_0} \left[\frac{j\beta^2}{r_0} \right] e^{-j\beta r_0} \quad \text{--- (1)}$$

$$E_1 = \frac{I_0 L \sin\theta}{4\pi r_1 \epsilon_0} \left[\frac{j\beta^2}{r_1} \right] e^{-j\beta r_1} \cdot e^{-j\beta d} \quad \left[\text{but } r_1 = r_0 - d \cos\theta \right]$$

$$= \frac{I_0 L \sin\theta}{4\pi r_0 \epsilon_0} \left[\frac{j\beta^2}{r_0} \right] e^{-j\beta(r_0 - d \cos\theta)} e^{-j\beta d}$$

$$E_1 = E_0 e^{j\beta d(\cos\theta - 1)} \Rightarrow E_1 = E_0 e^{j\psi} \quad \left[\psi = \beta d(\cos\theta - 1) \right]$$

$$\text{Ily } E_2 = E_0 e^{j2\psi}$$

$$E_{n-1} = E_0 e^{j(n-1)\psi}$$

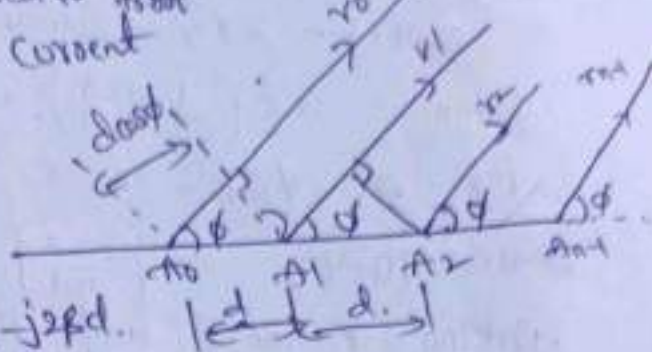
$$\text{Then } E_T = E_0 + E_1 + E_2 + \dots + E_{n-1}$$

$$= E_0 + E_0 e^{j\psi} + E_0 e^{j2\psi} + \dots + E_0 e^{j(n-1)\psi}$$

$$= E_0 (1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(n-1)\psi}) \quad \text{with n exponential series.}$$

$$E_T = E_0 \cdot \frac{1 - e^{jn\psi}}{1 - e^{j\psi}} \Rightarrow \frac{E_T}{E_0} = \frac{\sin n\psi/2}{\sin \psi/2} \cdot e^{j\frac{(n-1)\psi}{2}}$$

Consider only magnitude $\boxed{\frac{|E_T|}{|E_0|} = \frac{\sin n\psi/2}{\sin \psi/2}}$



Properties of End fire array.

① Major lobe. $\psi = \rho d(\cos\phi - 1) = 0 \Rightarrow \cos\phi = 1 \Rightarrow \phi = \cos^{-1}(1)$

$\boxed{\phi = 0^\circ}$ direction of principle maxima

② Magnitude of the major lobe

$$|\text{Major lobe}| = \lim_{\psi \rightarrow 0} \left\{ \frac{\frac{d}{d\psi} (\sin n\psi/2)}{\frac{d}{d\psi} (\sin \psi/2)} \right\} = \lim_{\psi \rightarrow 0} \left\{ \frac{\cos(n\psi/2)(n/2)}{\cos(\psi/2)(1/2)} \right\}$$

$$\boxed{|\text{Major lobe}| = n}$$

③ Nulls $\left| \frac{E}{E_0} \right| = \frac{\sin n\psi/2}{\sin \psi/2} = 0$

Condition of minima $\sin n\psi/2 = 0$, but $\sin \psi/2 \neq 0$.

$$\sin n\psi/2 = 0 \Rightarrow n\psi/2 = \pm m\pi, m=1,2,3, \dots$$

$$n \rho d(\cos\phi - 1) = \pm m\pi \quad \text{put } \left[\rho = \frac{2\pi}{\lambda} \right]$$

$$\frac{n d}{\lambda} (\cos\phi - 1) = \pm m \quad \left[(\cos\phi - 1) \text{ is always less than } 1 \right]$$

$$\frac{n d}{\lambda} (\cos\phi - 1) = -m \quad \text{ie } \cos\phi = 1 - \frac{m\lambda}{nd}$$

$$\boxed{\phi_{\min} = \cos^{-1} \left[1 - \frac{m\lambda}{nd} \right]} \quad \text{by } \boxed{\phi_{\min} = 2 \sin^{-1} \left[\pm \frac{m\lambda}{2nd} \right]}$$

④ Subsidiary maxima (or side lobe)

$$\sin(n\psi/2) = \pm 1, \quad n\psi/2 = \pm 3\pi/2, \pm 5\pi/2, \pm 7\pi/2, \dots$$

Since $n\psi/2 = \pm \pi/2$ is skipped bcz this value of $n\psi/2$, $\sin n\psi/2 = 1$, which is the direction of principle maxima.

we can write $n\psi/2 = \pm (2m+1)\pi/2, m=1,2,3, \dots$

$$n \rho d(\cos\phi - 1) = \pm (2m+1)\pi/2 \Rightarrow \rho d(\cos\phi - 1) = \pm (2m+1)\pi$$

$$\text{put } \rho = \frac{2\pi}{\lambda}, \quad \left[n \left(\frac{2\pi}{\lambda} \right) d(\cos\phi - 1) \right] = \pm (2m+1)\pi$$

$$\cos\phi - 1 = \pm (2m+1) \frac{\lambda}{2nd}$$

$$\boxed{\phi = \cos^{-1} \left[1 - \frac{(2m+1)\lambda}{2nd} \right]}$$

$(\cos\phi - 1)$ is always less than 0
So can't take -ve value.

⑤ Beamwidth of major lobe

Beamwidth = $2 \times$ Angle b/w first nulls & maximum of the major lobe.

$$\phi_{\min} = 2 \sin^{-1} \left[\pm \sqrt{\frac{md}{2nd}} \right] \quad \left[\frac{\sin \phi_{\min}}{2} \approx \frac{\phi_{\min}}{2} \text{ for } \phi_{\min} \text{ very small} \right]$$

$$\sin \frac{\phi_{\min}}{2} = \pm \sqrt{\frac{md}{2nd}}$$

$$\frac{\phi_{\min}}{2} = \pm \sqrt{\frac{md}{2nd}} \Rightarrow \phi_{\min} = \pm \sqrt{\frac{4md}{2nd}} = \pm \sqrt{\frac{2md}{nd}} \quad [nd=L]$$

$$\phi_{\min} = \pm \sqrt{\frac{2md}{L}} = \pm \sqrt{\frac{2m}{L/\lambda}}$$

$$\text{BWPN} = 2\phi_{\min} = \pm 2 \sqrt{\frac{2m}{L/\lambda}} \text{ rad} = \pm 2 \sqrt{\frac{2m}{L/\lambda}} \times 57.3$$

$$= \pm 114.6 \frac{2m}{L/\lambda} \text{ deg.}$$

⑥ Directivity

$$G_{\max} = \frac{U_{\max}}{U_0}$$

endfire array $U_{\max} = 1$,

$$G_{\max} = \frac{1}{\frac{\pi}{2npd}} = \frac{2npd}{\pi} = 2n \left(\frac{2\pi}{\lambda} \right) \left(\frac{d}{4\pi} \right) \quad [U_0 = \pi/2npd]$$

$$[nd=L]$$

$$G_{\max} = 4 \left(\frac{nd}{\lambda} \right) \Rightarrow 4 \left(\frac{L}{\lambda} \right)$$

element uniform linear broadside array & Endfire array	Broadside array.	Endfire array.
1) Direction of major lobe.	$\phi_{max} = 90^\circ$ (or) $\phi_{max} = 2\pi$	$\phi_{max} = 0$
2) Magnitude of major lobe.	n.	n
3) Direction of minor lobe.	$\phi_{min} = \cos^{-1}\left[\pm \frac{m\lambda}{nd}\right]$ where $m = 1, 2, 3$	$\phi_{min} = \cos^{-1}\left[1 - \frac{m\lambda}{nd}\right]$ where $m = 1, 2, 3$
4) Direction of side lobes (SLL)	$\phi = \cos^{-1}\left[\pm \lambda \frac{(2m+1)}{2nd}\right]$ where $m = 1, 2, 3$	$\phi = \cos^{-1}\left[1 - \frac{(2m+1)\lambda}{2nd}\right]$ where $m = 1, 2, 3$
5) BWFN	$BWFN = \frac{2\lambda}{L} = \frac{2\lambda}{L/\lambda} = \frac{2}{L/\lambda}$	$BWFN = \pm 2 \sqrt{\frac{2m}{L/\lambda}} = \pm 11.46 \sqrt{\frac{2m}{L/\lambda}}$
6) HPBW.	$HPBW = \frac{BWFN}{2} = \frac{1}{L/\lambda} \text{ rad} = \frac{57.3}{L/\lambda} \text{ deg}$	$HPBW = \pm \sqrt{\frac{2m}{L/\lambda}} \text{ rad} = \pm 57.3 \sqrt{\frac{2m}{L/\lambda}} \text{ deg}$
7) Directivity	$D = 2\left(\frac{nd}{\lambda}\right) = 2(L/\lambda)$	$D = 4(L/\lambda)$

① a broadside array of identical antennas consists of 8 isotropic radiators separated by distance $\lambda/2$. find radiation field in the plane containing the line of array showing directions of maxima & null.

given $n=8, d=\lambda/2$

① Major lobe. $\phi = 0, \text{ or } 180^\circ$

② Magnitude of major lobe $\left| \frac{E}{E_0} \right| = n = 8$

③ Nulls $\phi_{\min} = \cos^{-1} \left[\pm \frac{m\lambda}{nd} \right]$ where $m=1, 2, 3, \dots$

④ If $m=1$ $\phi_{\min} = \cos^{-1} \left[\pm \frac{m\lambda}{nd} \right] = \cos^{-1} \left[\pm \frac{\lambda}{(8)(\lambda/2)} \right] = \cos^{-1}(\pm 1/4) = 75.52^\circ$
 104.47°

⑤ If $m=2$, $\phi_{\min} = \cos^{-1} \left[\pm \frac{2\lambda}{(8)(\lambda/2)} \right] = 68.19^\circ$ & 121.81°

⑥ If $m=3$, $\phi_{\min} = \cos^{-1} \left[\pm \frac{3\lambda}{(8)(\lambda/2)} \right] = 41.4^\circ$ & 138.6°

⑦ Side lobe $\phi = \cos^{-1} \left[\pm \frac{d(2m+1)}{2nd} \right]$

a) If $m=1$, $\phi_{\min} = \cos^{-1} \left[\pm \frac{\lambda(2+1)}{2 \times 8 \times \lambda/2} \right] = \cos^{-1} \left[\pm 3/8 \right] = 61.91^\circ$
 118.09°

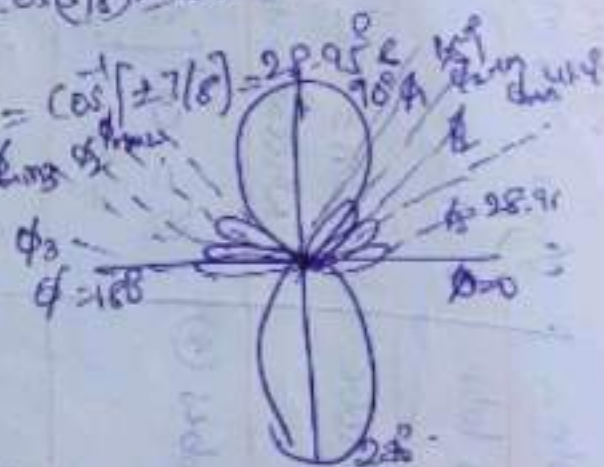
b) If $m=2$, $\phi_{\min} = \cos^{-1} \left[\pm \frac{\lambda(4+1)}{2 \times 8 \times \lambda/2} \right] = \cos^{-1} \left[\pm 5/8 \right] = 51.31^\circ$ & 128.69°

c) $m=3$, $\phi_{\min} = \cos^{-1} \left[\pm \frac{\lambda(6+1)}{2 \times 8 \times \lambda/2} \right] = \cos^{-1} \left[\pm 7/8 \right] = 29.94^\circ$ & 170.06°

⑧ $BWFN = \frac{2\lambda}{L} \left[L = nd = 8 \times \frac{\lambda}{2} = 4\lambda \right]$
 $= \frac{2\lambda}{4\lambda} = \frac{1}{2} = 0.5 \text{ rad.}$

⑨ $HPBW = \frac{BWFN}{2} = \frac{0.5}{2} = 0.25$

⑩ $G_{\text{max}} = 2 \left(\frac{L}{\lambda} \right) = 2 \left(\frac{4\lambda}{\lambda} \right) = 8$



- ② Find the minimum spacing b/w the elements in a broadside array of 10 isotropic radiators to have directivity of 10.

$$G_{\text{broad}} = 10, \quad n = 10.$$

$$G_{\text{max}} = 10 \log_{10} [G_{\text{max}}]$$

$$7 = 10 \log_{10} [G_{\text{max}}] \Rightarrow G_{\text{max}} = 5.0118$$

$$G_{\text{max}} = 2 \left(\frac{L}{\lambda} \right) = 2 \left(\frac{n d}{\lambda} \right) \Rightarrow 5.0118 = 2 \left(\frac{10 \times d}{\lambda} \right) = 0.25 \lambda$$

- ③ Find the length and BFW for broadside and endfire array if the directive gain is 15

$$G_{\text{max}} = 15$$

① For broadside array $G_{\text{max}} = 2 \left(\frac{L}{\lambda} \right) \Rightarrow 15 = 2 \left(\frac{L}{\lambda} \right) \Rightarrow L = 7.5 \lambda$

② $B_{\text{broad}} = \frac{114.6}{\left(\frac{L}{\lambda} \right)} \text{ deg} = \frac{114.6}{\left(\frac{7.5 \lambda}{\lambda} \right)} = 15.28^\circ$

Endfire array ① $G_{\text{max}} = 4 \left(\frac{L}{\lambda} \right) \Rightarrow 15 = 4 \left(\frac{L}{\lambda} \right) \Rightarrow L = 3.75 \lambda$

② $B_{\text{end}} = 114.6 \sqrt{\frac{2}{\left(\frac{L}{\lambda} \right)}} \text{ deg} = 114.6 \sqrt{\frac{2}{\left(\frac{3.75 \lambda}{\lambda} \right)}} = 83.6^\circ$

- ④ A uniform linear array consists of 16 isotropic sources with spacing of $\lambda/4$. If the phase difference is 90° , calculate
 a) Directivity index b) Beam solid angle c) Effective aperture

① HPBW

$$n = \text{no. of total elements} = 16$$

$$d = \text{spacing b/w adjacent elements} = \lambda/4$$

$$L = \text{total length of array} = (n-1)d = (16-1) \frac{\lambda}{4} = 15 \lambda/4$$

② HPBW $= 57.3 \sqrt{\frac{2m}{\left(\frac{L}{\lambda} \right)}} \text{ deg} \Rightarrow 57.3 \sqrt{\frac{2(1)}{\frac{15 \lambda}{4} \cdot \frac{1}{\lambda}}} = \sqrt{\frac{41.84}{15}} \left[\begin{matrix} m=1, \\ L=15 \frac{\lambda}{4} \end{matrix} \right]$

③ Directivity $D = 4 \left(\frac{L}{\lambda} \right) = 4 \left(\frac{15 \lambda}{4 \lambda} \right) = 15 \Rightarrow 10 \log_{10} 15 = 11.76 \text{ dB}$

④ Beam solid angle $\Omega = \frac{4\pi}{D} = \frac{4 \times \pi}{15} = 0.8377 \text{ sr}$

⑤ $A_e = \frac{D \lambda^2}{4\pi} = \frac{15 \times \lambda^2}{4\pi} = 1.1936 \lambda^2$

⑤ calculate 1) HPBW 2) solid angle if a linear array having 10 isotropic point sources with $\lambda/2$ spacing and phase difference 90° .

$\Rightarrow$ phase difference 90° , $n=10$, $d=\lambda/2$

[$m=1$, $L=\frac{\lambda}{2}$]

$$L = (n-1)d = (10-1)\lambda/2 = 9\lambda/2$$

$$1) \text{ HPBW} = 57.3 \sqrt{\frac{2\pi}{(L/\lambda)}} \text{ deg} = 57.3 \sqrt{\frac{2\pi}{(9\lambda/2)/\lambda}} = 57.3 \sqrt{\frac{4}{9}} = 38.2^\circ$$

2) beam solid angle $\Omega = \frac{4\pi}{D}$ where $D = \text{Directivity}$

$$D = 4\left(\frac{L}{\lambda}\right) = 4\left(\frac{9\lambda/2}{\lambda}\right) = 18.$$

$$\Omega = \frac{4\pi}{D} = \frac{4\pi}{18} = 0.6981 \text{ sr}.$$

⑥ find the phasing required to steer a beam zenith to -40° for a 5 element array with 0.4λ inter element spacing.

$n = \text{no. of elements} = 5$, $d = \text{spacing} = 0.4\lambda$, $2\phi = \text{beamwidth} = -40^\circ$.

phase difference required for radiations of two adjacent points sources is given by $\psi = \frac{2\pi}{\lambda} d \cos\phi + \alpha$ [$\alpha=0$]

$$\psi = \frac{2\pi}{\lambda} d \cos\phi = \frac{2\pi}{\lambda} (0.4\lambda) \cos(-20^\circ) = 2.3667 \text{ rad}$$

⑦ Calculate the directivity of given linear endfire array with improved directivity. Hansen-Woodward (increased directivity) uniform array of 10 elements with a separation of $\lambda/4$ b/w the elements.

Endfire array with increased directivity $D = 1.789 \left[4 \left(\frac{L}{\lambda} \right) \right]$

$$= 1.789 \left[4 \left(\frac{n \cdot d}{\lambda} \right) \right] = 1.789 \left[4 \left(\frac{10 \times (\lambda/4)}{\lambda} \right) \right] = 17.89$$

$$D \text{ in dB} = 10 \log(17.89) = 12.526 \text{ dB}.$$

Directivity of Endfire Array with increased Directivity.

(19)

[Hansen-Woodward End-fire array]

For endfire array with increased directivity and minimum radiation in $\theta = 0^\circ$ direction, the radiation intensity for small spacing of elements ($d \ll \lambda$) is given by $U_0 = \frac{1}{npd} \left(\frac{\pi}{2}\right)^2 \left[\frac{\pi}{2} + \frac{2}{\pi} - 1.81r\right]$

$U_0 = \frac{0.878}{npd}$ multiplying numerator & denominator quantities by 2π

$$U_0 = \frac{0.878 \times 2\pi}{npd \times 2\pi} = \frac{1.756\pi}{2\pi npd} = \frac{1.756}{\pi} \left(\frac{\pi}{2npd}\right)$$

$$U_0 = 0.559 \left(\frac{\pi}{2npd}\right)$$

Directivity is given by

$$D = \frac{U_{max}}{U_0} = \frac{1}{0.559 \left(\frac{\pi}{2npd}\right)}$$

$$D = \frac{1}{0.559} \left(\frac{2npd}{\pi}\right)$$

$$\left[d = \frac{2\pi}{\lambda}\right]$$

$$= 1.789 \left[\frac{2n \left(\frac{2\pi}{\lambda}\right) d}{\pi} \right] = 1.789 \left[\frac{4nd}{\lambda} \right]$$

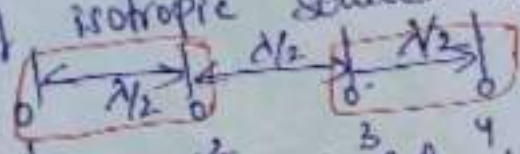
$$D = 1.789 \left[4 \left(\frac{n}{\lambda}\right) \right]$$

$$[L = (n-1)d \approx nd]$$

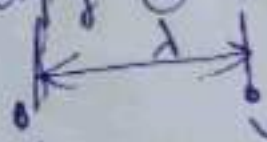
Pattern multiplication of 4 (element) pt sources.

→ with the help of this method, it is possible to sketch the radiation pattern of the arrays easily.

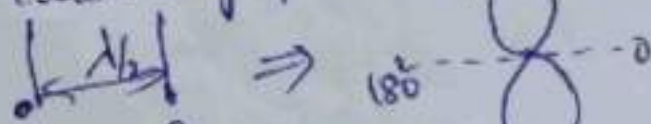
Array of isotropic sources separated by $\lambda/2$



We can consider group of 1 & 2 pts. Resultant is X.
group of 3 & 4 pt Resultant is Y.

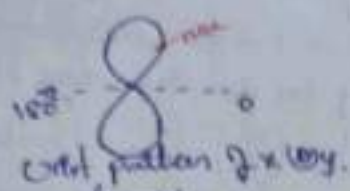


→ we will be calculating pattern of 'X'. i.e. group pattern of 1 & 2



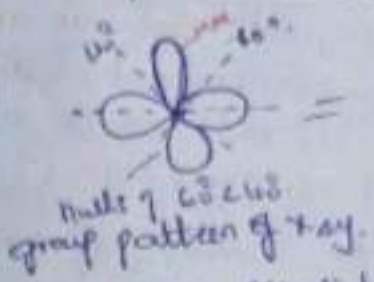
i.e. having nulls at 180°

Resultant pattern $E = \text{Unit pattern} \times (\cos y) \otimes \text{group pattern of } \cos y$



Radiation pattern of two antennas spaced at distance $\lambda/2$ and fed with 0°

$\otimes$

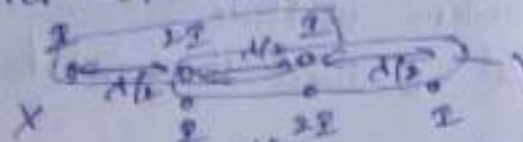


Rp of $\lambda/2$ distance



Resultant pattern
[max at 0 and 180 degrees, nulls at 90 and 270 degrees]

For unit binomial array.



Excitation with magnitude

$$E = \cos^3\left(\frac{\pi}{2} \cos \phi\right)$$

Resultant pattern = Unit pattern of $x(y) \otimes$ group pattern of $x \otimes y$.

$$180^\circ - 8 - 0 \oplus 180^\circ - 8 - 0 = 180^\circ - 8 - 0$$

There are no minor lobes

⇒ In general Electric field $E = \cos^{n-1}\left(\frac{\pi}{2} \cos \phi\right)$ where $n = 1, 2, \dots$ = no of elements

By Pascal's theorem (or) triangle calculate magnitude of elements.

Pascal's triangle giving coefficients of the binomial series.

						$n=1$
		1				$n=2$
	1		1			$n=3$
	1	2	1			$n=4$
	1	3	3	1		$n=5$
	1	4	6	4	1	$n=6$
	1	5	10	10	5	

Length of binomial array $L = (n-1)\lambda/2$

$$\text{HPBW} = \frac{1.06}{\sqrt{n-1}} = \frac{1.06}{\sqrt{24/2}} = 0.45 \text{ rad}$$

$$\Rightarrow \text{Directivity } D = 1.77\sqrt{n} = 1.77\sqrt{12} = 6.77 \text{ dB}$$

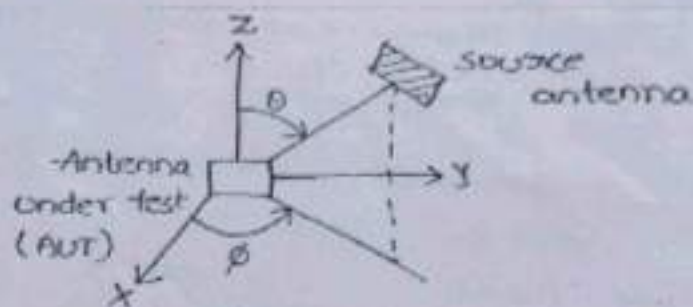
we can calculate magnitude of broad-beamed elements

$$180^\circ - 8 - 0 \oplus 180^\circ - 8 - 0 = 180^\circ - 8 - 0$$

Antenna Measurements

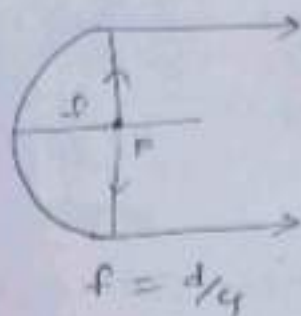
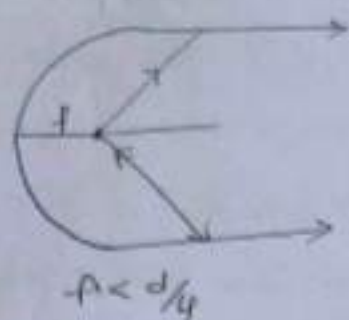
Basic Concept of Antenna Measurements:

In general, the important measurement parameters of the antenna are gain, directivity, radiation pattern etc.



TYPICAL CONFIGURATION UP FOR MEASUREMENT OF RADIATION PROPERTIES.

The Antenna Under Test (AUT) is considered to be located at origin of the coordinate system. The source antenna is placed at different locations with respect to the AUT. Note that the source antenna may be transmitting or receiving. At different locations the number of samples of the pattern are obtained. To achieve different locations, generally AUT is rotated. To achieve sharp sample of pattern, it is necessary that there exists single direct signal path b/w the AUT and source antenna.



When the focal point lies on the plane of the open mouth of the paraboloid by the geometry the focal length f is one fourth of the open mouth diameter d . This condition gives maximum gain pencil shaped radiation equal in horizontal and vertical plane.

When the focal length is too large, the focal point lies beyond the open mouth of the paraboloid as shown in fig above. Here it is difficult to direct all the radiations from the source on the reflector.

Note For practical applications, the value of the focal length to diameter ratio lies between 0.25 and 0.5

Practically it is observed that some of the rays are not fully captured by reflector, such non-captured rays form 'Spill over'.

While receiving spill over, the noise pick up increases which is troublesome. In addition to this, few radiations originated from the primary radiations are observed in forward direction. Such radiations get added with desired parallel beam. This is called 'back lobe radiation'.

Reciprocal Relationship between Transmitting and Receiving properties of Antenna:

Generally antenna can act either as a transmitter or receiver. There exists a reciprocal relationship between the transmitting and receiving properties of the antenna. This reciprocal relationship is very useful in the antenna measurements.

It is necessary to study two important consequences helpful in antenna measurements

The consequences are,

1. The transmitting and receiving patterns of antenna are same.
2. The power flow is the same in transmitting and receiving mode.

Thus antenna under test discussed earlier in the section can be used in either transmitting or receiving mode. When the AUT is used in a huge transmitter or receiver, the direction of the signal can be defined easily. Practically while using reciprocity relationship following conditions must get fulfilled.

1. The emfs at the terminals of the transmitting and receiving antennas should be of same frequency.
2. The power flow should be equal to that due to matched impedances.
3. The media should be linear, isotropic and passive.

Sources of Error in Antenna Measurement:

1. Error due to finite measurement distance

When the distance between the antennas is very small, then the field received by the AOT at different points will be with different phases causing quadratic phase error. The quadratic phase error affects by reducing the measured gain and increasing the side lobe as compared to the ideal uniform plane wave condition.

Due to the small distances between the antennas, the amplitude gets affected. The amplitude errors are of two types transverse amplitude error and longitudinal amplitude error. In transverse plane, the amplitude of the field is small

at the edges of the AUT, while slightly greater away from the edges. This causes the transverse amplitude errors.

2. Reflections from surroundings: The reflections from

Surroundings is another important source of the error because reflections cause amplitude ripple as well as phase ripple.

The ripples occur in a region, due to the interference between the direct wave and reflected wave.

3. Errors due to coupling in the reactive Near field:

The reactive near field causes significant error at low frequencies. If the distance is greater than 10λ , then the coupling is negligible.

4. Errors due to misalignment of antenna: Basically

the antenna measurement is a 3-dimensional vector measurement so any misalignment of the source antenna causes amplitude errors. Due to the misalignment the pattern cut can not be properly taken.

5. Errors due to manmade interface: On out-

door ranges, when the man-made interference

gets coupled with receiver at the frequency same as the measurement frequency or any other frequency harmonic distortion takes place.

On indoor ranges, in anechoic chambers, the reflections from the walls, floor and ceiling are significant.

6. Errors due to atmospheric effects: Due to the atmospheric effects, such as variation of refractive index of atmosphere, multihop propagation takes place which finally results in significant amplitude variation during measurement. At higher frequencies, the attenuation of the atmosphere is very high which results in amplitude variation.

7. Errors due to cables: If the cables used for the connection do not have proper shielding, leakage occurs and the cables act as antenna producing measurement errors. The incorrect use of cables also cause errors.

8. Errors due to impedance mismatch: If the antenna impedance is not properly matched with the instrument impedance, errors occur in

the gain measurement.

9. Factors due to Imperfections of Instruments:

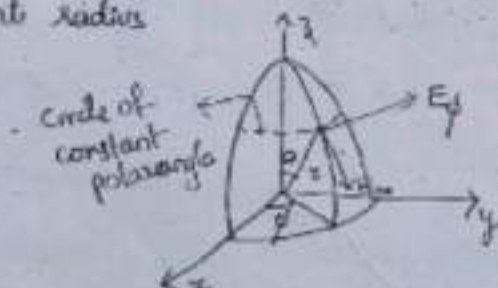
Due to the imperfections of the instruments in the measurements such as transmitters, receivers, positioners etc., the measurement error occurs.

Measurement of Radiation pattern:-

The Radiation capabilities of an antenna are characterised by the characteristics of an antenna such as the

- (1) Radiation pattern [Amplitude and phase pattern]
- (2) polarization
- (3) gain.

→ All these quantities are measured on the surface of a sphere with constant radius.



→ Basically for representation of a point on the surface, only θ and ϕ specifications are sufficient because of a sphere with constant radius is considered.

→ Thus the radiation characteristics of the antenna is a function of θ and ϕ for constant radius and frequency is called radiation pattern of an antenna.

for horizontal antenna following patterns are required.

- (i) The ϕ component of electric field as a function of ϕ measured in $x-y$ plane ($\theta=90^\circ$). The field component can be then represented as $E_\phi(\theta=90^\circ, \phi)$ and it is called E -plane pattern.
- (ii) The θ component of electric field as a function of θ measured in $x-z$ plane ($\phi=90^\circ$). The field component can be then represented as $E_\theta(\theta, \phi=90^\circ)$ and it is called H -plane pattern.

for vertical antenna

- (i) The θ component of electric field as a function of ϕ measured in $x-y$ plane ($\theta=90^\circ$). The field component can be represented as $E_\theta(\theta=90^\circ, \phi)$ and it is called H -plane pattern.
- (ii) The ϕ component of the electric field as a function of ϕ measured in $x-z$ plane ($\theta=90^\circ$). The field component can be represented as $E_\phi(\theta=90^\circ, \phi)$ and it is called E -plane pattern.

Measurement of Gain of an antenna:

- The performance of any antenna can be decided on orientation & in terms of figure of merit i.e. gain of an antenna.
- Depending upon the frequency of operation various methods can be used for the measurement of gain of an antenna.

Basically, there are two standard methods used for the measurement of gain of an antenna.

- (i) Gain-transfer method or direct comparison method.
- (ii) Absolute-gain method.

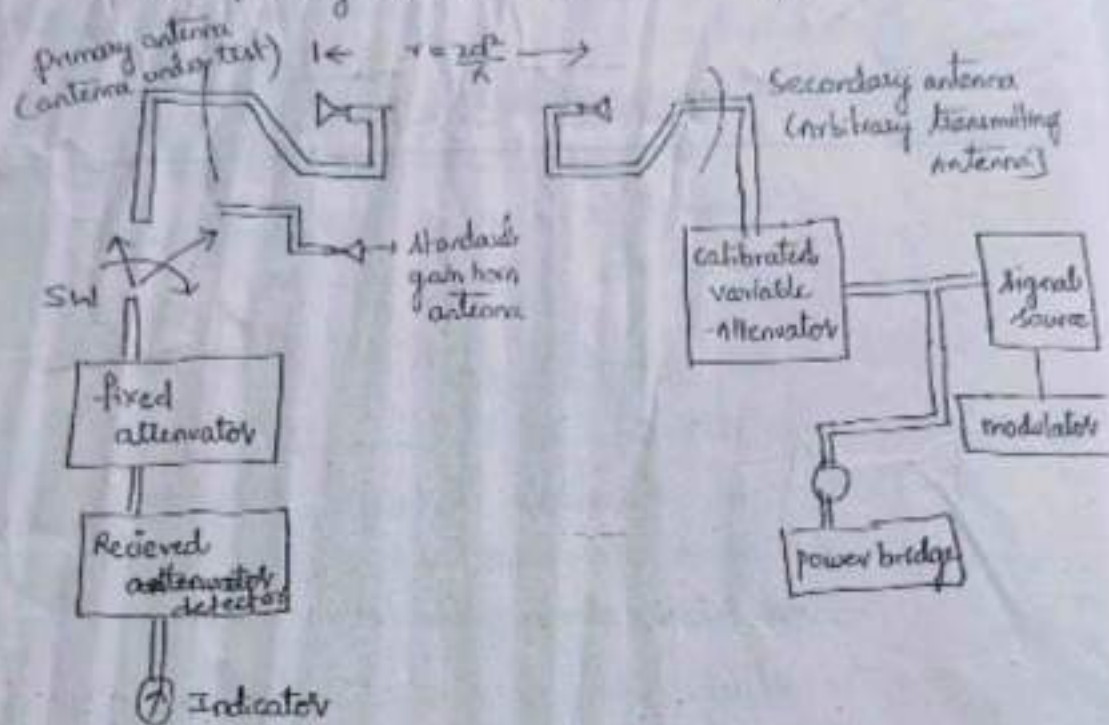
Gain measurement by Direct comparison method.

- The gain measurement is done by comparing the strengths of the signals transmitted or received by the antenna under test and the standard gain antenna.
- The antenna whose gain is accurately known and can be used for the gain measurement of other antenna is called standard gain antenna.
- At high frequency, the universally accepted standard gain antenna is the horn antenna.

Procedure

- This method uses two antennas termed as primary antenna and secondary antenna. The secondary antenna is arbitrary transmitting antenna.

- the knowledge of gain of the secondary antenna is not necessary
- The primary antenna consists two different antennas separated through a switch sw.
- The first primary antenna is the standard gain antenna (horn antenna)



Set up of gain measurement by gain comparison method

- The two primary antennas are located with sufficient distance of separation in b/w. so to avoid interference and coupling b/w the two antennas.
- while the primary and secondary antennas are separated by with a distance greater than or equal to $\frac{2d^2}{\lambda}$ to minimize the reflection b/w them to great extent.

→ To ensure almost frequency stability at the transmitter, the power bridge circuit is used.

The gain measurement by the gain-comparison method is two step procedure.

1) through the switch SW, the standard gain antenna is connected to the receiver. The antenna is adjusted in the direction of the secondary antenna to get the maximum signal intensity.

→ The input connected to the secondary or transmitting antenna is adjusted to required level.

→ For this input corresponding primary antenna reading at the receiver is recorded.

→ Corresponding attenuator and power bridge readings are recorded as A_1 and P_1 .

2) Secondly the antenna under test is connected to the receiver by changing the position of the switch SW. To get the same reading at the receiver, the same attenuator is adjusted. Then corresponding attenuator and power bridge readings are recorded as A_2 and P_2 .

Case 2:- If $P_1 = P_2$, then no correction needed to be applied and the gain of the subject antenna under test is given by

$$\text{Power gain} = G_p = \frac{A_2}{A_1}$$

Taking log on both sides

$$\log_{10} G_p = \log_{10} \left(\frac{A_2}{A_1} \right)$$

$$G_{p(dB)} = A_{2(dB)} - A_{1(dB)}$$

Case II :- If $P_1 \neq P_2$, then the correction need to be included

$$\frac{P_1}{P_2} = r, \text{ then}$$

$$\log_{10} \frac{P_1}{P_2} = r(dB)$$

Hence power gain is given by

$$G = G_p \times \frac{P_1}{P_2} = \frac{A_2}{A_1} \frac{P_1}{P_2}$$

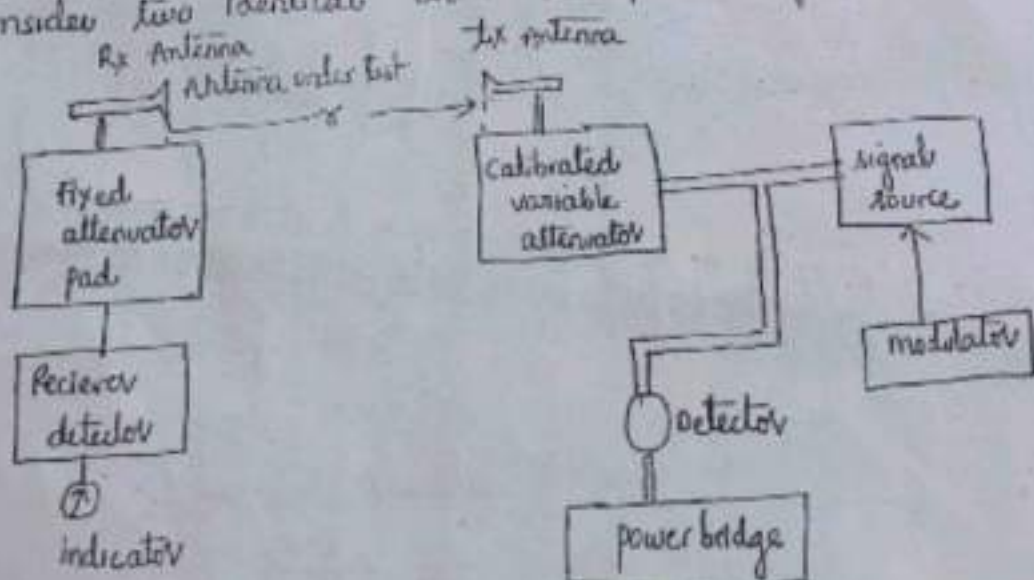
$$G = G_p \frac{P_1}{P_2}$$

$$\log G = \log G_p + \log_{10} (P_1/P_2)$$

$$G_{(dB)} = G_{p(dB)} + P_1/P_2(dB)$$

Measurement of absolute gain

Consider two identical antennas separated by distance r



Let the transmitted power be denoted by P_t and the received power by P_r . Let the effective apertures of the transmitting and receiving antennas be A_{et} and A_{er} respectively.

→ Two Antennas are identical identical then

$$A_{et} = A_{er} = \frac{G_0 \lambda^2}{4\pi}$$

From this equation, we can write

$$\frac{P_r}{P_t} = \frac{A_{er} \cdot A_{et}}{\lambda^2 \cdot r^2} = \left[\frac{G_0 \lambda^2}{4\pi} \right] \left[\frac{G_0 \lambda^2}{4\pi} \right] \frac{1}{\lambda^2 \cdot r^2}$$

$$\frac{P_r}{P_t} = \frac{4\pi/r}{\lambda} \left[\frac{G_0 \lambda}{4\pi r} \right]^2$$

$$\frac{G_0 \lambda}{4\pi r} = \sqrt{\frac{P_r}{P_t}}$$

$$G_0 = \frac{4\pi r}{\lambda} \sqrt{\frac{P_r}{P_t}}$$

By knowing wave length λ , distance b/w two antennas r , and measuring the radiated and received powers, the absolute gain of the antenna is obtained.

Measurement of Directivity

- Directivity is obtained from the radiation pattern of the antenna
- The Directivity of antenna is defined as the ratio of maximum power density to the avg power radiated

$$G_{\text{max}} = \frac{E_{\text{max}}}{\frac{P_{\text{rad}}}{4\pi r^2}} = D$$

Basically the directivity of an antenna is a dimensionless quantity. The directivity can be expressed in terms of the electric field intensity as

$$D = G_{\text{max}} = \frac{4\pi |E_{\text{max}}|^2}{\int_0^{2\pi} \int_0^\pi |E(\theta, \phi)|^2 \sin\theta d\theta d\phi}$$

$$D = G_{\text{max}} = \frac{4\pi}{\int_0^{2\pi} \int_0^\pi \frac{|E(\theta, \phi)|^2}{|E_{\text{max}}|^2} \sin\theta d\theta d\phi}$$

$$D = G_{\text{max}} = \frac{4\pi}{\int_0^{2\pi} \int_0^\pi f(\theta, \phi) \sin\theta d\theta d\phi}$$

where $f(\theta, \phi)$ is the relative radiation intensity as a function of space angles θ and ϕ

$$D = \frac{41,253}{\theta_1 \cdot \theta_2} \quad \text{where}$$

$\theta_1 \rightarrow$ HPBW of E-plane (H-plane)

$\theta_2 \rightarrow$ HPBW of H-plane (E-plane)

$$\text{or } D = \frac{72,815}{\theta_1^2 + \theta_2^2}$$

$\rightarrow$ The main drawback of this method is the least accuracy in the measurement

Near field & Far field

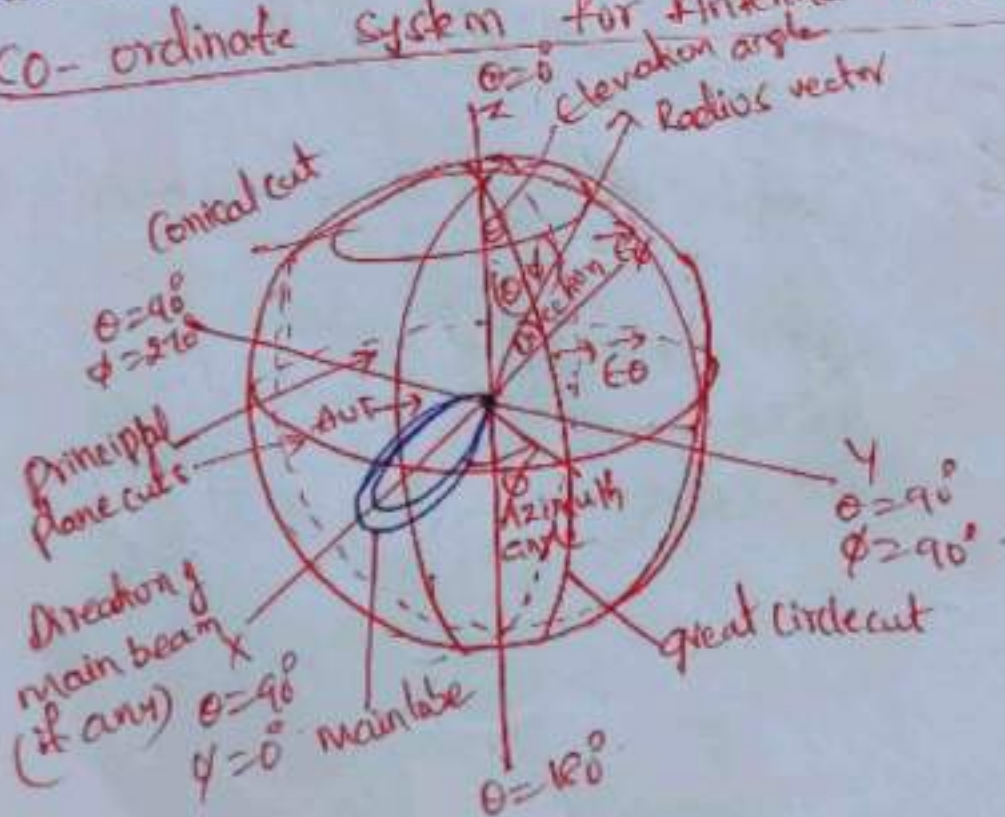
The advantages of the measurement in the far field are

- 1) Coupling and multiple reflections are least significant in the far field region.
- 2) In the far field region, only power measurement services the purpose for obtaining power pattern.
- 3) At any pt in the rest far field region, the field pattern measured is valid.

→ But the major drawback of the farfield measurement is that it requires large distance b/w Tx & Rx antennas. Distance increases far field region suffers by atmospheric attenuation.

→ Near field region is located very close to the Ant. Also due to the mutual impedance as a result of the reactive coupling b/w two antennas, the measurement becomes complicated. Hence practically, the reactive nearfield region is also not used for antenna measurement.

Co-ordinate system for Antenna Measurement.



- angle measured from the z -axis is called elevation angle and it is denoted by θ .
- The angle measured from the projection of the radio vector to the horizontal $x-y$ plane is called azimuth angle which is denoted by ϕ .
- Depending upon the mechanical structure of the antenna the coordinate system is defined such that the peak radiation takes place along z -axis in general.
- when the source antenna is moved along lines of const θ , the cuts obtained are called Conical cuts or θ -cuts.
- when source antenna is moved along lines of const ϕ , the cuts obtained are called great circle cuts or ϕ -cuts. If the cut is taken along the equator with $\theta = \pi/2$ then such a cut is called θ -cut as well as ϕ -cut.
- The two principle plane cuts are the orthogonal great circle cuts through the axis of the main lobe.



Fringing effect in MSA

→ Fringing effect is main role in designing of antenna.

→ Because of fringing effect it radiation goes in air.

→ when fed antenna w/o ground plane EM waves coupling from dielectric to ground plane when it comes from edges to ground it goes into air.

→ Because of fringing MSA radiates in the space. & thus can increase the fringing effect by 3 ways. $[E_r \rightarrow \text{Permittivity of dielectric substrate}]$

1) To increase the fringing by 3 ways

1) Reducing the E_r increase fringing

2) Increase height of substrate we can ↑ fringing

3) By increasing w we can ↑ fringing

Designing of MSA (Rectangular)

⇒ Effective length $L_{eff} = \frac{L + 2\Delta L}{2}$ $L = L_{eff} - 2\Delta L$

⇒ operating freq $f = \frac{c}{2L\sqrt{E_r}} = \frac{c}{2L_{eff}\sqrt{E_{eff}}}$

$$E_{eff} = \frac{E_r + 1}{2} + \frac{E_r - 1}{2} \left[1 + \frac{h}{w} \right]^{1/2}$$

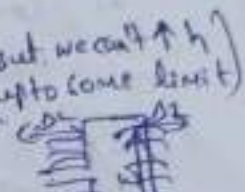
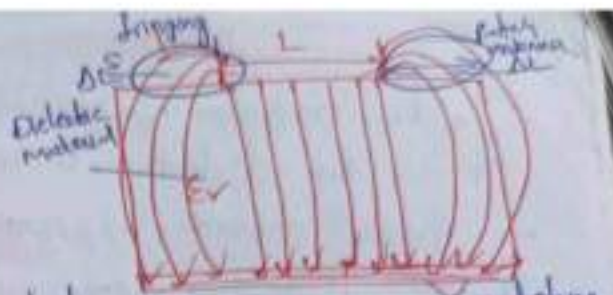
⇒ $\Delta L = h(0.412) \left(\frac{E_{eff} + 0.3}{E_{eff} - 0.258} \right) \left(\frac{\frac{w}{h} + 0.264}{\frac{w}{h} + 0.8} \right)$

⇒ Determine width it doesn't depend on the fringing

$$w = \frac{c}{2fr} \sqrt{\frac{2}{E_r + 1}}$$

⇒ Find Actual length L $L_{eff} = L + 2\Delta L \Rightarrow L = L_{eff} - 2\Delta L$

$$L = \frac{c}{2fr\sqrt{E_{eff}}} - 2\Delta L$$



2) Probe Feed MSA

→ outer conductor is connected to ground plane below the dielectric material.

→ Inner conductor is connected to patch

→ to provide impedance matching inner conductor moving from center

TOP VIEW



SIDE VIEW



Advantages

- 1) Easy to fabricate
- 2) Easy to match
- 3) Simple to model
- 4) Low spurious radiation.

disadvantages

- 1) Low Bandwidth (2%)
- 2) cross polarization (for impedance matching shift inner conductor from center)

② Proximity Coupled Feed MSA

ϵ_{r1} → one dielectric material

ϵ_{r2} → 2nd "

→ patch mounted on top of dielectric material

→ feed line that is send which is 2 dielectric material

→ Below 2nd dielectric material there is ground plane

→ Impedance matching provide by change length & width of feed line.

→ Design will be Symmetrical about axis.

Advantages

- 1) Low cross polarization
- 2) High Bandwidth (13%)
- 3) Low spurious radiation

disadvantages

- 1) Difficult to fabricate (because two $\epsilon_{r1}, \epsilon_{r2}$)
- 2) cost is more



④ Aperture Coupled Feed MSA

→ There will be slot coupled to line ground dw

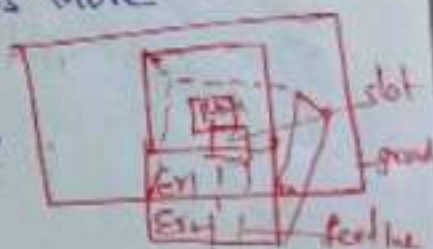
→ Symmetrical w.r.t to axis of line

→ Advantages

- 1) Low cross polarization
- 2) Easy to model
- 3) moderate spurious radiation

Disadvantages

- 1) It is most difficult to fabricate
- 2) It has narrow bandwidth.



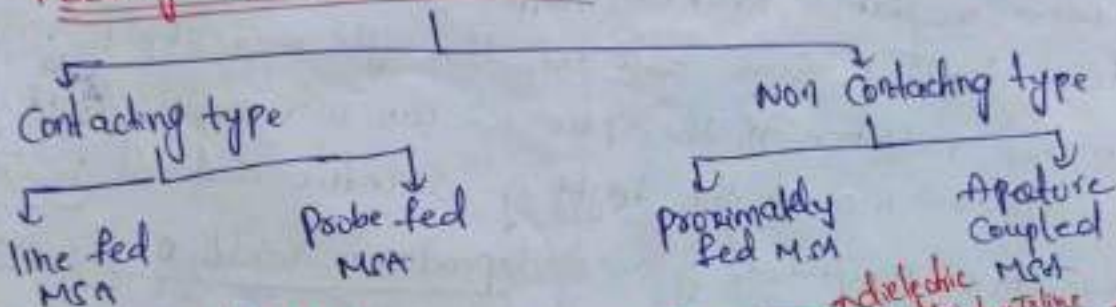
$$E_\theta = \frac{\sin\left[\frac{k w \sin\theta \sin\phi}{2}\right]}{\frac{k w \sin\theta \sin\phi}{2}} \cos\left(\frac{k L}{2} \sin\theta \cos\phi\right) \cos\phi$$

$$E_\phi = -\frac{\sin\left[\frac{k w \sin\theta \sin\phi}{2}\right]}{\frac{k w \sin\theta \sin\phi}{2}} \cos\left(\frac{k L}{2} \sin\theta \cos\phi\right) \sin\phi$$

width controls

- Based on width electric field value changed.
- length value l is fixed $L = \lambda/2$
- w controls I/P impedance
- $\text{I/P impedance} \propto \frac{1}{w}$ $w \uparrow \text{imp} \downarrow, w \downarrow \text{imp} \uparrow$
- w controls the bandwidth of antenna (Bandwidth $\propto w$)
- $\text{It controls Radiation pattern.}$

Feeding methods in MSA

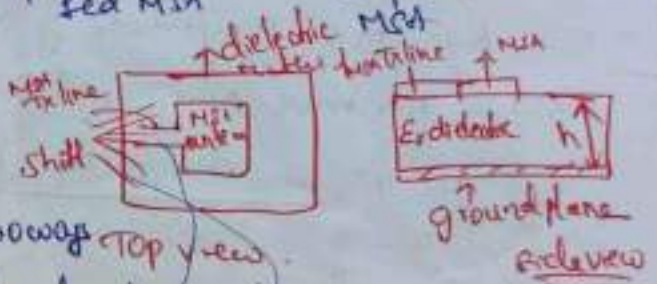


① line fed MSA

- Microstrip mounted on the dielectric material
- For impedance matching there are two ways
- 1) By shifting line position in this direction $\approx$
- 2) Insect feeding (not symmetrical to axis of MSA)
- Line Connected directly to the patch antenna.

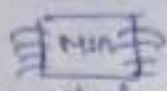
Advantages

- 1) easy to fabricate
- 2) It is simple to match
- 3) Simple to model



- #### Disadvantages
- 1) It has low Bandwidth (2%)
 - 2) It has cross polarization.

Rectangular microstrip antenna

- It is having microstrip patch placed on dielectric material support with ground plane.
- Microstrip TX line feed to this microstrip antenna
- Impedance matching of MSA by varying position of microstrip line. not connected with center line it would be somewhere.
- to provide impedance matching by shifting microstrip line like $\leftarrow$ w.r.t to center.
- dimension L (length) of a patch is $L = d/2$ d is the width of MSA. radiation happens from this side by 
- Height of dielectric material justify the value of magnitude which is the radiation higher the height of dielectric substrate.
- More radiation which will happen with increase the dielectric material height & some extended then after MSA stop to radiate.
- more radiation in the space we can increase the height of dielectric.
- we can't increase the height of dielectric material ($h < 0.05\lambda$)
- Radiation magnitude of MSA depends on width of the MSA.
- If you have more width radiation will increase.
- Impedance matching depends on width of MSA.

Analysis of MSA

operating freq $f_0 = \frac{c}{2L\sqrt{\epsilon_r}}$

Instead of ϵ_r we use ϵ_{eff}
electric field of MSA $E_0 = \sin$

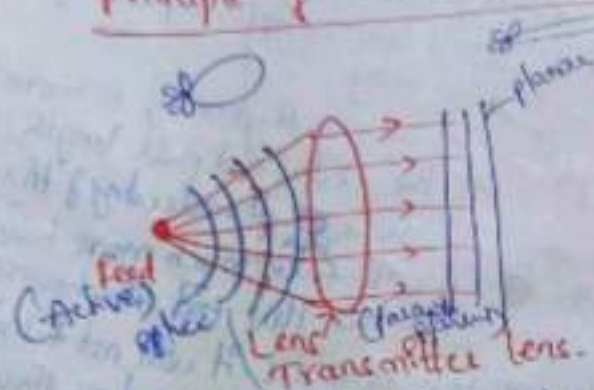
Lens Antennas

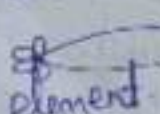
Basics of Lens antenna.

- It is an antenna which consists an electromagnetic lens with feed
- It converges spherical wavefront to planar wavefront and planar wavefront to focus at feed.
- It is typically thicker, heavier and more difficult to construct
- It has one advantage over Reflector antenna, blockage is not happening.

Principle of Lens Antenna.

By using lens antenna improve the radiation characteristics.



- 1) This lens antenna is used as Tx. This feed provide spherical wavefronts. going through lens antenna. This lens convert to planar waves.
- 2) If you see R. char like this . If we use after lens using R. Conner is li. . lens is parasitic element we don't provide feed to lens.
- 3) Because it converts spherical to planar at the Tx lens.
- 5) After using lens the directivity of beam increases.

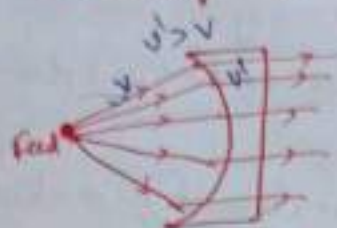
This lens antenna is used as Rx when signal is coming from very long distance. signal will be a planar wave passing through a lens antenna it will be converting planar wave to spherical wave front. ^{But this feed aperture is less even though highly point} let Receiver signal getting.

3) Lens antenna

Here the operation of Rx lens it converts planar waves to spherical waves even the aperture size is less. wave to be focus to feed at feed.

Types of Lens Antenna

Conducting type & Type.



1) This lens working as accelerated when EM waves propagating through conducting lens type material. If v' say velocity v' propagating through space the velocity is v .

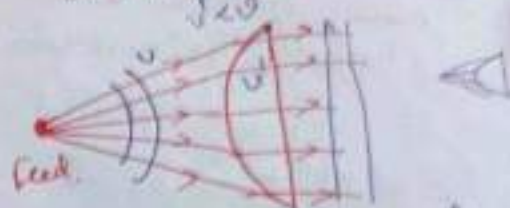
→ If propagating through this material the velocity increases i.e. $v' > v$.

So the wave propagating (passing) through material is getting acceleration. The need of acceleration why the need of acceleration? The total path length which is imaging the path length is high. ^{because there is low propagation velocity} We need to accelerated need to spherical wavefront to planar wavefronts.

→ velocity of wave increases when passing through this material.

→ There is higher path length ^{there is} wave need to provide ^{more} acceleration. It will ^(higher acceleration) convert spherical wavefronts to planar wavefronts.

Dielectric type



2) If v' say velocity inside is v' outside is v .

3) If it is this $v' < v$.

② In that case we observe the path length of this wave is low compare to path length of this way.

③ If we see thickness of is more at the center and path length is less ^{than} decelerating of this wave.

④ It will take much more time while at this end its having less thickness it will not taking much time to propagating through material. Ultimately the

⑤ spherical wavefront converts to planar wavefronts.

⑥ $v' < v$ means

thickness is more in middle and path length is less.

Zoned lens antenna. when we use lens antenna it is very bulky. So it reduce width of lens (is called) zoning. So we cutting width of lens we can reduce size.

→ In this case this lens are working similar to the lens which we see in last topic.

→ Now



Advantages of Lens Antenna

- No blockage due to feed and feed support (primary feeding is not coming, secondary wave lens)
- more EM can be received with respect to parabolic reflector
- Low noise
- Higher gain compared to Reflector antenna.

Disadvantages

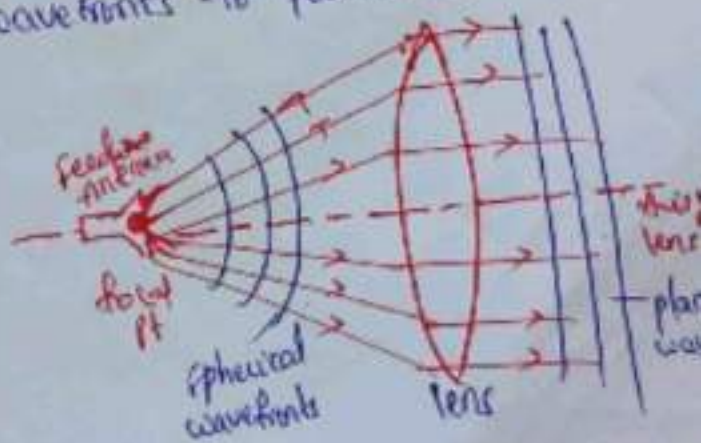
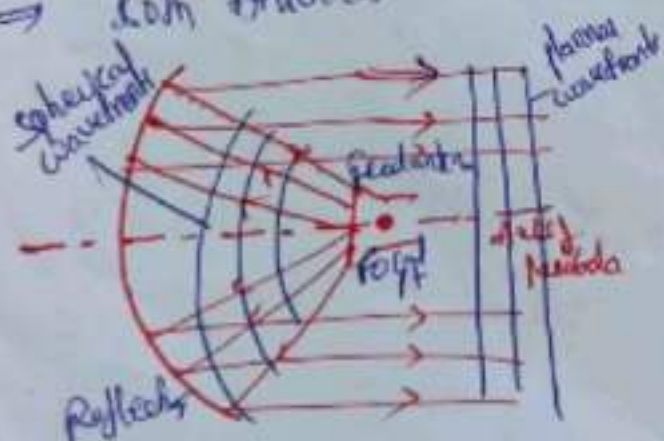
- Lens are heavy.
- Complex to construct
- Costlier compared to Reflector Antenna.

Applications

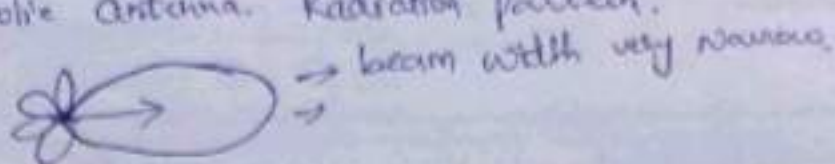
- 1) For narrow beam width
- 2) microwave transmission.

Lens Antenna

- It operates based on Refraction.
- It doesn't have blockage issue.
- It has issue of cross-polarization.
- It doesn't have isolation.
- It is complex structure.
- It's weight is high.
- Cost is high.
- Both antennas convert spherical wavefronts to planar wavefronts.



Parabolic Antenna. Radiation pattern.



Applications

- 1) Radio astronomy, microwave freq, Satellitecom
- Deep space communication.

Examples on Reflector Antenna

For parabolic reflector with uniform illumination

$$1) \text{ FNBW} \approx \frac{140\lambda}{D} \text{ (deg)}$$

$$2) \text{ HPBW} \approx \frac{58\lambda}{D} \text{ (deg)}$$

$$3) \text{ Power gain } G_p \approx 6 \left(\frac{D}{\lambda} \right)^2$$

$$4) \text{ Directivity } D = \frac{G}{k}$$

Calculate gain, FNBW and HPBW of a parabolic reflector of 2m diameter at 5GHz.

for $D = 2\text{m}$, $f = 5\text{GHz}$, $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{5 \times 10^9} = 0.06\text{m}$

$$1) \text{ gain } G = 6 \left(\frac{D}{\lambda} \right)^2 \Rightarrow 6 \left(\frac{2}{0.06} \right)^2 = 6666.67$$

$$2) \text{ FNBW} \approx \frac{140\lambda}{D} \Rightarrow \frac{140 \times 0.06}{2} = 4.2 \text{ deg}$$

$$3) \text{ HPBW} \approx \frac{58\lambda}{D} \Rightarrow \frac{58 \times 0.06}{2} = 1.74 \text{ deg}$$

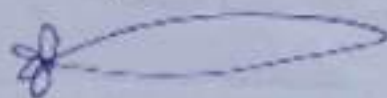
② Calculate gain, FNBW, HPBW of parabolic reflector of 10m diameter at 10GHz.

$D = 10\text{m}$, $f = 10\text{GHz}$. $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10 \times 10^9} = 0.03\text{m}$

$$1) G = 6 \left(\frac{D}{\lambda} \right)^2 \Rightarrow 6 \left(\frac{10}{0.03} \right)^2 = 6.67 \times 10^5$$

$$2) \text{ FNBW} \approx \frac{140\lambda}{D} \Rightarrow \frac{140 \times 0.03}{10} = 0.42 \text{ deg}$$

$$4) \text{ HPBW} \approx \frac{58\lambda}{D} \Rightarrow \frac{58 \times 0.03}{10} = 0.174 \text{ deg}$$





- It is highly directional Antenna
- It is used to very long distance communication, such as satellite communications
- It is applicable to microwave frequency (1-100GHz) and beyond that
- It consists two types of elements
 - 1) Active elements (Feed Antenna)
 - 2) Passive element (Reflector)

Types of Reflector Antenna

Antenna
Radiates
Plane wave
Rf

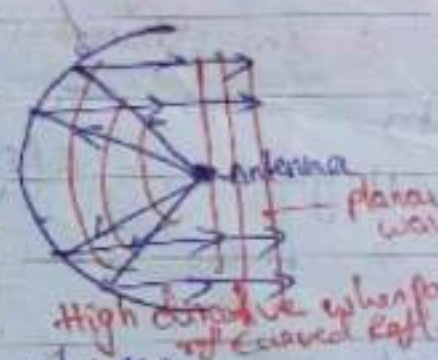


Plane Reflector



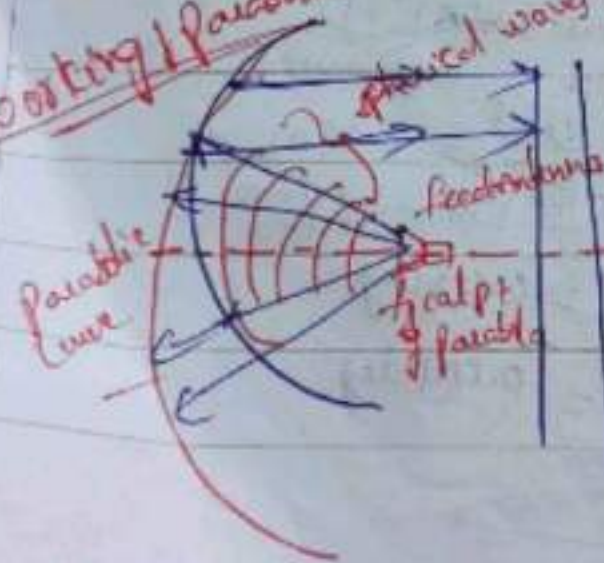
Corner Reflector

Curved Reflector



Based on Reflector shapes there are 2 types

Working of Parabolic

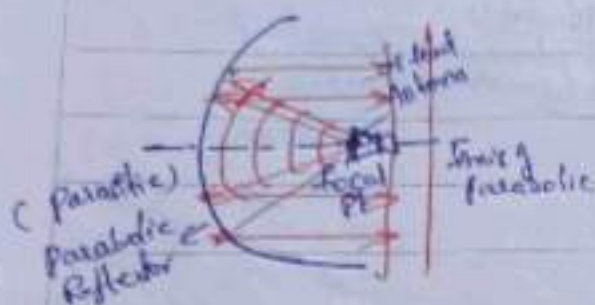


→ Parabolic Antenna converts spherical wavefronts into planar wavefronts
→ Due to that it is highly directional antenna.



Feeding mechanism of Parabolic Reflector

1) Center Fed Parabolic Reflector



Disadvantages
1) It is difficult to use for low noise applications due to isolation.

2) Blockage due to feed.

Advantages, it has less cross polarization.

2)

To overcome these two disadvantages by using offset feed.

2) Offset Feed P.R.

Advantages

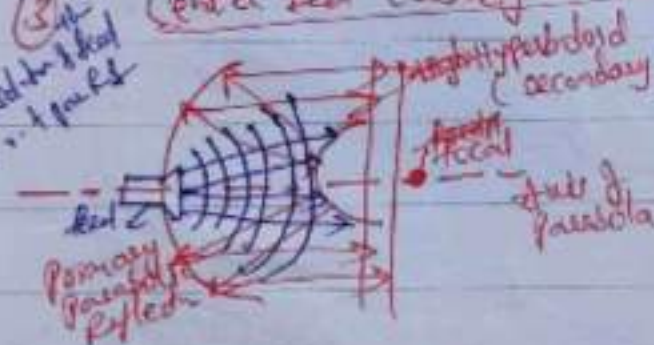
1) No blockage due to feed.



Disadvantages
1) Cross polarization
(Antenna design not symmetrical to axis)

3) Center Feed Cassegrain P.R.

Feed at focal point



Advantages

1) Isolation - feed leads it's use in low noise applications

2) Directivity is high.

3) Low cross polarization.

Disadvantages

1) Blockage due to secondary reflector

2)

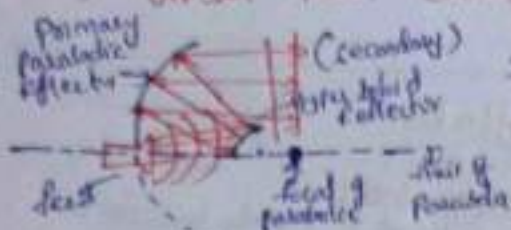
$$\frac{6 \times 10^3}{0.8} = \frac{5.81}{D}$$

$$\frac{6}{0.8} = \frac{140 \lambda}{D}$$



Name: _____

v) offset feed Cassegrain parabolic reflector



- Advantages
- 1) It provides isolation.
 - 2) No blockage due to secondary reflector.

- Disadvantages
- 1) Cross polarization due to not symmetrical to axis of parabola.



Ex Design a microstrip patch with dimensions w & L over a single substrate whose center freq is 10GHz the dielectric constant of the substrate is 10.2 and the height of the substrate is 0.127 cm.

Ans given data $f_r = 10\text{GHz}$, $\epsilon_r = 10.2$, $h = 1.27 \times 10^{-3}\text{m}$
 $w = ?$, $L = ?$

$$1) w = \frac{c}{2f_r \sqrt{\frac{\epsilon_r + 1}{2}}} = \frac{3 \times 10^8}{2 \times 10 \times 10^9 \sqrt{\frac{10.2 + 1}{2}}} = 6.33 \times 10^{-3}\text{m} \Rightarrow \underline{0.633\text{cm}}$$

$$\begin{aligned} 2) \epsilon_{eff} &= \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[1 + 12 \left(\frac{h}{w} \right) \right]^{-1/2} \\ &= \frac{10.2 + 1}{2} + \frac{10.2 - 1}{2} \left[1 + 12 \left(\frac{1.27 \times 10^{-3}}{6.33 \times 10^{-3}} \right) \right]^{-1/2} \\ &= 5.6 + 4.6 (1 + 2.4)^{-1/2} = \underline{8.09} \end{aligned}$$

$$\begin{aligned} 3) \Delta L &= h (0.412) \left(\frac{\epsilon_{eff} + 0.3}{\epsilon_{eff} - 0.258} \right) \left(\frac{\frac{w}{h} + 0.264}{\frac{w}{h} + 0.8} \right) \\ &= 1.27 \times 10^{-3} (0.412) \left(\frac{8.09 + 0.3}{8.09 - 0.258} \right) \left(\frac{\frac{6.33 \times 10^{-3}}{1.27 \times 10^{-3}} + 0.264}{\frac{6.33 \times 10^{-3}}{1.27 \times 10^{-3}} + 0.8} \right) \\ &= 1.27 \times \left(0.412 \times \frac{8.39}{7.832} \times \frac{5.25}{5.78} \right) \times 10^{-3} \end{aligned}$$

$$\Delta L = 0.5 \times 10^{-3} \Rightarrow \underline{5 \times 10^{-4}\text{m}} \text{ Ans.}$$

$$4) L_{eff} = \frac{c}{2f_r \sqrt{\epsilon_{eff}}} = \frac{3 \times 10^8}{2 \times 10 \times 10^9 \sqrt{8.09}} = 0.52 \times 10^{-2} = 5.2 \times 10^{-3}\text{m}$$

$$\begin{aligned} 5) \text{Actual length } L &= L_{eff} - 2\Delta L \\ &= 5.2 \times 10^{-3} - 2(5 \times 10^{-4}) \\ &= 5.2 \times 10^{-3} - 1 \times 10^{-3} \Rightarrow \underline{4.2 \times 10^{-3}\text{m}} \end{aligned}$$

$$L = \underline{0.42\text{cm}} \text{ Ans.}$$

Ant TV Unit

There are 3 antenna field zone

- 1) Reactive near field region
- 2) Radiating near field region
- 3) Far field region.

Reactive near field Region

→ It is that portion of the near field region immediately surrounding the antenna where in the reactive field predominates.

→ For most of the antennas, the outer boundary of this region is

$$R < 0.62 \sqrt{\frac{L^3}{\lambda}}$$

→ But for a very short dipole radiator the outer boundary

$$R < \lambda / (2\pi)$$

→ In general objects within this region will result in coupling with the antenna and distortion of the ultimately field antenna pattern.

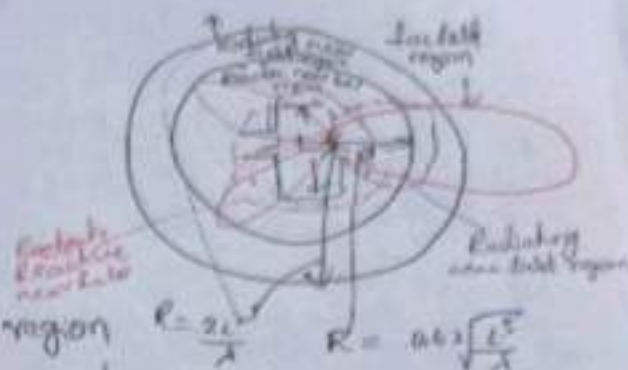
→ In general objects large conductor within this distance will couple with the antenna and 'disturb' it. The result can be an altered resonant freq, radiation resistance and Radiation pattern.

2) It is that region of the field of antenna b/w the reactive near field region and the far field region.

→ For this region, the distance from the antenna R is

$$0.62 \sqrt{\frac{L^3}{\lambda}} < R < \frac{2L^2}{\lambda}$$

This region is also called transition region.



Properties of this region are

- 1) The antenna pattern is taking shape but is not truly ^{completely} developed.
- 2) The radiation field predominates the reactive field.
- 3) The radiated wave front is still clarty curved.
- 4) $\vec{E}$ field & magnetic field ^{vectors} are not orthogonal.

③ Far-field

→ It is that region of the field of an antenna where the angular field distribution is essentially independent of the distance from the antenna.

→ For this region, the distance from the antenna R is

$$R > \frac{2L^2}{\lambda}$$

Properties of this region are

- 1) The wavefront becomes approximately planar.
- 2) The radiation pattern is completely formed and does ~~vary~~ vary with distance.
- 3) $\vec{E}$ & magnetic field vectors are orthogonal each & other.

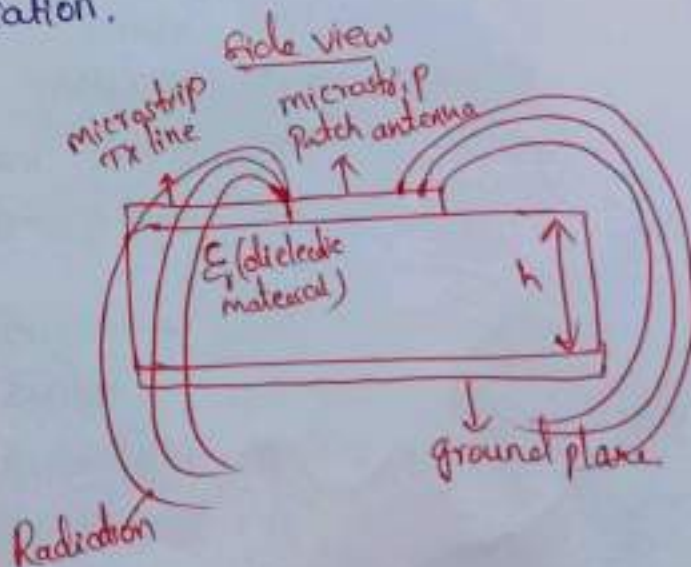
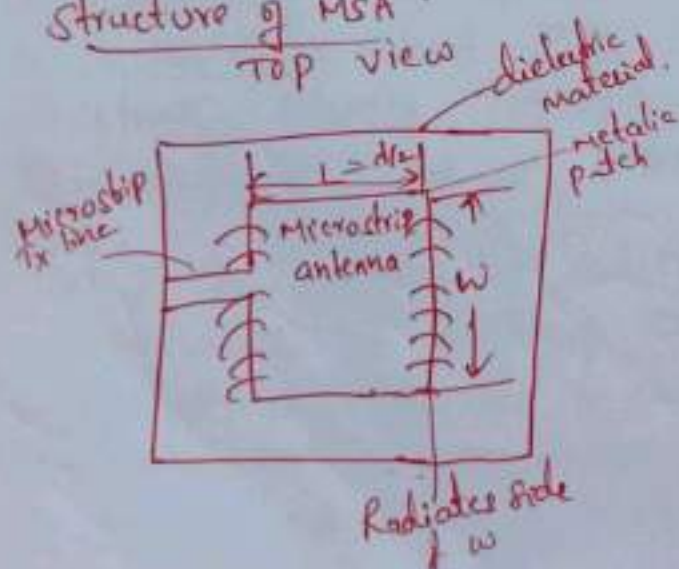
UNIT - III

VHF and UHF antennas - II

Microstrip Antennas Introduction :-

- patch antennas are assigned different names such as printed antennas, microstrip patch antennas (or) simply microstrip antennas (MSA).
- In telecommunication, a microstrip antenna means an antenna fabricated using microstrip techniques on a printed circuit board. It is a kind of internal antenna. They are used at microwave frequencies.
- microstrip antennas have become very popular in now-days.
- patch antennas can be directly printed onto a PCB board, these are becoming increasingly popular within the mobile phone market.
- They are low cost, have a low profile and are easily fabricated.
- It is a metallic patch placed on dielectric material and supported by ground plane.
- It could be easily fabricated on printed circuit board.
- It is most widely used antenna.
- Installation is very easy due to low size, weight, & cost.
- used in every mobile application.

Structure of MSA



Features of MSA

1) A patch antenna basically is a metal patch suspended over a ground plane. Patch antennas are simple to fabricate, easy to modify and closely related to microstrip antennas. These are constructed on a dielectric substrate, usually employing the same sort of lithographic patterning as used to fabricate printed circuit boards.

2) A patch antenna is a narrow band, wide-beam antenna fabricated by etching the antenna element pattern in metal trace bonded to an insulating dielectric substrate with a continuous metal layer bonded to the opposite side of the substrate which forms a ground plane.



UNIT - III

VHF and UHF antennas - II

- 1) It is suitable patch printed on substrate
- 2) It is a suspended by ground plane
- 3) It could be easily fabricated on PCB
- 4) It is most widely used antenna
- 5) Installation is very easy due to its low weight & cost
- 6) Used in many applications

Microstrip antennas : Introduction -

Patch antennas are assigned different names such as Printed antennas, microstrip patch antennas or simply microstrip antennas (MSA).

In telecommunication, a microstrip antenna means an antenna fabricated using microstrip techniques on a Printed circuit board. It is a kind of internal antenna. They are mostly used at microwave frequencies.

Microstrip antennas have become very popular in recent decades due to their thin planar profile which can be incorporated into the surfaces of consumer products, aircraft and missiles and their ease of fabrication using Printed circuit techniques.

Since patch antennas can be directly printed onto a circuit board, these are becoming increasingly popular within the mobile phone market. They are low cost, have a low profile and are easily fabricated.

Features of MSA :

1. A patch antenna basically is a metal patch suspended over a ground plane. Patch antennas are simple to fabricate, easy to modify and customize and closely related to microstrip antennas. These are constructed on a dielectric substrate, usually employing the same sort of lithographic patterning as used to fabricate printed circuit boards.
2. A patch antenna is a narrowband, wide-beam antenna fabricated by etching the antenna element pattern in metal trace bonded to an insulating dielectric substrate with a continuous metal layer bonded to the opposite side of the substrate which forms a ground plane.

3. One of the key drawbacks of such antennas is their narrow bandwidth. In order to achieve wider bandwidth, a relatively thick substrate is used.

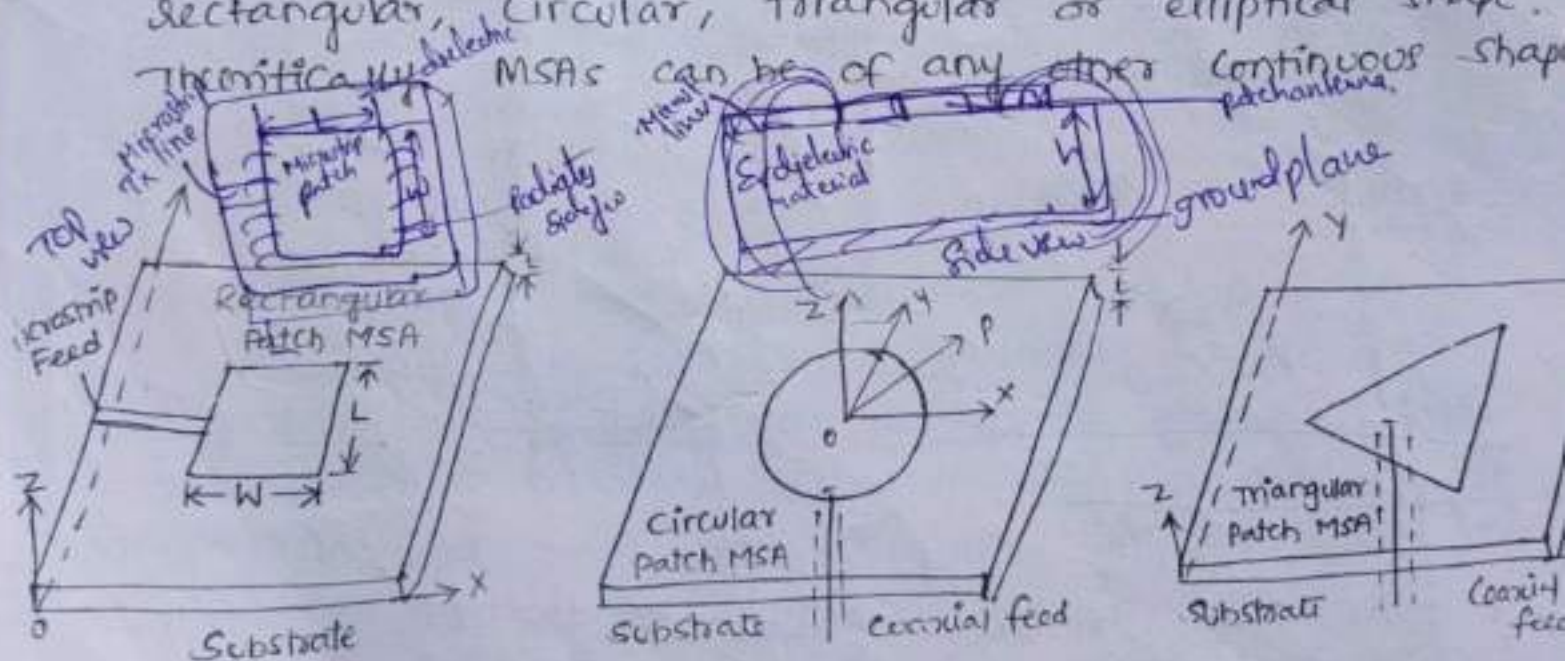
4. The microstrip antenna was first proposed by G. Deschamps in 1953. The proposed concept of microstrip antennas to transmit radio frequency signals could not gain much ground till the 1970s.

Further, the development of the printed circuit board (PCB), microwave techniques, and many kinds of low-attenuating media materials made the use of microstrip antennas more practical.

5. Microstrip patch antennas are often used where thickness and conformability to the surface of mount or platform are the key requirements.

The primary limitation of this type of antenna is the bandwidth, which is less than 5% for most single-substrate designs. However, a second substrate can be added to create a dual band design or a broadband design with a bandwidth of up to 35%.

6. The microstrip antennas may have a square, rectangular, circular, triangular or elliptical shape.



7. The size of microstrip antenna is inversely proportional to its frequency. At frequencies lower than for an AM radio at 1 MHz, the microstrip patch would be of the size of a football field. For a microstrip antenna designed to receive

an FM radio at 100 MHz, its length would be of the order of 1 meter which is still very large for any type of substrate. At X-band, the microstrip antenna size will be of the order of 1 cm

15

Advantages and limitations of microstrip antenna
Advantages: ① small size ② low weight ③ easy to fabricate ④ linear & circular polarization possible ⑤ it could be easily printed on a substrate

- Microstrip antennas are of light weight, smaller size and lesser volume. They operate at microwave frequencies where traditional antennas are not feasible to be designed.
- They are easily conformable to non-planar surface. They can be easily bolted or laminated to the metallic surface such as an aircraft, missile or automotive as they are designed to operate from the ground plane on the back of a printed circuit board.
- As the fabrication process involved in manufacturing the microstrip antennas is simple the complete production process is easy and very cheap.
- By mounting the MSA on a rigid surface, we get the MSA is most suitable option where mechanical robustness is required.
- Thickness and conformability - to the surface of a platform are the main requirements.

Disadvantages

- The MSAs low gain and low efficiency antennas.
- The MSAs have narrow bandwidth of operation. Moreover they have lower power handling capacity.
- The size of microstrip antenna is inversely proportional to frequency. They can be used only for very high frequencies only. Because for low frequencies the size of MSA is impractical.

4. The narrow bandwidth is the major drawback of the MSA. By inherent nature, the MSAs are resonant in nature with high Q -factor values. As $Q = \frac{f_r}{B.W}$ the bandwidth is narrow as Q is higher. (ii)

The bandwidth can be increased by increasing the thickness of the substrate at the cost of reduction in Q -factor value.

5. Increasing bandwidth by using any suitable method increases the complexity of design of the MSA. Limitations: 1) Narrow bandwidth 2) gain around (5 to 10 dB) by single patch 3) Radiation into half plane 4) low power handling capacity 5) It excites surface wave (unwanted)

Applications

① Military applications

(i) The high velocity aircrafts, space crafts, missiles, rockets require low profile, light weight antennas which can be conformally mounted to the exterior surfaces of these vehicles. The microstrip antennas are best suited for above applications.

(ii) Other areas where the microstrip antennas are used include application areas such as missile guiding, fuzeing, telemetry, satellite communication, radars, altimeters, GPS etc

② Space Applications

(i) The microstrip antennas are used in the space programs such as Earth limb measurement satellite (ELMS), International Sun earth explorer (ISEE)

Shuttle Imaging Radar (SIR) - A, B, C Series, (3)
 Solar Mesospheric Explorer (SME), Cosmic Background
 Explorer, GEOSTAR, SEASAT and Mars Pathfinder. (5)

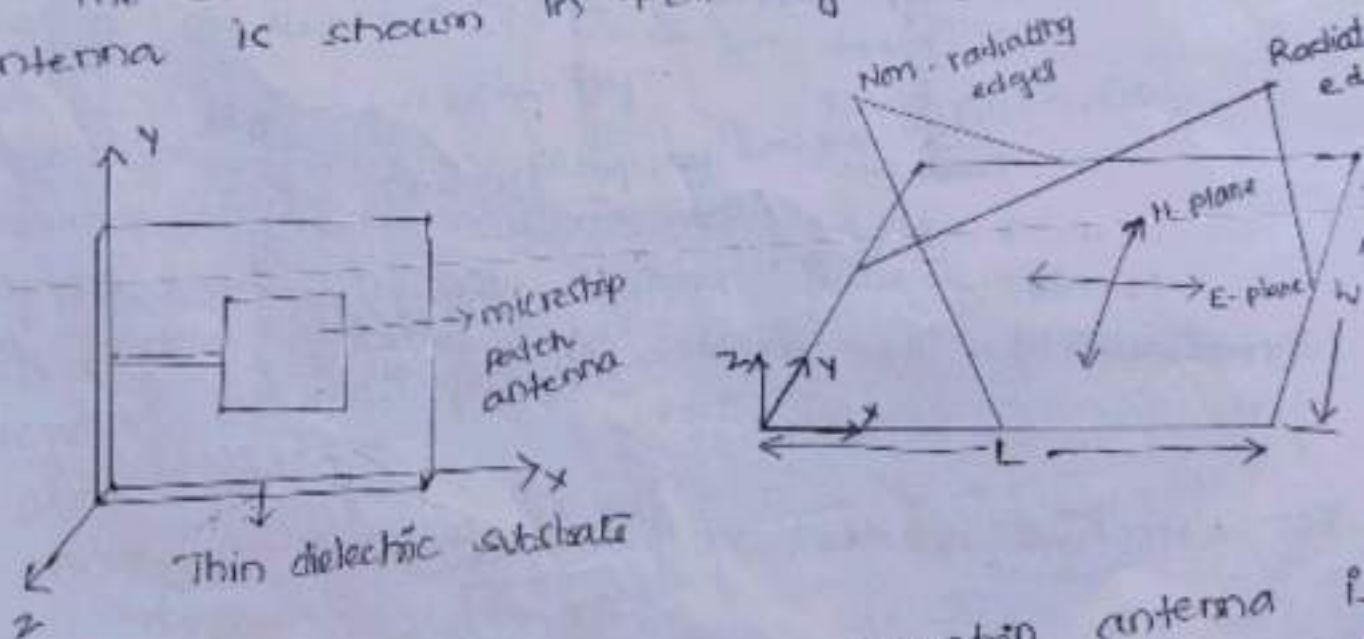
(ii) out of these space programs, the antennas
 are used for SEASAT and SIR-A, B, C series
 are all large panels consisting microstrip arrays
 nearly 10 meter in length at L and C band
 frequencies.
 applications: aircraft, missile, mobile, space crafts,
 satellite.

(3) Commercial applications.

The microstrip antennas are used commercially
 in applications such as mobile satellite communication,
 Direct Broadcast Satellite (DBS) services, Global
 Positioning System (GPS), Aeronautical and marine
 Radars and Earth Remote Sensing.

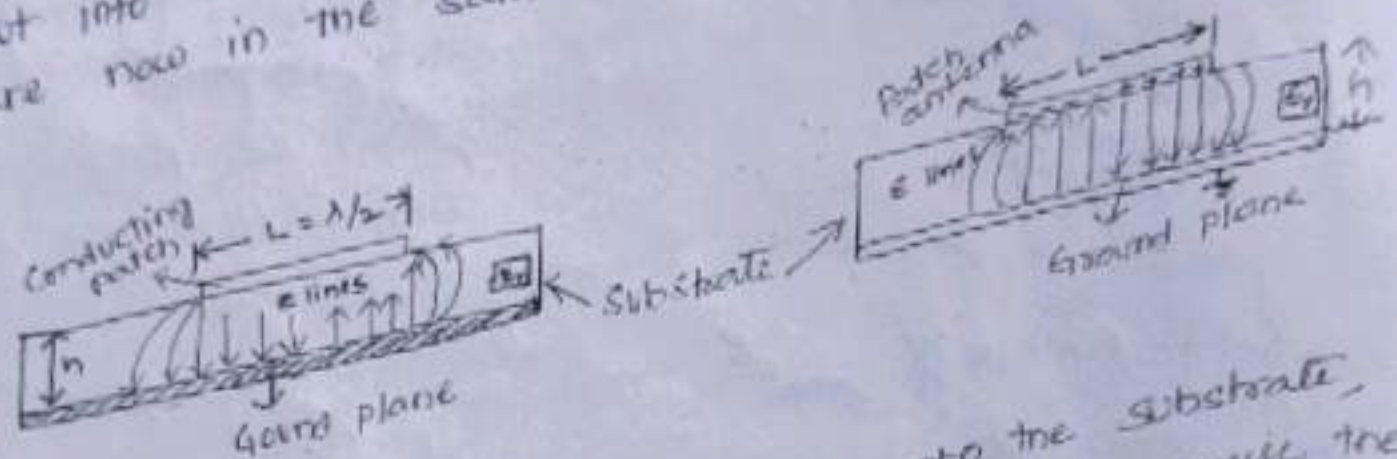
Rectangular Microstrip antennas

The Basic Structure of Rectangular microstrip
 antenna is shown in following figure.



The most commonly used microstrip antenna is
 rectangular MSA with a ground plane and dielectric is shown

4. The narrow band...
 The dimension L (length) of a...
 greater than dimension W (width) of a...
 At the ends of L dimension, there are radiation...
 edges which give single polarization.
 At the ends of W dimension, there are non-rad...
 edges which give cross polarization.
 When the patch length is half of wavelength
 i.e. $L = \lambda/2$, the electric field produced below the
 edges of L -dimension are of opposite polarity and
 when the field lines curve out and finally propaga...
 out into the direction normal to the substrate, they
 are now in the same direction (both facing left)

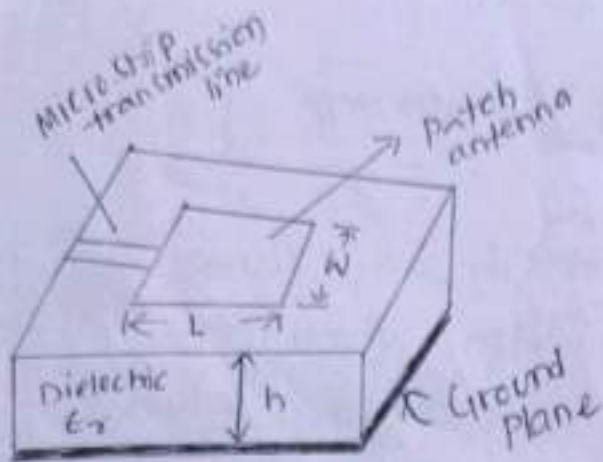


In the far-field perpendicular to the substrate,
 the radiation from the two sides adds up because the
 fields are in phase.
 The radiation intensity goes on decreasing as
 fields move away from edges and simultaneously
 two fields change phase also. At two angles
 particularly the fields are exactly out of phase
 and cancel each other.

For effective radiation of microstrip antenna:

- (i) Length $L = \lambda/2$
- (ii) The dielectric substrate should be sufficiently thicker
 dielectric constant.

Let us consider a rectangular microstrip antenna by a microstrip transmission line as shown in figure.



The frequency of operation of the patch antenna is in general determined by the length L . The critical or center frequency f_c can be approximately given by

$$f_c \approx \frac{c}{2L\sqrt{\epsilon_r}}$$

where c = velocity of light = $\frac{1}{\sqrt{\epsilon_0}}$

$$f_c = \frac{1}{2L\sqrt{(\epsilon_0\epsilon_r)\mu_0}}$$

where ϵ_0 - Permittivity of free space

μ_0 - permeability of free space

ϵ_r - permittivity of the dielectric substrate.

To obtain frequency of operation of a patch antenna accurately, we should consider dimension W and L also.

$$\therefore f_{r,nm} = \frac{c}{2\sqrt{\epsilon_{r,eff}}} \left[\left\{ \frac{n}{L+2\Delta L} \right\}^2 + \left\{ \frac{m}{W+2\Delta W} \right\}^2 \right]^{1/2}$$

$$\text{where } \epsilon_{r,eff} = \frac{4\epsilon_{re}\epsilon_{r,dyn}}{(\sqrt{\epsilon_{re}} + \sqrt{\epsilon_{r,dyn}})^2}$$

For dominant mode ($n=1, m=0$) operation expression reduces to,

$$f_{r, nm} = \frac{c}{2(L+2\Delta L)\sqrt{\epsilon_r, eff}}$$

In the above equations, ΔL and ΔW are the incremental length and width which account for the fringing of fields at the respective edges.

The width W of the patch is very important parameter as it controls the input impedance of an antenna. For a typical square shape patch antenna ($L=W$) the input impedance is typically 300Ω .

When the width is increased, the input impedance decreases. The width not only controls the input impedance but also controls the radiation pattern of a patch antenna.

Radiation Pattern of MSA :

The expressions for E field components E_θ and E_ϕ are given by,

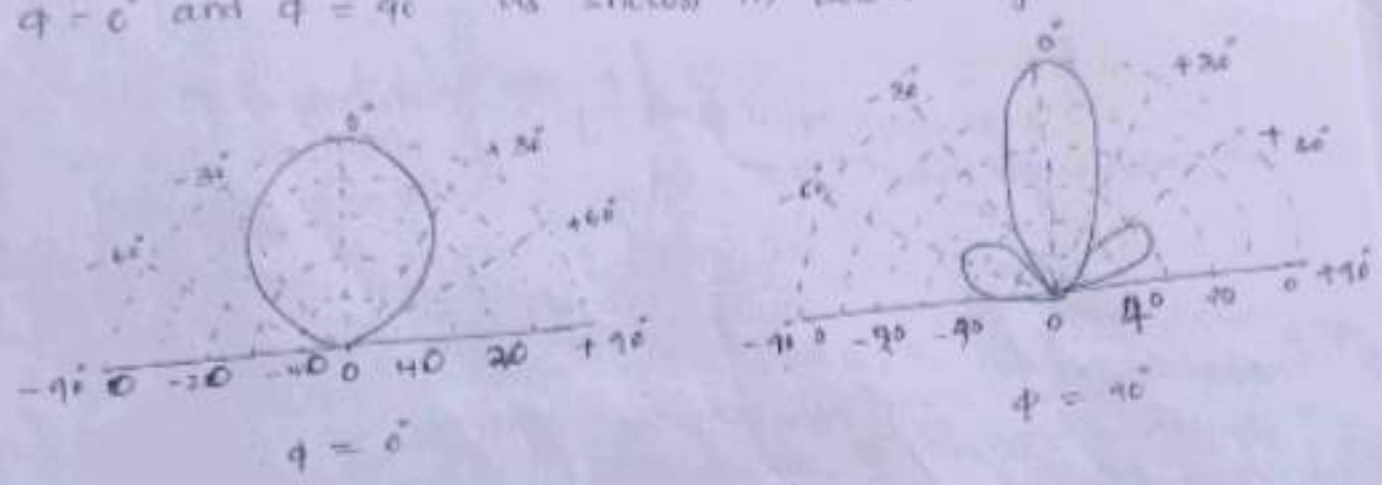
$$E_\theta = \frac{\sin[(Kw \sin\theta \sin\phi)/2]}{[(Kw \sin\theta \sin\phi)/2]} \cos\left[\frac{KL}{2} \sin\theta \cos\phi\right] \cos\phi$$

$$E_\phi = -\frac{\sin[(Kw \sin\theta \sin\phi)/2]}{[(Kw \sin\theta \sin\phi)/2]} \cos\left[\left(\frac{KL}{2}\right) \sin\theta \cos\phi\right] \sin\phi$$

The net magnitude of electric field at any point is a function of θ and ϕ and is given by

$$E(\theta, \phi) = \sqrt{(E_\theta^2 + E_\phi^2)}$$

The normalized pattern of the antenna can be obtained by plotting the field E_θ and E_ϕ , for $L = W = \lambda/2$ in $\phi = 0^\circ$ and $\phi = 90^\circ$ as shown in below figure.



Characteristics of microstrip antenna

① Radiation pattern:

Two radiation patterns in $\phi = 0$ (i.e. in azimuth) and $\phi = 90^\circ$ (in elevation) shown in above figure. The scale is logarithmic, so the power radiated at 180° is about 15 dB less than the power in the center of the beam, i.e. at 0° . The beamwidth is about 65° and the gain is about 9 dBi. An infinitely large ground plane would prevent any back radiation, but the real antenna has a fairly small ground plane, and power in the backward direction is only about 20 dB down from that in the main beam.

To understand the radiation pattern process in microstrip antennas, consider the side view of a patch antenna. In connection with the inset feed, it was mentioned the in an end-fed case the current will be low at the ends and high at the center of the antenna.

Since the patch is a conductor, the voltage is constant and the current is zero. The voltage and current are out of phase. Voltage will be maximum at the ends of the patch and minimum at its mid point.

② Beamwidth

MSA's generally have a very wide beam width, both in azimuth and elevation.

③ Directivity

In view of the cavity model of an MSA, the simple expression for directivity D for TM_{10} mode can be written as

$$D = \frac{2h^2 E_0^2 W'^2 K_0^2}{P_r \pi \eta_0}$$

where h is the thickness of the substrate, P_r is the radiated power, $W' = W + h$, $\eta_0 = 120\pi$, K_0 is the wave number and E_0 is the magnitude of the z -directed electric field intensity inside the cavity is given by

$$E_z = E_0 \cos \frac{m\pi x}{L} \cos \frac{n\pi y}{W}$$

Here L is the length of the patch along the x axis and W is the width of the patch along y axis.

④ Gain

Gain of a rectangular microstrip patch antenna with air dielectric is roughly estimated between 7-9 dB in view of the following counts-

1. Gain of the patch from the directivity relative to the vertical axis is normally about 2 dB, provided the length of the patch is half a wavelength.

If the patch is of square shape the pattern in the horizontal plane will be directional. Such a patch is equivalent to a pair of dipoles separated by half-wavelength. This counts for another 2 to 3 dB gain. (15)

3. If the addition of the ground plane cuts off most or all radiation behind the antenna, the power averaged over all directions is reduced by a factor of 2 and thus the gain is increased by 3 dB.

Bandwidth

The impedance bandwidth of a patch antenna is strongly influenced by the spacing between the patch and the ground plane. As the patch is moved closer to the ground plane, less energy is radiated and more energy is stored in the Patch Capacitance and Inductance. That is, the quality factor Q of the antenna increases and impedance bandwidth decreases.

The bandwidth decreases with the increase of Q in proportion to the dielectric constant of the substrate.

The feed structure also affects the bandwidth.

The relation between voltage standing wave ratio and the bandwidth is

$$\text{Bandwidth} = \frac{S-1}{Q_0 S} \quad ; \text{ where } S = \text{voltage standing ratio.}$$

From this relation, it can be concluded that as S increases the impedance bandwidth increases.

Quality factor $Q = \frac{\text{Energy stored}}{\text{Energy loss per period}} = 2\pi \times \frac{\text{high } Q \text{ means low loss}}{\text{Quality of the device}}$

MSA have a very high quality factor. The quality factor Q represents the losses associated with the antenna. A large Q leads to narrow bandwidth, and a low efficiency Q can be reduced by increasing the thickness of the dielectric substrate. But as the thickness increases, a large fraction of the total power delivered by the source transforms to surface wave.

Efficiency

The total loss factor for an MSA can be given by

$$L_T = L_c + L_d + L_r$$

where $L_r \rightarrow$ loss in radiation,

$L_c \rightarrow$ loss in conductor

$L_d \rightarrow$ loss in the dielectric.

The loss in the conductor and dielectric substrate results in the reduction of radiation efficiency which is given by

$$\eta = \frac{P_r}{P_c + P_d + P_r}$$

where $P_r \rightarrow$ radiated power

$P_c \rightarrow$ power dissipated due to conductor loss

$P_d \rightarrow$ power dissipated due to dielectric

Polarization

An inherent advantage of patch antennas is their ability to have polarization diversity. Patch antennas can easily be designed to have vertical, horizontal, right-hand circular (RHCP) or left-hand circular (LHCP) Polarizations, using multiple feed points, or a single-feed point with asymmetric patch structures. This unique property allows patch antennas to be used in many types of communication links that may have varied requirements.

Return loss

The return loss is defined as the ratio of the Fourier transforms of the incident pulse and the reflected signal. It is an important parameter to reckon with.

The bandwidth of patch antenna, is very small. The bandwidth of rectangular patch antenna is typically of the order of 3%.

The antenna designed to operate at 100 MHz is resonant at nearly 96 MHz. This shift is due to fringing fields around the antenna, which makes the patch appear a little longer.

Feed methods of parabolic reflector antenna.

A parabolic reflector antenna as a system consists two basic parts namely a source of radiation located at the focus and a reflector.

The source placed at the focus is called primary radiator, while the reflector is called secondary radiator. The primary radiator i.e. the source is commonly called feed radiator or simply feed.

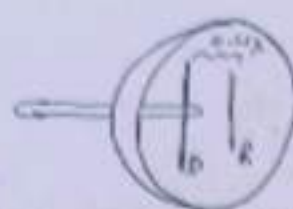
In case of a parabolic reflector a feed is said to be ideal feed if it radiates entire energy towards the reflector in such a way that the entire surface of reflector is illuminated and no energy is radiated in any unwanted direction. Practically there are number of possible feeds to the parabolic reflector antenna.

The secondary radiator used is most of the times a paraboloid.

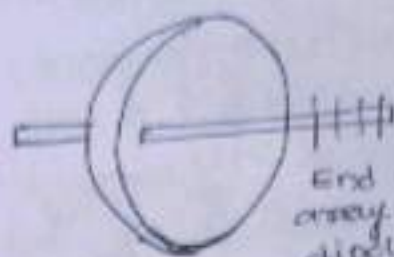
The simplest type of feed that can be used is a dipole antenna. But it is not a suitable feed for the parabolic reflector antenna.

In some cases, an end fire array of dipoles is used as feed radiator as shown in figure. The dipoles are spaced in such a way that the end fire pattern of an array, illuminates reflector.

The most widely used feed system in the parabolic reflector antenna is horn antenna. The horn antenna is fed with a waveguide. In case if circular polarization is required, then in place of a rectangular horn, a conical horn or helix antenna is used at the focus.

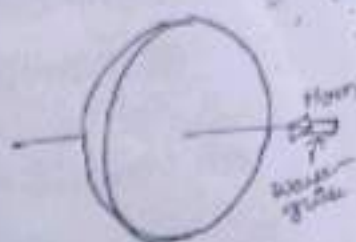


Dipole with Plane reflector



End fire array of dipoles

End fire array of dipole



Horn with waveguide

- Fig : Feed Systems

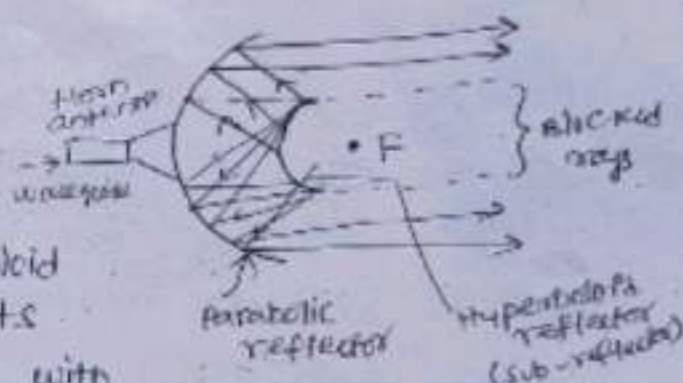
1. Cassegrain feed System

- This system of feeding paraboloid reflector is named after a mathematician prof. Cassegrain.

- In Cassegrain feed system, the feed radiator is placed at the vertex of the parabolic reflector, instead of placing it at the focus.

- This system uses a hyperboloid reflector placed such that its one of the foci coincides with the focus of the parabolic reflector.

This hyperboloid reflector is called Cassegrain Secondary reflector or sub-reflector.



- The primary radiator or feed radiator used is generally a horn antenna. It aims at the sub-reflector.

- When the feed radiator radiates towards the Cassegrain or sub-reflector, it radiates in all the directions and due to these radiations, the parabolic reflector

gets illuminated similar to the radiations from the feed placed at the focus. Then the parabolic reflector collimates all the radiations as previous feed systems. (16)

Advantages of Cassegrain feed system

1. It reduces the spill over and thus minor lobe radiations.
2. With this system greater focal length greater than the physical focal length can be achieved.
3. The system has ability to place a feed at convenient place.
4. Using this system, beam can be broadened by adjusting one of the reflector surfaces.

Disadvantages of Cassegrain feed system

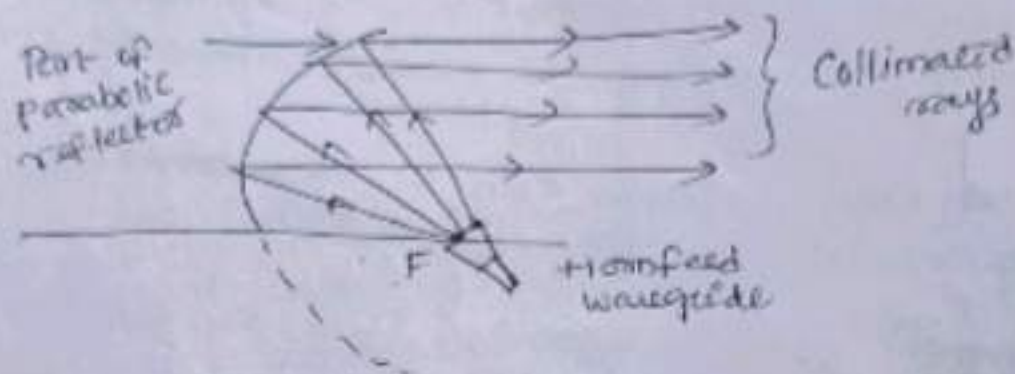
Some of the radiation from the parabolic reflector are obstructed or blocked by the hyperboloid reflector creating region of blocked rays. For small dimension parabolic reflector it is the main drawback of the Cassegrain feed system.

② Offset Feed System.

The paraboloidal reflector can be fed using $\lambda/2$ antenna with a small ground plane or a horn antenna. In both cases reflected wave from paraboloidal reflector causes mismatching and interaction at primary antenna. Also the primary antenna blocks central portion of the aperture which in creates minor lobes effectively.

To overcome the aperture blocking effect due to the dependence of the secondary reflector dimensions on the distance between feed and sub-reflector, the offset feed system as shown in figure.

Here feed radiator is placed at the focus as shown in figure. With this system all the rays are properly collimated without formation of the region of blocked rays.



Effect of variation of focal length

In paraboloid reflector, the ratio of the focal length f to the diameter of aperture is another important design constraint.

The three possible cases are as follows:

- (i) Focal point inside the aperture of paraboloid.
- (ii) Focal point along the plane of open mouth of paraboloid.
- (iii) Focal point beyond the open mouth of paraboloid.

When the focal length is very small, the focal point lies inside the open mouth of paraboloid as shown in figure. Here it is difficult to obtain uniform illumination over a wide angle.

Reflector Antennas

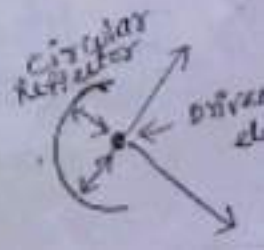
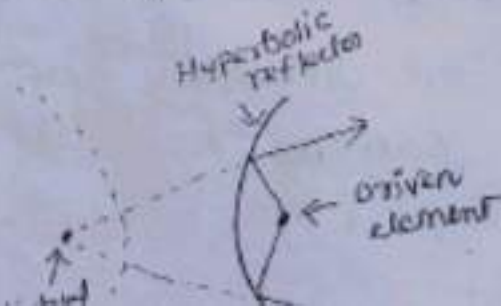
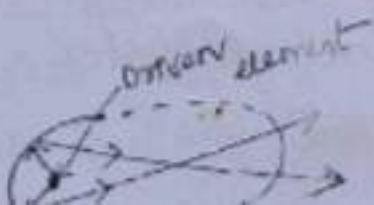
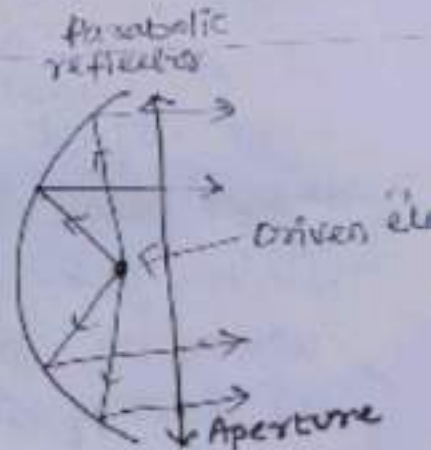
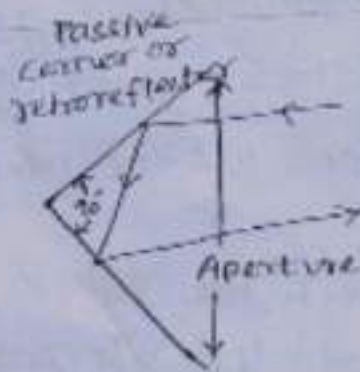
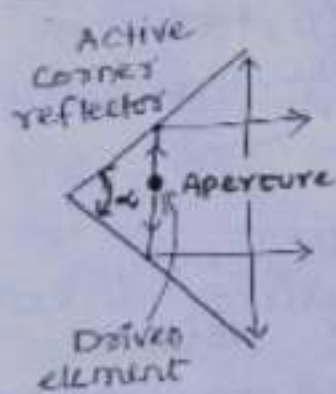
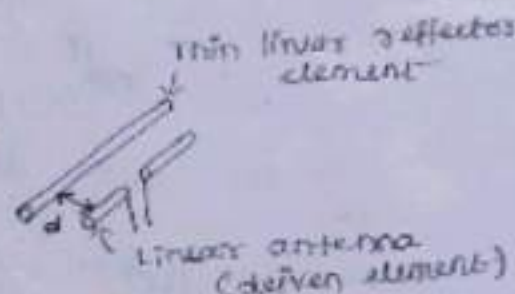
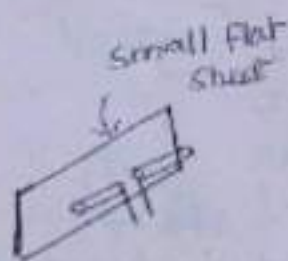
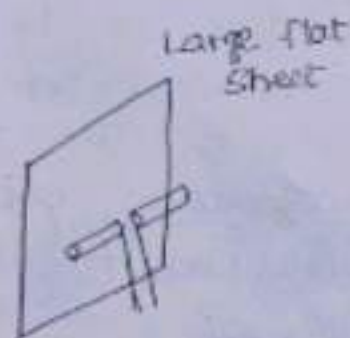
Reflectors are widely used to modify the radiation pattern of a radiating element. For example, the backward radiation from an antenna may be eliminated with a plane sheet reflector of large enough dimensions. In the more general case, a beam of predetermined characteristics may be produced by means of a large, suitably shaped, and illuminated reflector surface.

The Reflector antennas are most important in microwave radiation applications.

At microwave frequencies the physical size of the high gain antenna becomes so small that practically any suitably shaped reflector can produce desired directivity.

Reflector antenna requires another antenna to excite. Hence the antennas such as dipole, horn, slot antennas which excite the reflector antenna is called primary antenna while the reflector antenna is called secondary antenna.

Several reflector types are shown in the figure 1



Reflector antenna can be of any shape, but most used are

1. plane reflectors
2. corner reflectors
3. curved or parabolic reflectors.

The arrangement in large flat sheet reflector near a linear dipole antenna to reduce the backward radiation. With small spacings between the antenna and sheet reflector may be in this arrangement also yields a substantial gain in the forward direction.

With two flat sheets intersecting at an angle $\alpha < 180^\circ$, a sharper radiation pattern than from a flat sheet reflector ($\alpha = 180^\circ$) can be obtained.

This arrangement, called an active corner reflector antenna. A corner reflector without an exciting antenna can be used as a passive reflector or target for radio waves.

When it is feasible to build antennas with apertures of many wavelengths, parabolic reflectors can be used to provide highly directional antennas. The parabola reflects the waves originating from a source at the focus in a parallel beam, the parabola transforming the curved wave front from the feed antenna at the focus into a plane wave front.

Flat Sheet Reflectors or plane reflectors

1. The plane reflector is the simplest form of the reflector antenna.
2. When the plane reflector is kept in front of the feed, the energy is radiated in the desired direction.
3. To increase the directivity of the antenna, a large flat sheet can be kept as plane reflector in front of a half dipole as shown below.

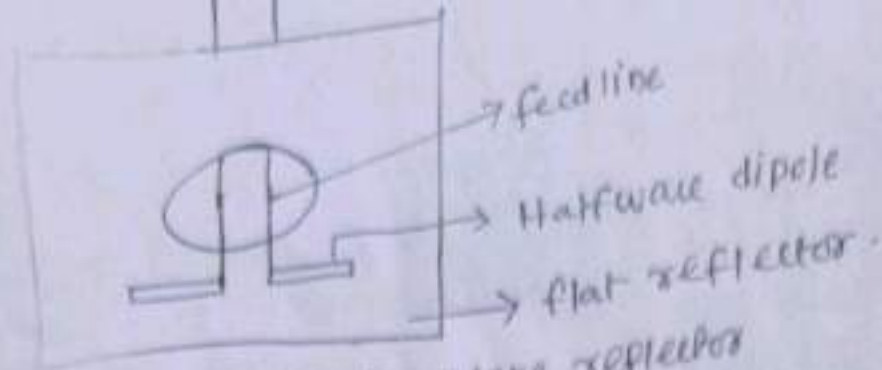
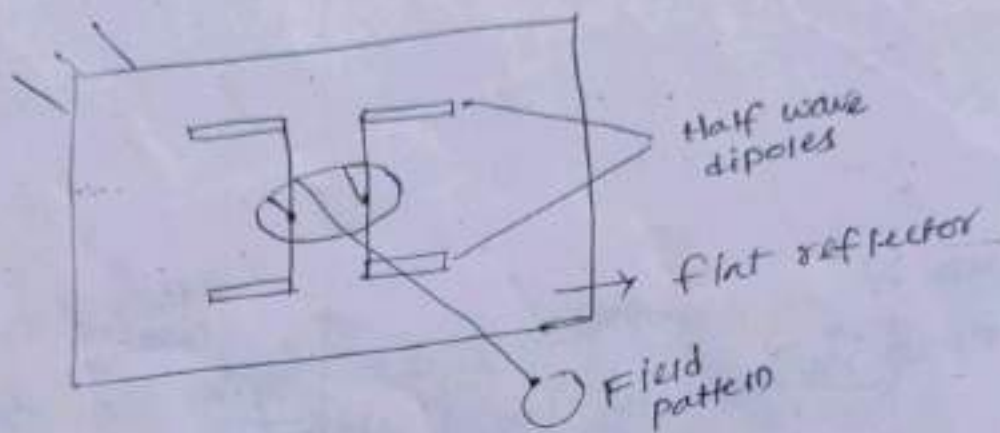


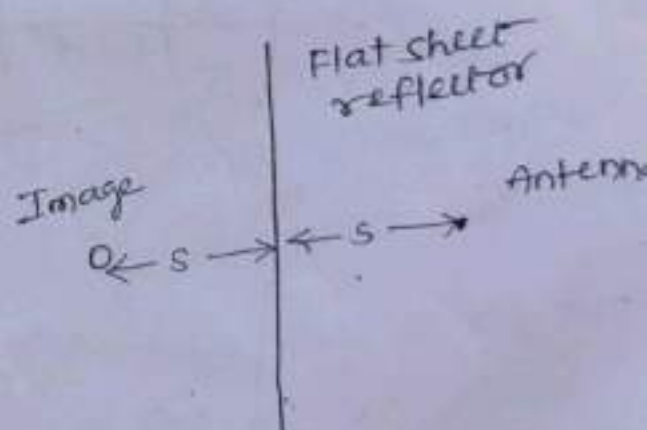
Fig. 1.1/2 dipole with plane reflector

- The main advantage of the plane reflector is that for the dipole backward radiations are reduced and gain in the forward direction increases.
5. To increase the directivity further, we can use array of two half wave dipoles in front of a flat plane reflector as shown below.



6. Instead of using flat sheet, only a reflector element can be used to increase the directivity.
7. The analysis of flat sheet reflector can be done with the help of method of images. In this method, reflector is replaced by image of an antenna at a distance as from feed antenna.

8. For an infinite plane-reflector, assuming zero reflector losses, the gain of a $\lambda/2$ dipole antenna at a distance s is given by

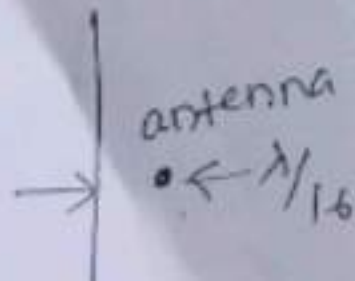
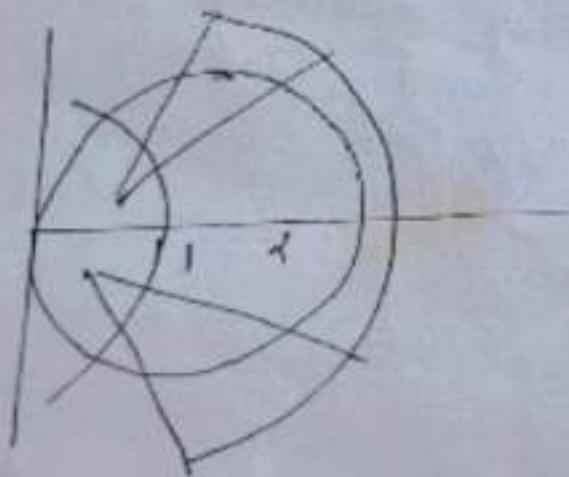
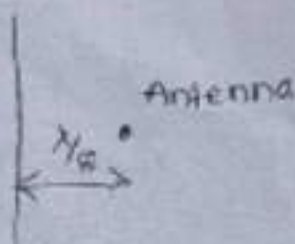
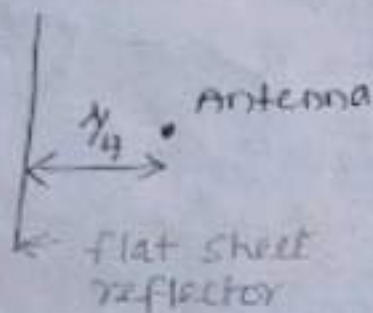
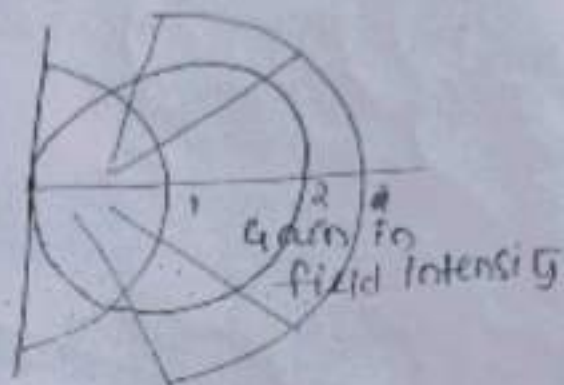
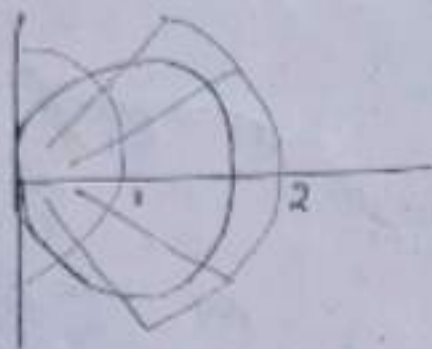


$$\sqrt{R_{11} + R_{loss} - R_{12}}$$

where $S_{12} = \left[\frac{2\pi}{\lambda} \right] S$

The gain of reflector relative to the half wave dipole antenna is a function of the spacing between flat sheet and half wave dipole antenna.

9. When spacing between the half wave dipole and infinite sheet decreases, the gain decreases.



Corner Reflectors

A corner reflector antenna is a type of directional antenna used at VHF and UHF frequencies. It was invented by John D. Kraus in 1938.

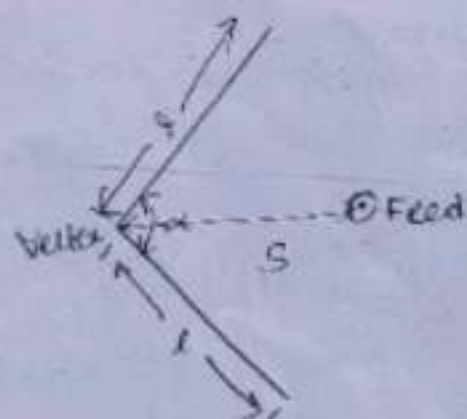
When the flat sheet (180° included angle) is folded into a square (90°) corner then it corresponds to a higher gain.

Thus the corner reflector developed as an extension of the ^{array of $16 \frac{1}{2}$ dipoles with} flat sheet reflector.

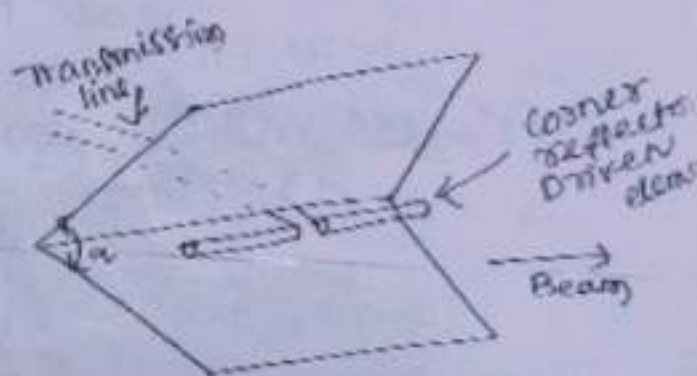
3. The disadvantage of plane reflector is that there is radiation in back and side directions. Hence in order to overcome this limitation, the shape of the plane reflector is modified so that the radiation is in forward direction only.

4. The modified arrangement consists of two plane reflectors which are joined to form corner with some angle. This type of reflector is called corner reflector.

5. The angle at which two plane reflectors are joined is called included angle (α) or corner angle.



Top view of corner reflector

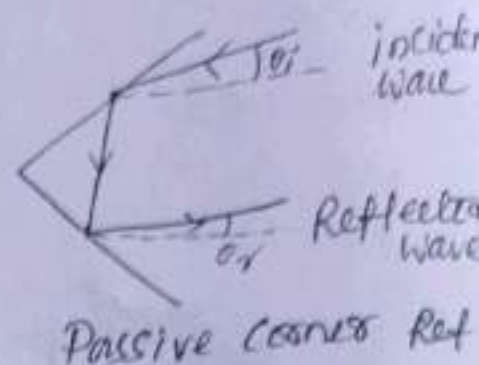


Corner reflector antenna side view.
(or)
Active corner ref. ant.

A corner reflector with two flat conducting sheets at a corner angle α and a driven antenna is called 'active corner reflector antenna' or simply corner reflector antenna.

7. Corner reflector antenna consists of only two flat conducting sheets at a corner angle α , with out an driven element then it is called passive corner ref.

Such a passive corner reflector antenna is shown in the figure.



8. The analysis for the radiated field of the source with the corner reflectors is found to be useful with included angle

$$\alpha = \frac{\pi}{N} \text{ where } N \text{ is integer.}$$

Design equations for corner reflector :-

The important dimensions in the corner reflector antenna are as follows.

D_A = dimension of aperture,

l = side length of the reflector sheet.

S = Distance between feed and the vertex of the reflector.

It is a general practice, the side length l is selected equal to twice the distance S . Hence design equations are as follows.

$$l = 2S \quad \text{--- (1)}$$

$$\text{Now } D_A = \sqrt{l^2 + l^2} = \sqrt{2} l = 1.414 l$$

$$\text{But } l = 2S$$

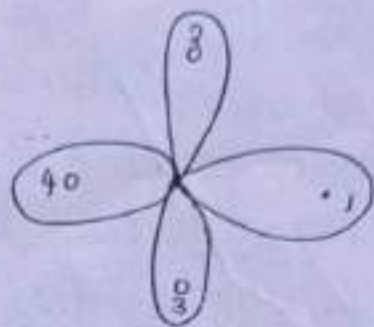
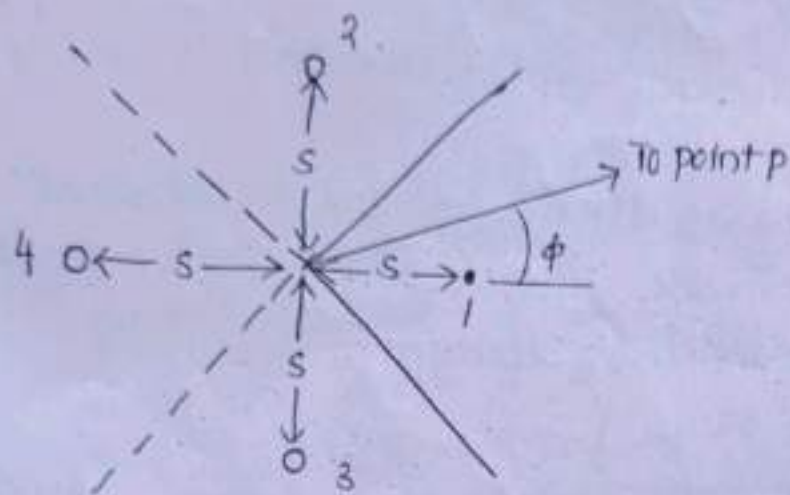
$$\therefore D_A = 1.414 (2S)$$

$$D_A = 2.828 S \quad \text{--- (2)}$$

Equations (1) & (2) are the design equations for corner reflector antenna.

of infinite extent, the method of images can be applied to analyze the corner reflector antenna for angles $\alpha = 180^\circ/n$, where n is any positive integer. Corner angles of $180^\circ, 90^\circ, 60^\circ$ etc. can be treated in this way.

In the analysis of the 90° corner reflector there are 3 image elements, 2, 3 and 4, located as shown in figure.



The driven antenna 1 and the 3 images have currents of equal magnitude. The phase of the currents in 1 and 4 is the same. The phase of the currents in 2 and 3 is the same but 180° out of phase with respect to the currents in 1 and 4. All elements are assumed to be $\lambda/2$ long.

At a point p at a large distance D from the antenna, the field intensity is

$$E(\phi) = 2KI_1 [\cos(S_r \cos \phi) - \cos(S_r \sin \phi)] \quad \text{--- (1)}$$

Where

I_1 = Current in each element

S_r = Spacing of each element from the corner,

$$= \left(\frac{2\pi}{\lambda} \right) s$$

including the distance D , etc.

The emf V_1 at the terminals at the center of the driven elements is

$$V_1 = I_1 Z_{11} + I_1 R_{1L} + I_1 Z_{14} - 2I_1 Z_{12} \quad \text{--- (2)}$$

where Z_{11} - self impedance of driven element
 R_{1L} - equivalent loss resistance of driven element
 Z_{12} - mutual impedance of elements 1 and 2
 Z_{14} - mutual impedance of elements 1 and 4

Similar expressions can be written for the emf's at the terminals of each of the images. Then if P is the power delivered to the driven element, we have from symmetry that

$$I_1 = \sqrt{\frac{P}{R_{11} + R_{1L} + R_{14} - 2R_{12}}} \quad \text{--- (3)}$$

Substituting (3) in (1)

$$\therefore E(\phi) = 2K \sqrt{\frac{P}{R_{11} + R_{1L} + R_{14} - 2R_{12}}} \left| [\cos(\phi_0 \cos \phi) - \cos(\phi_0 \sin \phi)] \right| \quad \text{--- (4)}$$

The field intensity at the point P at a distance D from the driven $\lambda/2$ element with the reflector removed is

$$E_{HN}(\phi) = K \sqrt{\frac{P}{R_{11} + R_{1L}}} \quad \text{--- (5)}$$

This is the relation for field intensity of a $\lambda/2$ dipole antenna in free space with a power input P and provides convenient reference for the corner reflecting antenna.

Thus, dividing (4) by (5), we obtain the gain in field intensity of a corner reflector antenna over a single $1/2$ antenna in free space with the same power input,

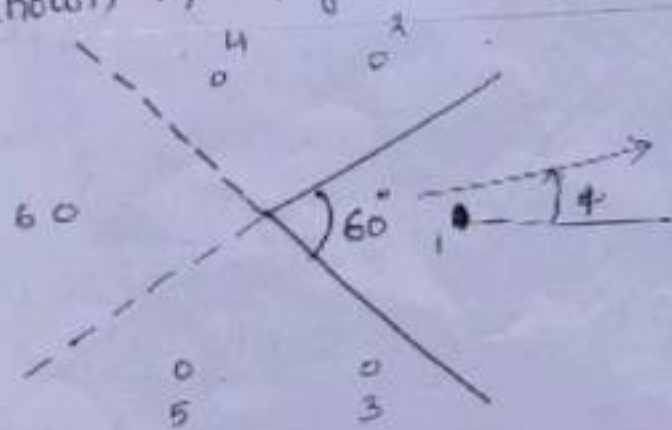
$$G_f(\phi) = \frac{E(\phi)}{E_{HW}(\phi)}$$

$$= 2 \sqrt{\frac{R_{11} + R_{11}}{R_{11} + R_{11} + R_{14} - 2R_{12}}} \left| [\cos(S_1 \cos \phi) - \cos(S_2)] \right| \quad (6)$$

Where the expression in brackets is the pattern factor and the expression included under the radical sign is the coupling factor. The pattern shape is a function of both the angle ϕ and the antenna to corner spacing S . The pattern calculated by equation (6) has 4 lobes as shown in figure. However, only one of the lobes is real.

Expressions for the gain in field intensity of corner reflectors with corner angles of 60° , 45° , etc can be obtained in a similar manner.

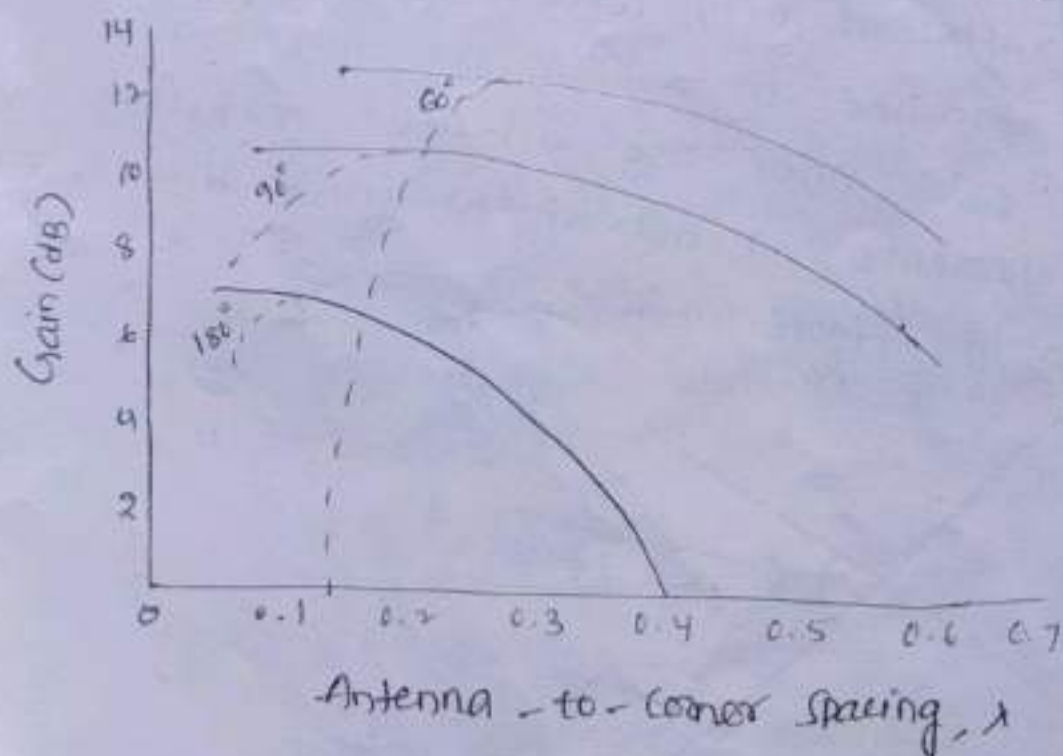
For 60° corner the analysis requires a total of 6 elements, 1 actual antenna and 5 images, as shown in figure.



Gain pattern expressions for corner reflectors of 90° and 60° are listed below.

Corner angle, deg	Number of elements in analysis	Gain in field intensity over $\lambda/2$ antenna in free space with same power input
180	2	$2 \sqrt{\frac{R_{11} + R_{1L}}{R_{11} + R_{1L} - R_{12}}} \sin(s_r \cos \phi)$
90	4	$2 \sqrt{\frac{R_{11} + R_{1L}}{R_{11} + R_{1L} + R_{14} - 2R_{12}}} \left \cos(s_r \cos \phi) - \cos(s_r \sin \phi) \right $
60	6	$2 \sqrt{\frac{R_{11} + R_{1L}}{R_{11} + R_{1L} + R_{14} - 2R_{12} - R_{16}}} \times \left\{ \sin(s_r \cos \phi) - \sin[s_r \cos(60^\circ - \phi)] - \sin[s_r \cos(60^\circ + \phi)] \right\}$

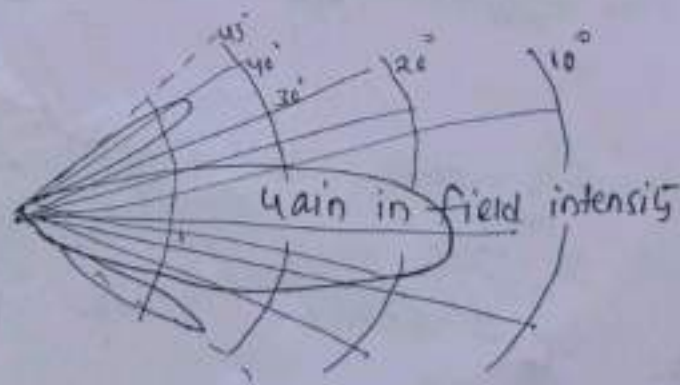
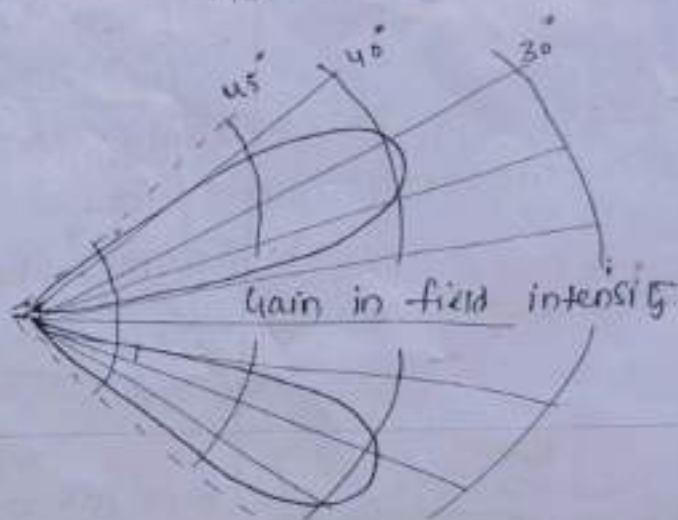
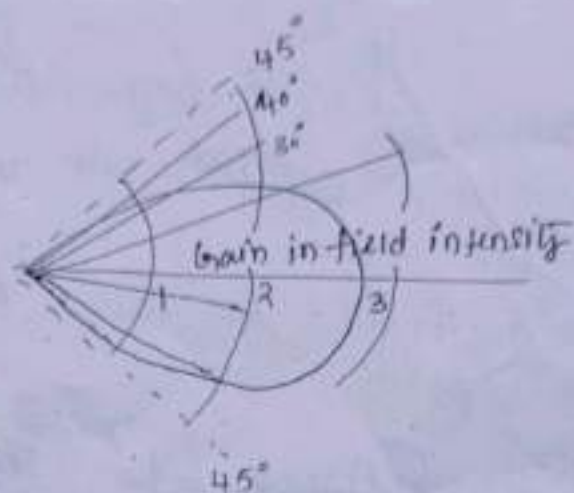
Gain of corner reflector antennas over a $\lambda/2$ dipole antenna in free space with the same power input as a function of the antenna to corner spacing.



The solid curve in each case is computed for zero losses ($R_{1L}=0$), while the dashed curve is for an assumed loss resistance $R_{1L}=1\Omega$.

The calculated pattern of a 90° corner reflector (12) with antenna-to-corner spacing $s = 0.5\lambda$ is shown in figure below.

If s exceeds a certain value, a multilobed pattern may be obtained. For example, a square-corner reflector with $s = 1.0\lambda$ has a 2 lobed pattern. If the spacing is increased to 1.5 , the pattern shown in below figure, is obtained with the major lobe in the $\phi = 0$ direction but with minor lobes present.



corner angle, α

90°

180° (flat sheet)

corner to dipole spacing, s

$0.25 - 0.7\lambda$

$0.1 - 0.3\lambda$

Parabolic reflector antenna

To improve the overall radiation characteristics of the reflector antenna, the parabolic structure is oftenly used.

Referring to the following figure, the parabolic curve may be defined as follows. The distance from any point P on a parabolic curve to a fixed point F , called the focus, is equal to the perpendicular distance to a fixed line called the directrix. Thus from figure (b) $PF = PQ$.

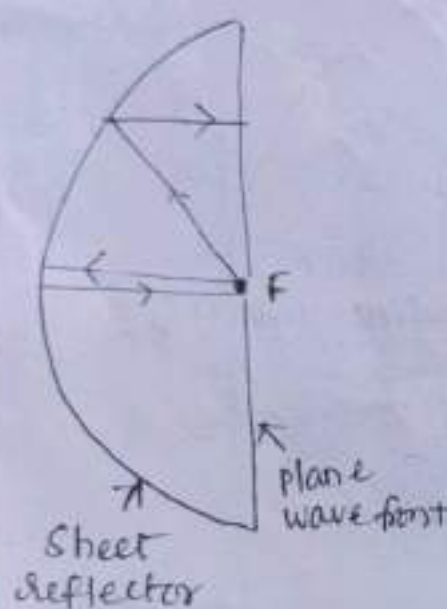
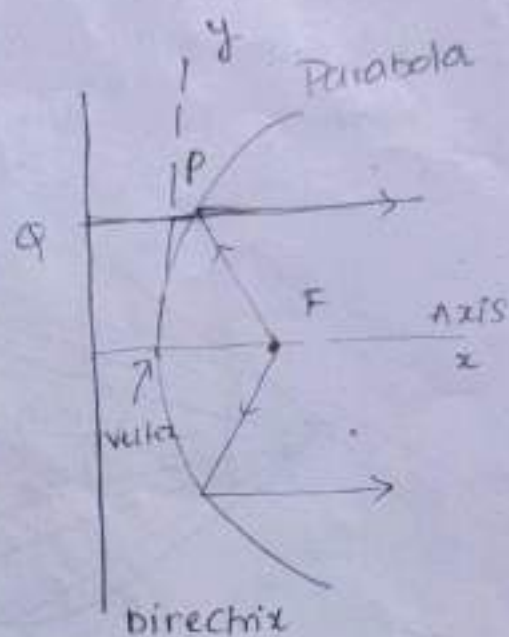


Fig (a)



(b)



(c)

Aperture plane

Referring now to fig (c), let AA' be a line normal to the axis at an arbitrary distance QS from the directrix. Since $PS = QS - PQ$ and

Let AA' be a line normal to the axis at an arbitrary distance QS from the directrix. Since $PS = QS - PQ$ and $PF = PQ$, it follows that the distance from the focus to S is

$$PF + PS = PF + QS - PQ = QS \quad (\because PF = PQ)$$

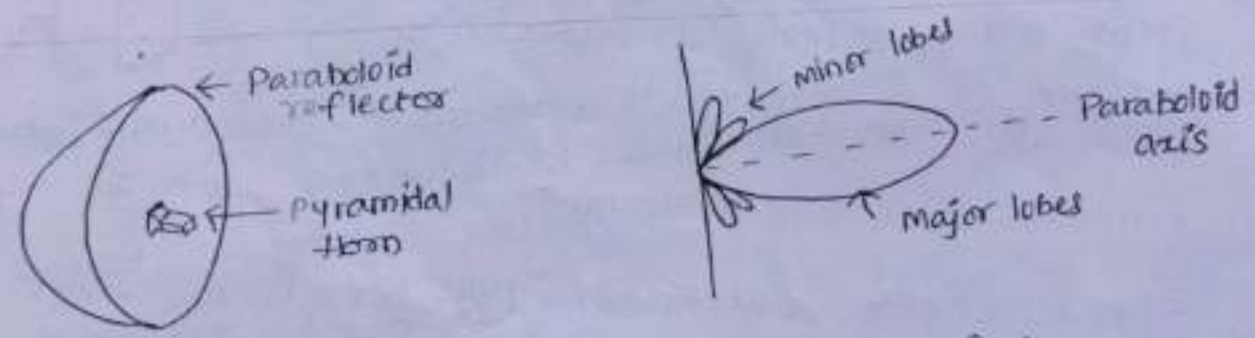
Thus, a property of a parabolic reflector is that all waves from an isotropic source at the focus that are reflected from the parabola arrive at a line AA' with equal phase. The image of the focus is the directrix, and the reflected field along the line AA' appears as though it originated at the directrix as a plane wave. The plane BB' at which a reflector is cut off is called the aperture plane.

Paraboloid Reflector

The parabolic reflector for practical applications is a three dimensional structure.

The three dimensional structure of the parabolic reflector can be obtained by rotating the parabola around its axis and it is called paraboloid.

The paraboloid is shown in the figure below.



Paraboloid with pyramidal horn as feed.

The parallel, beam produced are of the circular cross-section.
The radiation pattern consists very sharp major lobe and smaller minor lobes.

Types of Paraboloid reflectors

Depending upon the use, the paraboloid is modified in various types of the structures as follows.

① Truncated paraboloid or cut paraboloid :-

This type of the paraboloid is formed by cutting some of the portion of the paraboloid to meet the requirements.

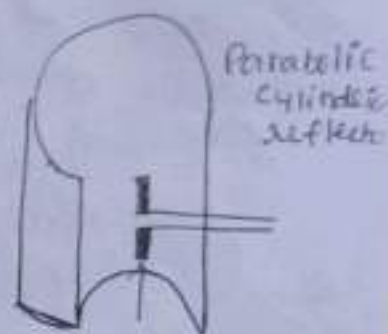
As the portion of the paraboloid is cut away or truncated as shown in figure below, the paraboloid is called cut paraboloid or truncated paraboloid.



Truncated
or
cut paraboloid

② Parabolic right cylinder :-

The right cylindrical structure of the parabolic reflector is shown in figure. This structure is obtained by moving the parabola side ways. This parabolic structure has focal line instead of a focal point and similarly a vertex line instead of a vertex.



Dipole (feed antenna)

③ Pill box or cheese antenna :-

These cheese antenna or pill box is a short parabolic right cylinder enclosed by parallel plates as shown in the figure. This antenna is useful in producing wide beam in one of the planes while a narrow in other.



Pill box
antenna

paraboloid is called as Microwave dish antenna: -
The paraboloid is called microwave dish which produces sharp major lobe and smaller minor lobes.

Consider that the power gain of the paraboloid, with circular mouth or aperture, with respect to half wave dipole is given by,

$$G_p = \frac{4\pi A_0}{\lambda^2}$$

where

A_0 - Aperture area

$A_0 < A$ (actual area of the mouth)

$$\therefore G_p = \frac{4\pi(KA)}{\lambda^2}$$

$$A_0 = KA$$

where K - constant dependent on feed antenna used.

$K = 0.65$ for dipole

$$= \frac{4\pi \times 0.65A}{\lambda^2}$$

The actual area with circular aperture with diameter d is given by,

$$A = \pi \left(\frac{d}{2} \right)^2 = \frac{\pi d^2}{4}$$

$$\therefore G_p = \frac{\pi^2 \times 0.65 d^2}{\lambda^2}$$

$$G_p = 6 \left(\frac{d}{\lambda} \right)^2$$

From the above equation it is clear that the power gain of the paraboloid depends on d/λ ratio. is called aperture ratio.

Consider $\lambda = 0.02 \text{ m}$ and $d = 1 \text{ m}$, $G_p = 1500$

Assuming large circular aperture, the beamwidth between first null can be expressed as

$$\text{BW}_{FN} = \frac{140\lambda}{d} \text{ degree}$$

Similarly BWFN for uniformly illuminated rectangular aperture is given by,

$$\text{BWFN} = \frac{11.5\lambda}{L} \text{ degree}$$

where L - length of rectangular aperture.

The half power beamwidth for large circular aperture can be expressed as,

$$\text{HPBW} = \frac{58\lambda}{d} \text{ degree}$$

Similarly the directivity of the uniformly illuminated aperture is given by

$$D = \frac{4\pi A_e}{\lambda^2}$$

For circular aperture

$$A_e = \pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$$

$$\therefore D = \frac{4\pi \left(\frac{\pi d^2}{4}\right)}{\lambda^2}$$

$$\therefore D = \pi^2 \left(\frac{d}{\lambda}\right)^2 = 9.87 \left(d/\lambda\right)^2$$

UNIT-IV

Antenna Arrays

In some wireless communication applications, we need to have narrow beam for long distance communication. - it is possible

by two ways.

- 1) Increasing the size of antenna
- 2) Using antenna array

TO increase the gain of antenna
TO have narrow beam

→ TO increase the field strength in the desired direction by using group of antennas is called array of antennas (or) Antenna array

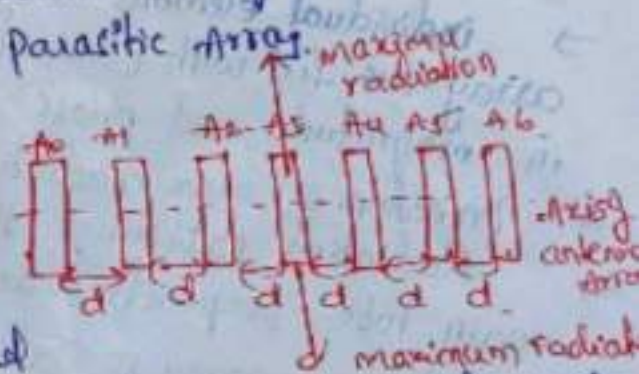
→ Antenna array can be defined as the system of similar antennas directed to get required high directivity in the desired direction.

There are 4 types of Antenna arrays. These are

- 1) Broadside Array
- 2) Endfire array
- 3) Collinear Array
- 4) Parasitic Array

1) Broadside Array.

→ Broadside array is the array of antennas in which all the elements are placed parallel to each other and the direction of maximum radiation is always perpendicular to the plane containing elements.

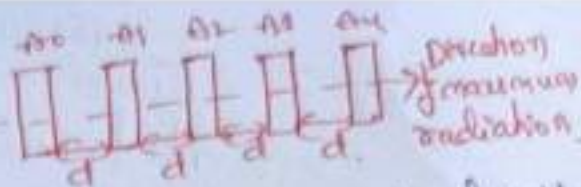


→ All the individual antennas are spaced equally along the axis of antenna array.

- The spacing between any two elements is denoted by 'd'.
- All the elements are fed with currents with equal magnitude and same phase.
- radiation pattern for the broadside array is bidirectional because
- arrangement of antenna in which maximum radiation is in the direction perpendicular to the axis of array and plane containing the elements of array.

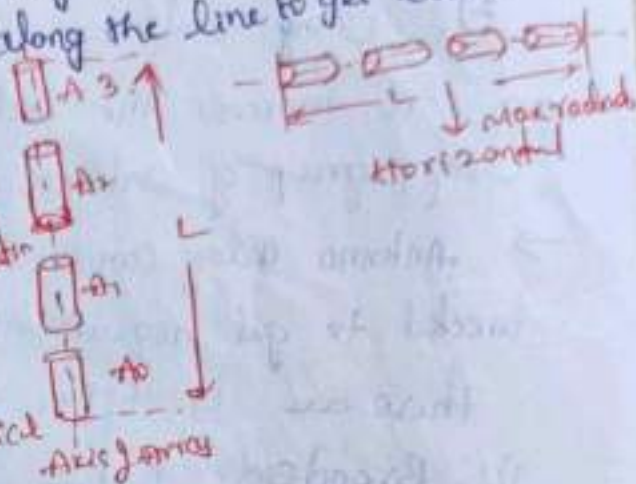
② End fire Array

- Endfire Array is very similar to the broadside array from the point of view of arrangement.
- Difference is in the direction of maximum radiation (radiation along the axis of array).
- direction of the maximum radiation is along the axis of array.
- Antennas are spaced equally along a line.
- All the antennas are fed individually with currents of equal magnitude but their phases vary progressively along the line to get entire arrangement unidirectional finally.



③ Collinear Array

- Antennas are arranged co-axially i.e. the antennas are arranged end to end along a single line.
- individual elements in the collinear array are fed with currents equal in magnitude and phase. Similar to broadside array.
- Direction of max radiation is perpendicular to the axis of array.
- Radiation pattern of this array has circular symmetry with main lobe perpendicular everywhere to the principle axis.
- Collinear array is also called omnidirectional array (or) broadside array.
- gain of the collinear array is maximum if the spacing b/w the elements is of the order of 0.5λ to 0.7λ . Small spacing gives feeding problems.
- To overcome this the elements of the array are operated with their ends very close to each other by connecting ends by an insulator.
- power gain of the collinear array doesn't increase in proportion with no. of elements. E.g. 2-element array power gain is 1.9 dB but for 4-elements is not equal to twice of 2-element but it is equal to 4.2 dB.
- Collinear array with more than 4-element is not practically used as power gain is not sufficient. Practically use 2-element array.



④ Parasitic Array.

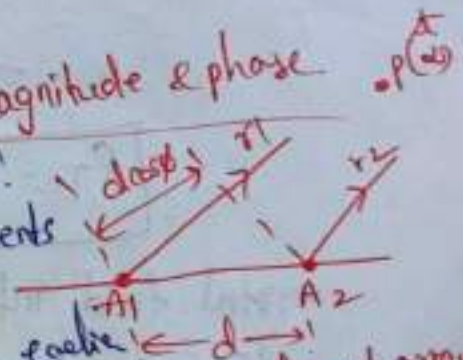
- Parasitic array consists one driven element and one parasitic element
- In multielement parasitic array, there may be one or more driving elements and also one or more parasitic elements.
- So in general multielement parasitic array is the array with at least one driven element and one or more parasitic elements.
- Ex: 2 parasitic array with half wave dipoles as elements
- Array is Yagi-Uda.
- Amplitude and phase of the current induced in the parasitic element depends on the spacing b/w the driven element and parasitic element.
- Radiation pattern Unidirectional
- phase of the currents are changed by adjusting the spacing b/w elements.

Array of Point Sources

- Point source in this an antenna is regarded as point source or volume radiator.
- Depending upon the magnitude of and phase currents given to two point sources we obtain three cases.
 - 1) Equal amplitude and phase
 - 2) " " and opposite phase
 - 3) Unequal amplitude and " " and opposite phase

① Two point sources with currents equal in magnitude & phase

- Consider two pt sources A_1 & A_2 separated by 'd'.
- both the point sources are supplied with currents equal in magnitude & phase $I_1 = I_2 = I$
- radiation from the pt source A_2 will reach earlier at pt P than that from pt source A_1 because of path difference.
- pt P far away from the array. distance b/w pt P & point sources A_1 & A_2 be r_1 & r_2 respectively.



Hence path difference is given by.

$$\text{path difference} = d \cos \phi \quad \text{--- (1)}$$

path difference can be expressed in terms of wavelength as

$$\text{path diff} = \frac{d \cos \phi}{\lambda} \quad \text{--- (2)}$$

Hence the phase angle ψ is given by.

$$\text{phase angle} = \psi = 2\pi (\text{path difference}) \\ = 2\pi \left(\frac{d \cos \phi}{\lambda} \right)$$

$$\boxed{\psi = \frac{2\pi}{\lambda} d \cos \phi \text{ rad}} \quad \text{--- (3)}$$

But phase shift $= \beta = \frac{2\pi}{\lambda}$, then eq (3) becomes.

$$\psi = \frac{2\pi}{\lambda} d \cos \phi \Rightarrow \boxed{\beta d \cos \phi \text{ rad} = \psi} \quad \text{--- (4)}$$

$E_1 \rightarrow$ the field at a distance point P due to point source A_1 .

$$\text{ly } E_2 = \frac{1}{\sqrt{2}} e^{j\psi/2}$$

$$E_1 = E_0 e^{-j\psi/2}, \quad E_2 = E_0 e^{j\psi/2}$$

Amplitude of both the field components is E_0 as currents are same & the 2 sources are identical.

$\rightarrow$ Total field at Pt P is given by.

$$E_T = E_1 + E_2 \Rightarrow E_0 e^{-j\psi/2} + E_0 e^{j\psi/2}$$

$$E_T = E_0 (e^{-j\psi/2} + e^{j\psi/2})$$

$$= 2E_0 \left(\frac{e^{-j\psi/2} + e^{j\psi/2}}{2} \right)$$

$$= 2E_0 \cos(\psi/2) \quad (\text{Sub } \psi \text{ value})$$

$$\boxed{E_T = 2E_0 \cos \left(\frac{\beta d \cos \phi}{2} \right)}$$

Total field intensity at Pt P due to two point sources having currents same amp & phase. Total amplitude is $2E_0 \cos \left(\frac{\beta d \cos \phi}{2} \right)$

Maxima direction.

Total field is max when $\cos\left(\frac{\beta d \cos\phi}{2}\right)$ is maximum.

Condition for max. is $\cos\left(\frac{\beta d \cos\phi}{2}\right) = \pm 1$

$$\cos\left(\frac{\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cos\phi}{2}\right) = \pm 1 \Rightarrow \cos\left(\frac{\pi}{2} \cos\phi\right) = \pm 1$$

$$\frac{\pi}{2} \cos\phi = \cos^{-1}(\pm 1)$$

when $n = 0, 1, 2, \dots$

$$\frac{\pi}{2} \cos\phi_{\max} = \pm n\pi$$

If $n=0$, then $\frac{\pi}{2} \cos\phi_{\max} = 0 \Rightarrow \cos\phi_{\max} = 0$

$n=1$, then $\frac{\pi}{2} \cos\phi_{\max} = \pm \pi \Rightarrow \cos\phi_{\max} = \pm 1$

$$\phi_{\max} = 90^\circ \text{ (or) } 270^\circ$$

minima direction.

$$\cos\left(\frac{\beta d \cos\phi}{2}\right) = 0$$

$$\frac{\pi}{2} \cos\phi_{\min} = \cos^{-1}(0) \Rightarrow \frac{\pi}{2} \cos\phi_{\min} = \pm (2n+1)\frac{\pi}{2}$$

If $n=0$, $\frac{\pi}{2} \cos\phi_{\min} = \pm \frac{\pi}{2}$

$$\cos\phi_{\min} = \pm 1 \Rightarrow \phi_{\min} = \cos^{-1}(\pm 1)$$

$$\phi_{\min} = 0^\circ \text{ (or) } 180^\circ$$

2) Half power point directions.

$$\cos\left(\frac{\beta d \cos\phi}{2}\right) = \pm \frac{1}{\sqrt{2}}$$

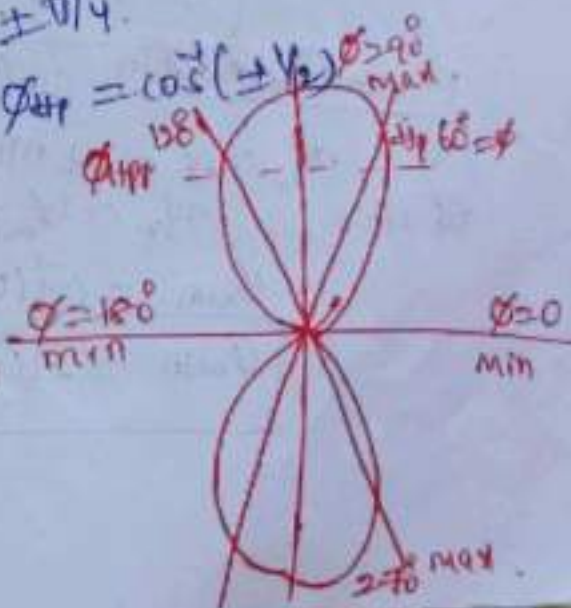
$$\frac{\pi}{2} \cos\phi = \cos^{-1}\left(\pm \frac{1}{\sqrt{2}}\right) = \pm (2n+1)\frac{\pi}{4}$$

If $n=0$, then $\frac{\pi}{2} \cos\phi = \pm \frac{\pi}{4}$

$$\cos\phi_{\text{HPP}} = \pm \frac{1}{\sqrt{2}} \Rightarrow \phi_{\text{HPP}} = \cos^{-1}\left(\pm \frac{1}{\sqrt{2}}\right)$$

$$\phi_{\text{HPP}} = 60^\circ \text{ (or) } 120^\circ$$

Field pattern for two point sources with spacing $d = \lambda/2$ & fed with currents equal in mag & phase.



Two point sources with currents equal in magnitude but opposite in phase.

All the conditions are exactly same except the phase of the currents is opposite i.e. 180° . Total field at P is given by -

$$E_T = -E_1 + E_2$$

$$E_1 = E_0 e^{j\psi/2}, \quad E_2 = E_0 e^{j\psi/2}$$

$$E_T = -E_0 e^{-j\psi/2} + E_0 e^{j\psi/2}$$

$$E_T = E_0 \left(-e^{-j\psi/2} + e^{j\psi/2} \right)$$

$$= 2jE_0 \left(\frac{-e^{-j\psi/2} + e^{j\psi/2}}{2j} \right)$$

$$\frac{e^{j0} - e^{-j0}}{2j} = \sin \psi/2$$

$$E_T = 2jE_0 \sin \psi/2$$

phase angle $\psi = \beta d \cos \phi$

$$E_T = j(2E_0) \sin \left(\frac{\beta d \cos \phi}{2} \right)$$

Maxima direction

$$\sin \left(\frac{\beta d \cos \phi}{2} \right) = \pm 1$$

$$\sin \left(\frac{\pi \lambda}{\lambda} \cos \phi \right) = \pm 1 \Rightarrow \sin \left(\pi/2 \cos \phi \right) = \pm 1$$

$$\pi/2 \cos \phi = \sin^{-1}(\pm 1) \Rightarrow \pi/2 \cos \phi = \pm (2n+1)\pi/2$$

If $n=0$, then $\pi/2 \cos \phi_{\max} = \pm \pi/2$

$$\cos \phi_{\max} = \pm 1 \Rightarrow \phi_{\max} = \cos^{-1}(\pm 1)$$

$$\phi_{\max} = 0^\circ \text{ or } 180^\circ$$

Minima direction

$$\sin(\pi/2 \cos \phi) = 0 \Rightarrow \pi/2 \cos \phi = \sin^{-1}(0)$$

$$\pi/2 \cos \phi = \pm n\pi \Rightarrow$$

If $n=0$, $\pi/2 \cos \phi_{\min} = 0 \Rightarrow \cos \phi_{\min} = 0$

$$\phi_{\min} = \cos^{-1}(0) \Rightarrow$$

$$\phi_{\min} = 90^\circ \text{ (or) } -90^\circ$$

Half power point Direction (HPD)

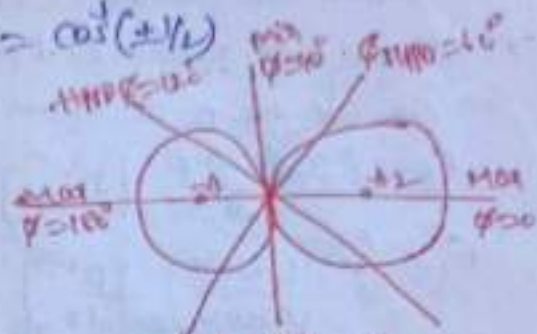
$$\sin(\pi/2 \cos \phi) = \pm \frac{1}{\sqrt{2}}$$

$$\pi/2 \cos \phi = \sin^{-1}(\pm 1/\sqrt{2}) = \pm (2n+1)\pi/4$$

$$\text{Let } n=0, \quad \pi/2 \cos \phi = \pm \pi/4$$

$$\cos \phi = \pm 1/2 \Rightarrow \phi = \cos^{-1}(\pm 1/2)$$

$$\boxed{\theta_{HPD} = 60^\circ \text{ (or) } 120^\circ}$$



(3) Two point sources with unequal magnitude and with any phase

→ Here we consider a general condition, where amplitude of two point sources are not equal and they have phase diff say α .

$$\psi = \frac{2\pi}{\lambda} d \cos \theta + \alpha$$

→ let us also assume that source 1 is taken as reference point

$$\text{then } E_r = E_1 e^{j\psi} + E_2 e^{j\psi}$$

$$\text{where } \frac{E_2}{E_1} = k \quad \text{Here } E_1 > E_2 \\ k < 1 \\ 0 \leq k \leq 1$$

$$E_r = E_1 \left[1 + \frac{E_2}{E_1} e^{j\psi} \right]$$

$$E_r = E_1 [1 + k e^{j\psi}]$$

$$= E_1 [1 + k(\cos \psi + j \sin \psi)]$$

$$|E_r| = E_1 \sqrt{(1 + k \cos \psi)^2 + (k \sin \psi)^2}$$

$$\angle E_r = \tan^{-1} \left[\frac{k \sin \psi}{1 + k \cos \psi} \right]$$

phase angle b/w two fields at the far point p is

given by

$$\boxed{\theta = \tan^{-1} \frac{k \sin \psi}{1 + k \cos \psi}}$$

N Element Uniform Linear array with equal spacing and currents equal in magnitude & phase — Broadside array
 Max radiation occurs in the directions normal to the line of array. Hence such an array is known as Uniform Broadside array.

→ Electric field produced at pt P due to an element r_0 is given by

$$E_0 = \frac{\pi d \sin \theta}{4\pi r_0 \epsilon_0} \left[\frac{p^r}{r_0} \right] e^{-j\beta r_0} \quad \text{--- (1)}$$

$$E_1 = \frac{\pi dL \sin \theta}{4\pi r_0 \epsilon_0} \left[\frac{p^r}{r_0} \right] e^{-j\beta(r_0 - d \cos \theta)}$$

$$E_1 = E_0 e^{j\beta d \cos \theta} \quad \text{--- (2)}$$

$$E_2 = E_1 e^{j\beta d \cos \theta} = E_0 e^{j2\beta d \cos \theta} \quad \text{--- (3)}$$

$$E_{n-1} = E_0 e^{j(n-1)\beta d \cos \theta} \quad \text{--- (4)}$$

total electric field at pt P is given by.

$$E_T = E_0 + E_1 + E_2 + \dots + E_{n-1} \\ = E_0 + E_0 e^{j\beta d \cos \theta} + E_0 e^{j2\beta d \cos \theta} + \dots + E_0 e^{j(n-1)\beta d \cos \theta}$$

Let $\beta d \cos \theta = \psi$, then rewriting above eqn

$$E_T = E_0 + E_0 e^{j\psi} + E_0 e^{j2\psi} + \dots + E_0 e^{j(n-1)\psi} \quad \text{--- (5)}$$

$$= E_0 [1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(n-1)\psi}] \quad \text{--- (6)}$$

consider a series given by $S = 1 + r + r^2 + r^3 + \dots + r^{n-1}$ --- (7)

multiply with r to eq (7)

$$S \cdot r = r + r^2 + r^3 + r^4 + \dots + r^n \quad \text{--- (8)}$$

Subtracting (8) from (7) $S - S \cdot r = 1 - r^n$ --- (9)

$$S(1 - r) = 1 - r^n \Rightarrow S = \frac{1 - r^n}{1 - r} \quad \text{--- (10)}$$

Using eq (9) & eq (10) can be modified as

$$E_T = E_0 \left[\frac{1 - e^{jn\psi}}{1 - e^{j\psi}} \right] \Rightarrow \frac{E_T}{E_0} = \frac{e^{jn\psi/2} [e^{jn\psi/2} - e^{-jn\psi/2}]}{e^{j\psi/2} [e^{j\psi/2} - e^{-j\psi/2}]}$$

From the trigonometric identities

$$e^{j\theta} = \cos\theta - j\sin\theta, \quad e^{-j\theta} = \cos\theta + j\sin\theta$$

$$e^{j\theta} - e^{-j\theta} = -j2\sin\theta$$

Using above trigonometric identities eq (8) can be written as

$$\frac{E_T}{E_0} = \frac{e^{jn\psi/2} [-j2\sin(n\psi/2)]}{e^{j\psi/2} [-j2\sin(\psi/2)]}$$

$$\frac{E_T}{E_0} = e^{j(n-1)\psi/2} \left[\frac{\sin(n\psi/2)}{\sin(\psi/2)} \right] \quad \text{--- (9)}$$

exponential term in eq (9) is the phase shift. Now considering magnitudes of the electric fields, we can write

$$\left| \frac{E_T}{E_0} \right| = \frac{\sin \frac{n\psi}{2}}{\sin \frac{\psi}{2}} \quad \text{--- (10)}$$

properties of Broadside Array.

- ① Major lobe. Field is max in the direction normal to the axis of array. condition for max field at P is given by.

$\psi = 0$, i.e. $\cos\phi = 0$. i.e. $\cos\phi = 0 \Rightarrow \phi = \cos^{-1}(0)$
 $\phi = 90^\circ \text{ (or) } 270^\circ$ are called directions of principle max.

- ② Magnitude of major lobe. Max radiation occurs when $\psi = 0$,

$$\begin{aligned} |\text{Major lobe}| &= \left| \frac{E_T}{E_0} \right| = \lim_{\psi \rightarrow 0} \left\{ \frac{\frac{d}{d\psi} (\sin n\psi/2)}{\frac{d}{d\psi} (\sin \psi/2)} \right\} \\ &= \lim_{\psi \rightarrow 0} \left\{ \frac{\cos(\psi/2)(n\psi/2)}{\cos(\psi/2)(\psi/2)} \right\} = n. \end{aligned}$$

$|\text{Major lobe}| = n$

③ Nulls direction of minima, equating ratio of magnitude is zero.

$$\left| \frac{E}{E_0} \right| = \frac{\sin n\psi/2}{\sin \psi/2} = 0$$

Condition for minima is given by

$$\sin n\psi/2 = 0, \text{ but } \sin \psi/2 \neq 0$$

Hence we can write $\sin n\psi/2 = 0$.

$$\text{i.e. } n\psi/2 = \sin^{-1}(0) = \pm m\pi, \text{ where } m = 1, 2, 3, \dots$$

$$\text{Now } \psi = \beta d \cos \phi = \frac{2\pi}{\lambda} d \cos \phi$$

$$\frac{n}{2} \left(\frac{2\pi}{\lambda} d \cos \phi \right) = \pm m\pi$$

$$\frac{nd}{\lambda} \cos \phi_{\min} = \pm m \Rightarrow \boxed{\phi_{\min} = \cos^{-1} \pm \left(\frac{m\lambda}{nd} \right)} \quad \text{--- (1)}$$

where n = no. of elements in array, d = spacing b/w elements in array

$\lambda = w.v.$, $m = \text{const}$

Eqn (1) gives the direction of nulls.

④ Subsidiary maxima (or side lobes)

Side lobes can be obtained $\sin(n\psi/2) = \pm 1$

$$n\psi/2 = \pm 3\pi/2, \pm 5\pi/2, \pm 7\pi/2, \dots$$

Hence $\sin(n\psi/2) = \pm 1$ is not considered. Bcz if $n\psi/2 = \pi/2$ then

$\sin n\psi/2 = 1$, which is the direction of principle maxima.

So we can skip $\sin n\psi/2 = \pm \pi/2$ value.

Thus we get $\psi = \pm 2\pi/n, \pm 4\pi/n, \pm 6\pi/n, \dots$

$$\psi = \beta d \cos \phi = \left(\frac{2\pi}{\lambda} \right) d \cos \phi = \pm 2\pi/n, \pm 4\pi/n, \pm 6\pi/n, \dots$$

$$\cos \phi = \frac{\lambda}{2\pi d} \left[\pm \frac{(2m+1)\pi}{n} \right] \text{ where } m = 1, 2, 3, \dots$$

$$\phi = \cos^{-1} \left[\pm \frac{\lambda(2m+1)}{2nd} \right] \quad \text{--- (2)}$$

Eqn (2) represents directions of side lobes.

⑤ Beamwidth of major lobe.

⑥

beamwidth between first nulls is given by

$$\text{BWFN} = 2\gamma, \text{ where } \gamma = 90 - \phi$$

$$\phi_{\min} = \cos^{-1}\left(\pm \frac{m\lambda}{nd}\right), \text{ where } m=1, 2, 3$$

$$\text{Also } 90 - \phi_{\min} = \gamma, \text{ i.e. } 90 - \gamma = \phi_{\min}$$

$$\text{Hence } 90 - \gamma = \cos^{-1}\left(\pm \frac{m\lambda}{nd}\right)$$

Taking cosine of angle on both the sides.

$$\cos(90 - \gamma) = \cos\left[\cos^{-1}\left(\pm \frac{m\lambda}{nd}\right)\right], \sin \gamma = \pm \frac{m\lambda}{nd}$$

If γ is very small, $\sin \gamma \approx \gamma$, Sub in eq. $\gamma = \pm \frac{m\lambda}{nd}$

$$\text{Let } m=1, \gamma = \pm \frac{\lambda}{nd}$$

$$\text{BWFN} = 2\gamma = \frac{2\lambda}{nd}$$

$$[nd=L]$$

$$\text{BWFN} = \frac{2\lambda}{L} \text{ rad} = \frac{2}{(L/\lambda)} \text{ rad. mdeg BWFN} = \frac{114.6}{(L/\lambda)} \text{ deg.}$$

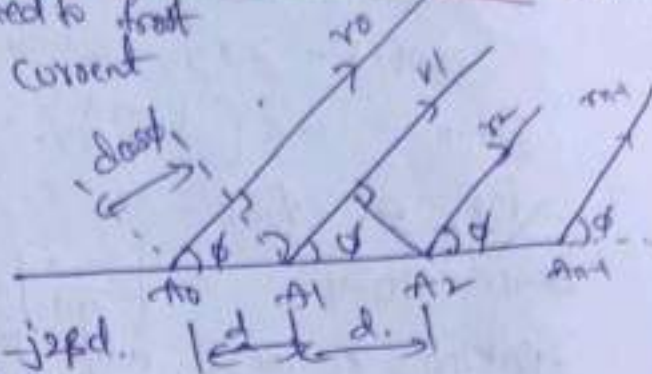
$$\text{HPBW} = \frac{\text{BWFN}}{2} = \frac{1}{(L/\lambda)} \text{ rad. HPBW} = \frac{57.3}{(L/\lambda)} \text{ deg.}$$

Directivity,

$$G_{\text{d max}} = \frac{\text{Maximum radiation intensity}}{\text{Average}} = \frac{U_{\text{max}}}{U_{\text{avg}}} = \frac{U_{\text{max}}}{U_0}$$

$$G_{\text{d max}} = 2\left(\frac{L}{\lambda}\right)$$

2) n elements with equal spacing and currents equal in Magnitude but with opposite phase — End Fire Array.
 Consider that the current supplied to first element A_0 be I_0 . Then the current supplied to A_n is given by.



$$I_1 = I_0 e^{-j\beta d}$$

$$\text{Ily } I_2 = I_1 e^{-j\beta d} \\ = [I_0 e^{-j\beta d}] e^{-j\beta d} = I_0 e^{-j2\beta d}$$

$$I_{n-1} = I_0 e^{-j(n-1)\beta d}$$

Electric field produced at pt P, due to A_0 is given by.

$$E_0 = \frac{I_0 d \sin\theta}{4\pi\omega\epsilon_0} \left[\frac{j\beta^2}{r_0} \right] e^{-j\beta r_0} \quad \text{--- (1)}$$

$$E_1 = \frac{I_0 d \sin\theta}{4\pi\omega\epsilon_0} \left[\frac{j\beta^2}{r_1} \right] e^{-j\beta r_1} \cdot e^{-j\beta d} \quad \left[\text{but } r_1 = r_0 - d \cos\phi \right]$$

$$= \frac{I_0 d \sin\theta}{4\pi\omega\epsilon_0} \left[\frac{j\beta^2}{r_0} \right] e^{-j\beta(r_0 - d \cos\phi)} e^{-j\beta d}$$

$$E_1 = E_0 e^{j\beta d(\cos\phi - 1)} \Rightarrow E_1 = E_0 e^{j\psi} \quad \left[\psi = \beta d(\cos\phi - 1) \right]$$

$$\text{Ily } E_2 = E_0 e^{j2\psi}$$

$$E_{n-1} = E_0 e^{j(n-1)\psi}$$

$$\text{Then } E_T = E_0 + E_1 + E_2 + \dots + E_{n-1}$$

$$= E_0 + E_0 e^{j\psi} + E_0 e^{j2\psi} + \dots + E_0 e^{j(n-1)\psi}$$

$$= E_0 (1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(n-1)\psi}) \quad \text{with n exponential series.}$$

$$E_T = E_0 \cdot \frac{1 - e^{jn\psi}}{1 - e^{j\psi}} \Rightarrow \frac{E_T}{E_0} = \frac{\sin n\psi/2}{\sin \psi/2} \cdot e^{j\frac{(n-1)\psi}{2}}$$

Consider only magnitude

$$\boxed{\left| \frac{E_T}{E_0} \right| = \frac{\sin n\psi/2}{\sin \psi/2}}$$

Properties of End fire arrays.

① Major lobe. $\psi = \rho d(\cos\phi - 1) = 0 \Rightarrow \cos\phi = 1 \Rightarrow \phi = \cos^{-1}(1)$

$\boxed{\phi = 0^\circ}$ direction of principle maxima

② Magnitude of the major lobe

$$|\text{Major lobe}| = \lim_{\psi \rightarrow 0} \left\{ \frac{\frac{d}{d\psi}(\sin n\psi/2)}{\frac{d}{d\psi}(\sin\psi/2)} \right\} = \lim_{\psi \rightarrow 0} \left\{ \frac{\cos(n\psi/2)(n/2)}{\cos(\psi/2)(1/2)} \right\}$$

$$\boxed{|\text{Major lobe}| = n}$$

③ Nulls $\left| \frac{E}{E_0} \right| = \frac{\sin n\psi/2}{\sin\psi/2} = 0$

Condition of minima $\sin n\psi/2 = 0$, but $\sin\psi/2 \neq 0$.

$$\sin n\psi/2 = 0 \Rightarrow n\psi/2 = \pm m\pi, m = 1, 2, 3, \dots$$

$$n \frac{\rho d(\cos\phi - 1)}{2} = \pm m\pi \quad \text{put } \left[\rho = \frac{2\pi}{\lambda} \right]$$

$$\frac{n d}{\lambda} (\cos\phi - 1) = \pm m \quad \left[(\cos\phi - 1) \text{ is always less than } 1 \right]$$

$$\frac{n d}{\lambda} (\cos\phi - 1) = -m \quad \text{ie } \cos\phi = 1 - \frac{m\lambda}{nd}$$

$$\boxed{\phi_{\min} = \cos^{-1} \left[1 - \frac{m\lambda}{nd} \right]} \quad \text{by } \boxed{\phi_{\min} = 2 \sin^{-1} \left[\pm \frac{m\lambda}{2nd} \right]}$$

④ Subsidiary maxima (or side lobe)

$$\sin(n\psi/2) = \pm 1, \quad n\psi/2 = \pm 3\pi/2, \pm 5\pi/2, \pm 7\pi/2, \dots$$

Hence $n\psi/2 = \pm \pi/2$ is skipped bcz this value of $n\psi/2$, $\sin n\psi/2 = 0$, which is the direction of principle maxima.

we can write $n\psi/2 = \pm (2m+1)\pi/2, m = 1, 2, 3, \dots$

$$\frac{n \rho d(\cos\phi - 1)}{2} = \pm (2m+1)\pi/2 \Rightarrow \frac{n \rho d(\cos\phi - 1)}{2} = \pm (2m+1)\pi$$

$$\text{put } \rho = \frac{2\pi}{\lambda}, \quad \left[\frac{n \left(\frac{2\pi}{\lambda} \right) d(\cos\phi - 1)}{2} \right] = \pm (2m+1)\pi$$

$$\cos\phi - 1 = \pm (2m+1) \frac{\lambda}{2nd}$$

$$\boxed{\phi = \cos^{-1} \left[1 - \frac{(2m+1)\lambda}{2nd} \right]}$$

$(\cos\phi - 1)$ is always less than 0
So can't take -ve value.

⑤ Beamwidth of major lobe

Beamwidth = $2 \times$ Angle b/w first nulls & maximum of the major lobe.

$$\phi_{\min} = 2 \sin^{-1} \left[\pm \sqrt{\frac{m\lambda}{2nd}} \right] \quad \left[\sin \frac{\phi_{\min}}{2} \approx \frac{\phi_{\min}}{2} \text{ for } \phi_{\min} \text{ very small} \right]$$

$$\sin \frac{\phi_{\min}}{2} = \pm \sqrt{\frac{m\lambda}{2nd}}$$

$$\frac{\phi_{\min}}{2} = \pm \sqrt{\frac{m\lambda}{2nd}} \Rightarrow \phi_{\min} = \pm \sqrt{\frac{4m\lambda}{2nd}} = \pm \sqrt{\frac{2m\lambda}{nd}} \quad [nd=L]$$

$$\phi_{\min} = \pm \sqrt{\frac{2m\lambda}{L}} = \pm \sqrt{\frac{2m}{L/\lambda}}$$

$$\text{BWFLN} = 2\phi_{\min} = \pm 2 \sqrt{\frac{2m}{L/\lambda}} \text{ rad} = \pm 2 \sqrt{\frac{2m}{(L/\lambda)}} \times 57.3$$

$$= \pm 114.6 \frac{2m}{(L/\lambda)} \text{ deg.}$$

⑥ Directivity

$$G_{\max} = \frac{U_{\max}}{U_0}$$

endfire array $U_{\max} = 1$,

$$G_{\max} = \frac{1}{\frac{\pi}{2n\beta d}} = \frac{2n\beta d}{\pi} = 2n \left(\frac{2\pi}{\lambda} \right) \left(\frac{d}{4\pi} \right) \quad U_0 = \pi / 2n\beta d$$

$$[nd=L]$$

$$G_{\max} = 4 \left(\frac{nd}{\lambda} \right) \Rightarrow 4 (L/\lambda)$$

element uniform linear broadside array & Endfire array	Broadside array.	Endfire array
1) Direction of major lobe	$\phi_{max} = 90^\circ$ (or) $\phi_{max} = 2\pi$	$\phi_{max} = 0$
2) Magnitude of major lobe	n	n
3) Direction of minor lobe	$\phi_{min} = \cos^{-1} \left[\pm \frac{m\lambda}{nd} \right]$ where $m = 1, 2, 3$	$\phi_{min} = \cos^{-1} \left[1 - \frac{m\lambda}{nd} \right]$ where $m = 1, 2, 3$
4) Direction of side lobes (SLL)	$\phi = \cos^{-1} \left[\pm \frac{\lambda(2m+1)}{2nd} \right]$ where $m = 1, 2, 3$	$\phi = \cos^{-1} \left[1 - \frac{(2m+1)\lambda}{2nd} \right]$ where $m = 1, 2, 3$
5) BWFN	$BWFN = \frac{2\lambda}{L} = \frac{2\lambda}{L/\lambda} = \frac{2}{L/\lambda}$	$BWFN = \pm 2 \sqrt{\frac{2m}{L/\lambda}} = \pm 1.19 \sqrt{\frac{2m}{L/\lambda}}$
6) HPBW	$HPBW = \frac{BWFN}{2} = \frac{1}{L/\lambda} \text{ rad} = \frac{57.3}{L/\lambda} \text{ deg}$	$HPBW = \pm \sqrt{\frac{2m}{L/\lambda}} \text{ rad} = \pm 1.19 \sqrt{\frac{2m}{L/\lambda}} \text{ deg}$
7) Directivity	$D = 2 \left(\frac{nd}{\lambda} \right) = 2(L/\lambda)$	$D = 4(L/\lambda)$

① A broadside array of identical antennas consists of 8 isotropic radiators separated by distance $\lambda/2$. Find radiation field in coplanar containing the line of array showing directions of maxima & null.

given $n=8, d=\lambda/2$

① Major lobe $\phi = 0, \text{ Beamwidth} = 0, \phi = 9.0^\circ \text{ to } 270^\circ$

② Magnitude of major lobe $\left| \frac{E_{\phi}}{E_0} \right| = n = 8$

③ Nulls $\phi_{\min} = \cos^{-1} \left[\pm \frac{m\lambda}{nd} \right]$ where $m=1, 2, 3, \dots$

③ If $m=1$ $\phi_{\min} = \cos^{-1} \left[\pm \frac{m\lambda}{nd} \right] = \cos^{-1} \left[\pm \frac{\lambda}{(8)(\lambda/2)} \right] = \cos^{-1}(\pm 1/4) = 75.52^\circ$
 104.47°

③ If $m=2$ $\phi_{\min} = \cos^{-1} \left[\pm \frac{2\lambda}{(8)(\lambda/2)} \right] = 68.19^\circ$ & 121.81°

③ If $m=3$ $\phi_{\min} = \cos^{-1} \left[\pm \frac{3\lambda}{(8)(\lambda/2)} \right] = 41.4^\circ$ & 138.6°

④ Side lobe $\phi = \cos^{-1} \left[\pm \frac{d(2m+1)}{2nd} \right]$

a) If $m=1$ $\phi_{\min} = \cos^{-1} \left[\pm \frac{\lambda(2+1)}{2 \times 8 \times d/2} \right] = \cos^{-1} \left[\pm 3/8 \right] = 61.92^\circ$
 118.08°

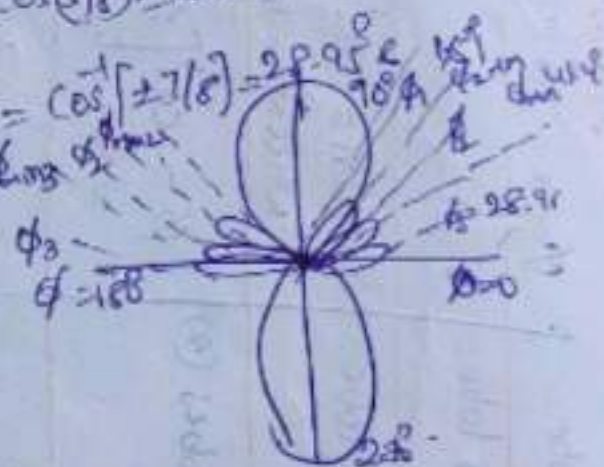
b) If $m=2$ $\phi_{\min} = \cos^{-1} \left[\pm \frac{\lambda(4+1)}{2 \times 8 \times d/2} \right] = \cos^{-1} \left[\pm 5/8 \right] = 51.31^\circ$ & 128.69°

c) $m=3$ $\phi_{\min} = \cos^{-1} \left[\pm \frac{\lambda(6+1)}{2 \times 8 \times d/2} \right] = \cos^{-1} \left[\pm 7/8 \right] = 29.94^\circ$ & 170.06°

⑤ $\text{BWPN} = \frac{2\lambda}{L} \left[L = nd = 8 \times \frac{\lambda}{2} = 4\lambda \right]$
 $= \frac{2\lambda}{4\lambda} = \frac{1}{2} = 0.5 \text{ rad.}$

⑥ $\text{HPBW} = \frac{\text{BWPN}}{2} = \frac{0.5}{2} = 0.25$

⑦ $G_{\text{max}} = 2 \left(\frac{L}{\lambda} \right) = 2 \left(\frac{4\lambda}{\lambda} \right) = 8$



- ② Find the minimum spacing b/w the elements in a broadside array of 10 isotropic radiators to have directivity of 7dB

$$G_{\text{broad}} = 7\text{dB}, n = 10$$

$$G_{\text{broad}} = 10 \log_{10} [G_{\text{max}}]$$

$$7 = 10 \log_{10} [G_{\text{max}}] \Rightarrow G_{\text{max}} = 5.0118$$

$$G_{\text{max}} = 2 \left(\frac{L}{\lambda} \right) = 2 \left(\frac{n d}{\lambda} \right) \Rightarrow 5.0118 = 2 \left(\frac{10 \times d}{\lambda} \right) = 0.25 \lambda$$

- ③ Find the length and BFW for broadside and endfire array if the directive gain is 15

$$G_{\text{max}} = 15$$

① For broadside array $G_{\text{max}} = 2 \left(\frac{L}{\lambda} \right) \Rightarrow 15 = 2 \left(\frac{L}{\lambda} \right) \Rightarrow L = 7.5 \lambda$

② $B_{\text{broad}} = \frac{114.6}{\left(\frac{L}{\lambda} \right)} \text{ deg} = \frac{114.6}{\left(\frac{7.5 \lambda}{\lambda} \right)} = 15.28^\circ$

Endfire array $G_{\text{max}} = 4 \left(\frac{L}{\lambda} \right) \Rightarrow 15 = 4 \left(\frac{L}{\lambda} \right) \Rightarrow L = 3.75 \lambda$

② $B_{\text{end}} = 114.6 \sqrt{\frac{2}{\left(\frac{L}{\lambda} \right)}} \text{ deg} = 114.6 \sqrt{\frac{2}{\left(\frac{3.75 \lambda}{\lambda} \right)}} = 83.6^\circ$

- ④ A uniform linear array consists 16 isotropic sources with spacing of $\lambda/4$. If the phase difference is 90° . Calculate
 1) Directivity index 2) Beam solid angle 3) Effective aperture

① HPBW

$$n = \text{no. of total elements} = 16$$

$$d = \text{spacing b/w adjacent elements} = \lambda/4$$

$$L = \text{total length of array} = (n-1)d = (16-1) \frac{\lambda}{4} = 15 \frac{\lambda}{4}$$

① $\text{HPBW} = 57.3 \sqrt{\frac{2m}{\left(\frac{L}{\lambda} \right)}} \text{ deg} \Rightarrow 57.3 \sqrt{\frac{2(1)}{\frac{15 \lambda}{4} \cdot \frac{1}{\lambda}}} = \boxed{41.84^\circ} \quad \left[\begin{array}{l} m=1, \\ L=15 \frac{\lambda}{4} \end{array} \right]$

② Directivity $D = 4 \left(\frac{L}{\lambda} \right) = 4 \left(\frac{15 \lambda}{4 \lambda} \right) = 15 \Rightarrow 10 \log_{10} 15 = 11.76 \text{ dB}$

③ Beam solid angle $\Omega = \frac{4\pi}{D} = \frac{4 \times \pi}{15} = 0.8377 \text{ sr}$

④ $A_e = \frac{D \lambda^2}{4\pi} = \frac{15 \times \lambda^2}{4\pi} = 1.1936 \lambda^2$

⑤ calculate 1) HPBW 2) Solid angle if a linear array having 10 isotropic point sources with $\lambda/2$ spacing and phase difference $\delta = 90^\circ$.
 $\Rightarrow$ phase difference $\delta = 90^\circ$, $n = 10$, $d = \lambda/2$.
 $L = (n-1)d = (10-1)\lambda/2 = 9\lambda/2$ [$m=1$, $L=\frac{\lambda}{2}$]

$$1) \text{ HPBW} = 57.3 \sqrt{\frac{2\pi}{(L/\lambda)}} \text{ deg} = 57.3 \sqrt{\frac{2\pi}{(9\lambda/2)/\lambda}} = 57.3 \sqrt{\frac{4}{9}} = 38.2^\circ$$

2) beam solid angle $\Omega = \frac{4\pi}{D}$ where $D = \text{Directivity}$

$$D = 4 \left(\frac{L}{\lambda} \right) = 4 \left(\frac{9\lambda/2}{\lambda} \right) = 18.$$

$$\Omega = \frac{4\pi}{D} = \frac{4\pi}{18} = 0.6981 \text{ sr}.$$

⑥ find the phasing required to steer a beam zenith to -40° for a 5 element array with 0.4λ inter element spacing.

$n = \text{no. of elements} = 5$, $d = \text{spacing} = 0.4\lambda$, $2\phi = \text{beamwidth} = -40^\circ$.

phase difference required for radiations of two adjacent points sources is given by $\psi = \frac{2\pi}{\lambda} d \cos\phi + \alpha$ [$\alpha = 0$]

$$\psi = \frac{2\pi}{\lambda} d \cos\phi = \frac{2\pi}{\lambda} (0.4\lambda) \cos(-20^\circ) = 2.3617 \text{ rad}$$

⑦ Calculate the directivity of given linear endfire array with improved directivity. Hansen-Woodward (increased directivity) uniform array of 10 elements with a separation of $\lambda/4$ b/w the elements.

Endfire array with increased directivity $D = 1.789 \left[4 \left(\frac{L}{\lambda} \right) \right]$

$$= 1.789 \left[4 \left(\frac{n \cdot d}{\lambda} \right) \right] = 1.789 \left[4 \left(\frac{10 \times (\lambda/4)}{\lambda} \right) \right] = 17.89$$

$$D \text{ in dB} = 10 \log(17.89) = 12.526 \text{ dB}.$$

Directivity of Endfire Array with increased Directivity.

(19)

[Hansen-Woodward End-fire array]

For endfire array with increased directivity and minimum radiation in $\theta = 0^\circ$ direction, the radiation intensity for small spacing d elements ($d \ll \lambda$) is given by $U_0 = \frac{1}{\eta \beta d} \left(\frac{\pi}{2} \right)^2 \left[\frac{\pi}{2} + \frac{2}{\pi} - 1.81r \right]$

$U_0 = \frac{0.878}{\eta \beta d}$ multiplying numerator & denominator with 2π

$$U_0 = \frac{0.878 \times 2\pi}{\eta \beta d \times 2\pi} = \frac{1.756\pi}{2\pi \eta \beta d} = \frac{1.756}{\pi} \left(\frac{\pi}{2\eta \beta d} \right)$$

Directivity is given by $D = \frac{U_{max}}{U_0} = \frac{1}{0.559 \left(\frac{\pi}{2\eta \beta d} \right)}$

$D = \frac{1}{0.559 \left(\frac{\pi}{2\eta \beta d} \right)} \quad \left[d = \frac{2\pi}{\lambda} \right]$

$$= 1.789 \left[\frac{2\pi \left(\frac{2\pi}{\lambda} \right) d}{\pi} \right] = 1.789 \left[\frac{4\pi d}{\lambda} \right]$$

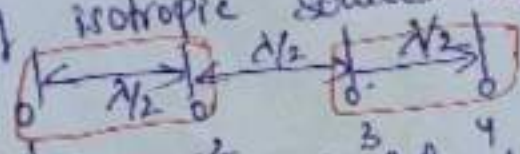
$$D = 1.789 \left[4 \left(\frac{\pi}{\lambda} \right) \right]$$

$$L = (n-1)d \sin \theta$$

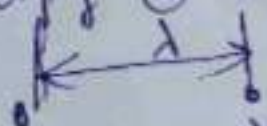
Pattern multiplication of 4 element pt sources.

→ with the help of this method, it is possible to sketch the radiation pattern of the arrays easily.

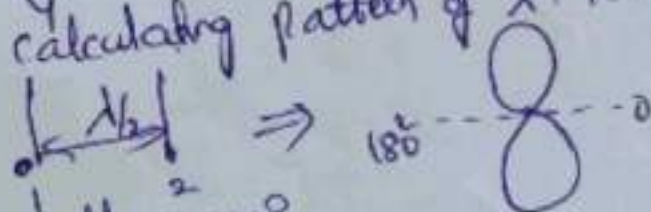
Array 1: isotropic sources separated by $\lambda/2$



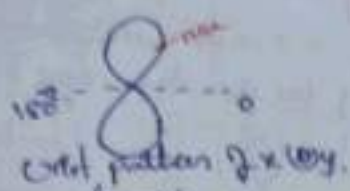
We can consider group of 1 & 2 pts. Resultant is X.
group of 3 & 4 pt Resultant is Y.



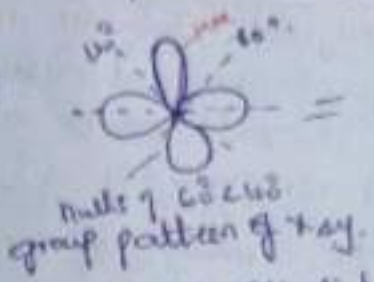
→ we will be calculating pattern of 'X' i.e. group pattern of 1 & 2
is having nulls at 180°



Resultant pattern $E = \text{Unit pattern} \times (\text{array}) \otimes \text{group pattern of array}$



$\otimes$



Radiation pattern of two antennas spaced at distance $\lambda/2$ and fed with 0°

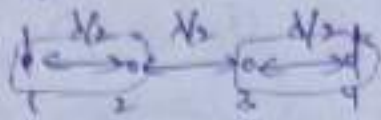
Rp of $\lambda/2$ distance

[max & min = max
max & min = min]

Binomial

This is an array with non uniform amplitude.
The amplitudes are arranged so that the radiation pattern has no minor lobes.

Unit p radiation pattern is $E = \cos(\pi b \cos \theta)$
 $d = \lambda/2$

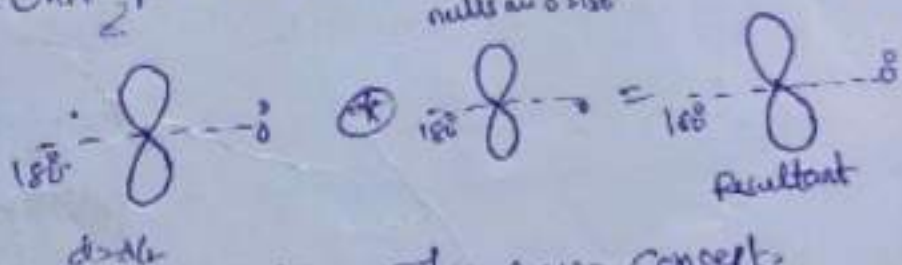


$\Rightarrow$ pattern of x = $\cos$ $\Rightarrow$ Resultant pattern is = Unit pattern of x (or) y $\otimes$ group pattern of x array.

So ultimately ① $\otimes$ ②

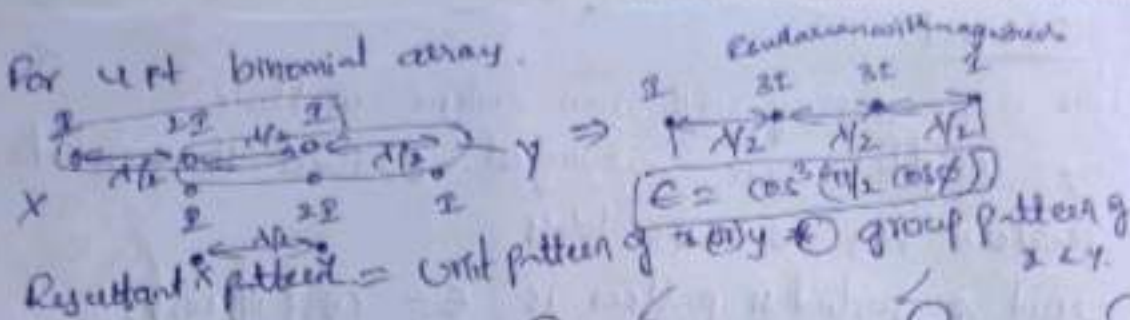
How we should reduce the basic arrangement is binomial away. $E = \cos(\pi b \cos \theta)$

Resultant pattern = Unit pattern of x (or) y $\otimes$ group pattern of x array.
 nulls are 0 and π



as we will be finding no minor lobes. This basic concept overlapping element ② $\otimes$ ③.

For unit binomial array.



There are no minor lobes

⇒ In general Electric field $E = \cos^{n-1}(\frac{\pi}{2} \cos \phi)$ where $n = 1, 2, \dots$ = no. of elements

⇒ In general electronic configuration

By Pascal's theorem (or) triangle calculate magnitude of elements.

Pascal's triangle displaying coefficients of the binomial series.

					$n=1$
1					$n=2$
1		1			$n=3$
1	2	1			$n=4$
1	3	3	1		$n=5$
1	4	6	4	1	$n=6$
1	5	10	10	5	1

Length of binomial array $L = (n-1)d/2$

HPBW $= \frac{1.06}{\sqrt{n-1}} \frac{1.06}{\sqrt{2\lambda/d}}$

$= \frac{0.75}{\sqrt{L/\lambda}}$

⇒ Directivity $D \approx 6.77 \sqrt{n}$

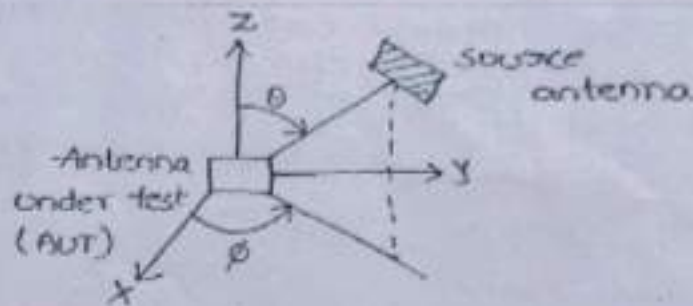
$= 6.77 \sqrt{1+2(L/\lambda)}$

We can calculate magnitude of directivity of elements

Antenna Measurements

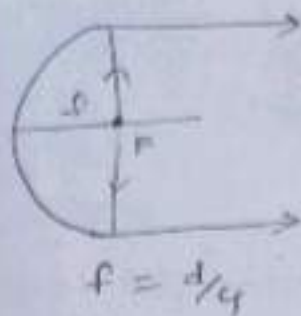
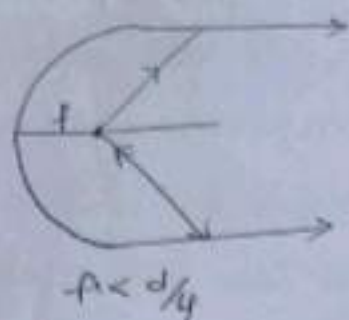
Basic Concept of Antenna Measurements:

In general, the important measurement parameters of the antenna are gain, directivity, radiation pattern etc.



TYPICAL CONFIGURATION UP FOR MEASUREMENT OF RADIATION PROPERTIES.

The Antenna Under Test (AUT) is considered to be located at origin of the coordinate system. The source antenna is placed at different locations with respect to the AUT. Note that the source antenna may be transmitting or receiving. At different locations the number of samples of the pattern are obtained. To achieve different locations, generally AUT is rotated. To achieve sharp sample of pattern, it is necessary that there exists single direct signal path b/w the AUT and source antenna.



When the focal point lies on the plane of the open mouth of the paraboloid by the geometry the focal length f is one fourth of the open mouth diameter d . This condition gives maximum gain pencil shaped radiation equal in horizontal and vertical plane.

When the focal length is too large, the focal point lies beyond the open mouth of the paraboloid as shown in fig above. Here it is difficult to direct all the radiations from the source on the reflector.

Note For practical applications, the value of the focal length to diameter ratio lies between 0.25 and 0.5

Practically it is observed that some of the rays are not fully captured by reflector, such non-captured rays form 'Spill over'.

While receiving spill over, the noise pick up increases which is troublesome. In addition to this, few radiations originated from the primary radiations are observed in forward direction. Such radiations get added with desired parallel beam. This is called 'back lobe radiation'.

Reciprocal Relationship between Transmitting and Receiving properties of Antenna:

Generally antenna can act either as a transmitter or receiver. There exists a reciprocal relationship between the transmitting and receiving properties of the antenna. This reciprocal relationship is very useful in the antenna measurements.

It is necessary to study two important consequences helpful in antenna measurements

The consequences are,

1. The transmitting and receiving patterns of antenna are same.
2. The power flow is the same in transmitting and receiving mode.

Thus antenna under test discussed earlier in the section can be used in either transmitting or receiving mode. When the AUT is used in a huge transmitter or receiver, the direction of the signal can be defined easily. Practically while using reciprocity relationship following conditions must get fulfilled.

1. The emfs at the terminals of the transmitting and receiving antennas should be of same frequency.
2. The power flow should be equal to that due to matched impedances.
3. The media should be linear, isotropic and passive.

Sources of Error in Antenna Measurement:

1. Errors due to finite measurement distance

between antennas: When the distance between the antennas is very small, then the field received by the AOT at different points will be with different phases causing quadratic phase error. The quadratic phase error affects by reducing the measured gain and increasing the side lobe as compared to the ideal uniform plane wave condition.

Due to the small distances between the antennas, the amplitude gets affected. The amplitude errors are of two types transverse amplitude error and longitudinal amplitude error. In transverse plane, the amplitude of the field is small

at the edges of the AUT, while slightly greater away from the edges. This causes the transverse amplitude errors.

2. Reflections from surroundings: The reflections from

surroundings is another important source of the error because reflections cause amplitude ripple as well as phase ripple.

The ripples occur in a region, due to the interference between the direct wave and reflected wave.

3. Errors due to coupling in the reactive Near field:

The reactive near field causes significant error at low frequencies. If the distance is greater than 10λ , then the coupling is negligible.

4. Errors due to misalignment of antenna: Basically

the antenna measurement is a 3-dimensional vector measurement so any misalignment of the source antenna causes amplitude errors. Due to the misalignment the pattern cut can not be properly taken.

5. Errors due to manmade interface: On out-

door ranges, when the man-made interference

gets coupled with receiver at the frequency same as the measurement frequency or any other frequency harmonic distortion takes place.

On indoor ranges, in anechoic chambers, the reflections from the walls, floor and ceiling are significant.

6. Errors due to atmospheric effects: Due to the atmospheric effects, such as variation of refractive index of atmosphere, multihop propagation takes place which finally results in significant amplitude variation during measurement. At higher frequencies, the attenuation of the atmosphere is very high which results in amplitude variation.

7. Errors due to cables: If the cables used for the connection do not have proper shielding, leakage occurs and the cables act as antenna producing measurement errors. The incorrect use of cables also cause errors.

8. Errors due to impedance mismatch: If the antenna impedance is not properly matched with the instrument impedance, errors occur in

the gain measurement.

Factors due to Imperfections of Instruments:

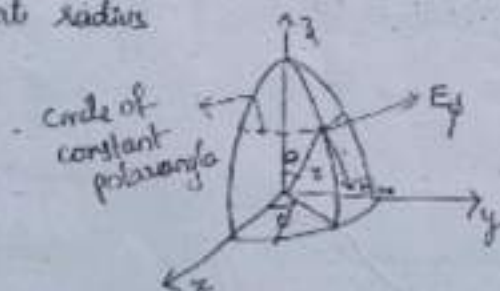
Due to the imperfections of the instruments in the measurements such as transmitters, receivers, positioners etc, the measurement error occurs.

Measurement of Radiation pattern:-

The Radiation capabilities of an antenna are characterised by the characteristics of an antenna such as the

- (1) Radiation pattern [Amplitude and phase pattern]
- (2) polarization
- (3) gain.

→ All these quantities are measured on the surface of a sphere with constant radius.



→ Basically for representation of a point on the surface, only θ and ϕ specifications are sufficient because of a sphere with constant radius is considered.

→ Thus the radiation characteristics of the antenna is a function of θ and ϕ for constant radius and frequency is called radiation pattern of an antenna.

for horizontal antenna following patterns are required.

- (i) The ϕ component of electric field as a function of ϕ measured in $x-y$ plane ($\theta = 90^\circ$). The field component can be then represented as $E_\phi(\theta = 90^\circ, \phi)$ and it is called E -plane pattern.
- (ii) The θ component of electric field as a function of θ measured in $x-z$ plane ($\phi = 90^\circ$). The field component can be then represented as $E_\theta(\theta, \phi = 90^\circ)$ and it is called H -plane pattern.

For vertical antenna

- (i) The θ component of electric field as a function of ϕ measured in $x-y$ plane ($\theta = 90^\circ$). The field component can be represented as $E_\theta(\theta = 90^\circ, \phi)$ and it is called H -plane pattern.
- (ii) The ϕ component of the electric field as a function of ϕ measured in $x-z$ plane ($\theta = 90^\circ$). The field component can be represented as $E_\phi(\theta = 90^\circ, \phi)$ and it is called E -plane pattern.

Measurement of Gain of an antenna:

- The performance of any antenna can be decided on performance in terms of figure of merit i.e. gain of an antenna.
- Depending upon the frequency of operation various methods can be used for the measurement of gain of an antenna.

Basically there are two standard methods used for the measurement of gain of an antenna.

- (i) Gain-transfer method or direct comparison method
- (ii) Absolute-gain method.

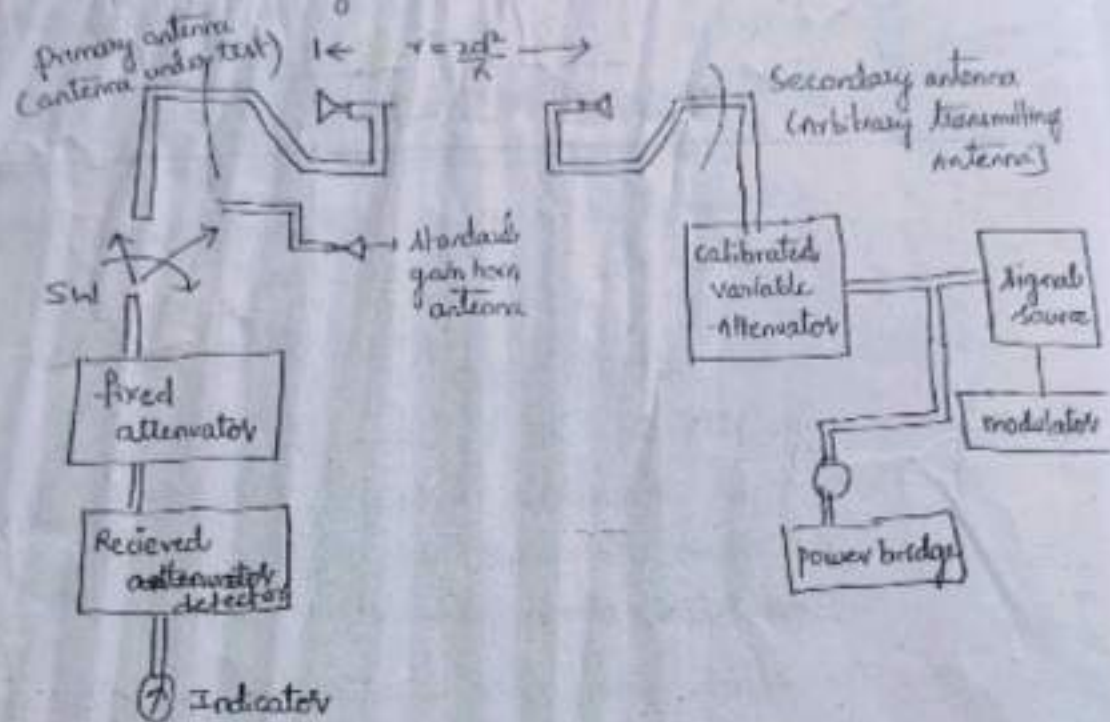
Gain measurement by Direct comparison method.

- The gain measurement is done by comparing the strengths of the signals transmitted or received by the antenna under test and the standard gain antenna.
- The antenna whose gain is accurately known and can be used for the gain measurement of other antenna is called standard gain antenna.
- At high frequency, the universally accepted standard gain antenna is the horn antenna.

Procedure

- This method uses two antennas termed as primary antenna and secondary antenna. The secondary antenna is arbitrary transmitting antenna.

- the knowledge of gain of the secondary antenna is not necessary
- The primary antenna consists two different antennas separated through a switch sw.
- The first primary antenna is the standard gain antenna (horn etc)



Set up of gain measurement by gain comparison method

- The two primary antennas are located with sufficient distance of separation in b/w. so to avoid interference and coupling b/w the two antennas.
- while the primary and secondary antennas are separated by with a distance greater than or equal to $\frac{2d^2}{\lambda}$ to minimize the reflection b/w them to great extent.

→ To ensure almost frequency stability at the transmitter, the power bridge circuit is used.

The gain measurement by the gain-comparison method is two step procedure.

1) through the switch SW, first standard gain antenna is connected to the Receiver. The Antenna is adjusted in the direction of the secondary antenna to get maximum signal intensity.

→ The input connected to the secondary or transmitting antenna is adjusted to required level.

→ For this input corresponding primary antenna reading at the receiver is recorded.

→ Corresponding attenuator and power bridge readings are recorded as A_1 and P_1 .

2) Secondly the antenna under test is connected to the receiver by changing the position of the switch SW. To get the same reading at the receiver, the auto attenuator is adjusted. Then corresponding attenuator and power bridge readings are recorded as A_2 and P_2 .

Case 2:- If $P_1 = P_2$, then no correction needed to be applied and the gain of the subject antenna under test is given by

$$\text{power gain} = G_p = \frac{A_2}{A_1}$$

Taking log on both sides

$$\log_{10} G_p = \log_{10} \left(\frac{A_2}{A_1} \right)$$

$$G_{p(dB)} = A_{e(dB)} - A_{i(dB)}$$

Case II :- If $P_1 \neq P_2$, then the correction need to be included

$$\frac{P_1}{P_2} = r, \text{ then}$$

$$\log_{10} \frac{P_1}{P_2} = r(dB)$$

Hence power gain is given by

$$G = G_p \times \frac{P_1}{P_2} = \frac{A_e}{A_i} \frac{P_1}{P_2}$$

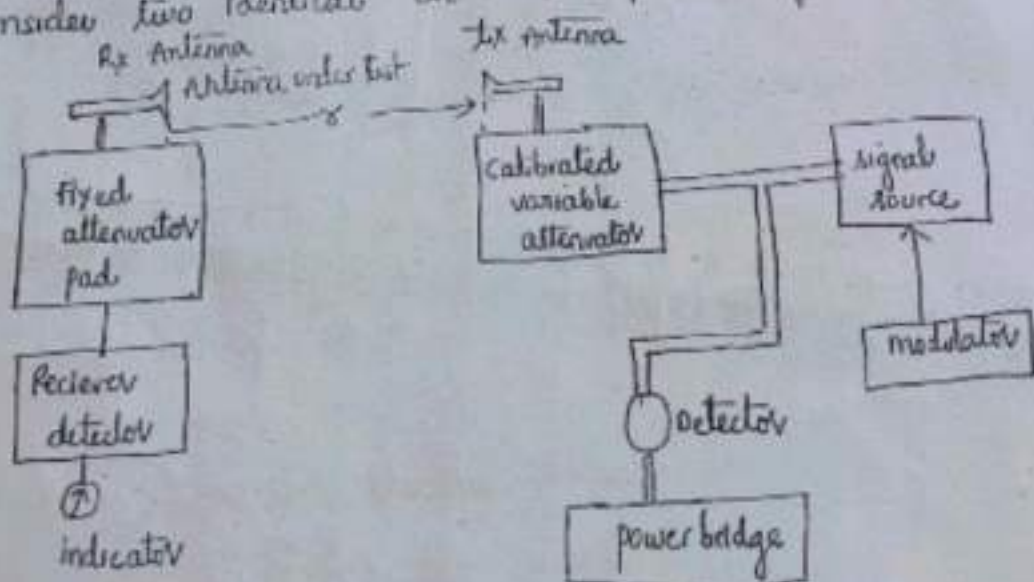
$$G = G_p \frac{P_1}{P_2}$$

$$\log G = \log G_p + \log_{10} (P_1/P_2)$$

$$G_{(dB)} = G_{p(dB)} + P_1/P_2(dB)$$

Measurement of absolute gain

Consider two identical antennas separated by distance r



Let the transmitted power be denoted by P_t and the received power by P_r . Let the effective apertures of the transmitting and receiving antennas be A_{et} and A_{er} respectively.

→ Two antennas are identical identical then

$$A_{et} = A_{er} = \frac{G_0 \lambda^2}{4\pi}$$

From Friis equation, we can write

$$\frac{P_r}{P_t} = \frac{A_{er} \cdot A_{et}}{\lambda^2 \cdot r^2} = \left[\frac{G_0 \lambda^2}{4\pi} \right] \left[\frac{G_0 \lambda^2}{4\pi} \right] \frac{1}{\lambda^2 \cdot r^2}$$

$$\frac{P_r}{P_t} = \frac{4\pi}{\lambda^2} \left[\frac{G_0 \lambda^2}{4\pi} \right]^2$$

$$\frac{G_0 \lambda}{4\pi r} = \sqrt{\frac{P_r}{P_t}}$$

$$G_0 = \frac{4\pi r}{\lambda} \sqrt{\frac{P_r}{P_t}}$$

By knowing wave length λ , distance b/w two antennas r , and measuring the radiated and received powers, the absolute gain of the antenna is obtained.

Measurement of Directivity

- Directivity is obtained from the radiation pattern of the antenna
- The Directivity of antenna is defined as the ratio of maximum power density to the avg power radiated

$$G_{\text{max}} = \frac{E_{\text{max}}}{\frac{P_{\text{rad}}}{4\pi r^2}} = D$$

Basically the directivity of an antenna is a dimensionless quantity. The directivity can be expressed in terms of the electric field intensity as

$$D = G_{\text{max}} = \frac{4\pi |E_{\text{max}}|^2}{\int_0^{2\pi} \int_0^\pi |E(\theta, \phi)|^2 \sin\theta d\theta d\phi}$$

$$D = G_{\text{max}} = \frac{4\pi}{\int_0^{2\pi} \int_0^\pi \frac{|E(\theta, \phi)|^2}{|E_{\text{max}}|^2} \sin\theta d\theta d\phi}$$

$$D = G_{\text{max}} = \frac{4\pi}{\int_0^{2\pi} \int_0^\pi f(\theta, \phi) \sin\theta d\theta d\phi}$$

where $f(\theta, \phi)$ is the relative radiation intensity as a function of space angles θ and ϕ

$$D = \frac{41,253}{\theta_1 \cdot \theta_2} \quad \text{where}$$

$\theta_1 \rightarrow$ HPBW of E-plane (H-plane)

$\theta_2 \rightarrow$ HPBW of H-plane (E-plane)

$$\text{or } D = \frac{71,815}{\theta_1^2 + \theta_2^2}$$

$\rightarrow$ The main drawback of this method is the least accuracy in the measurement

Near field & Far field

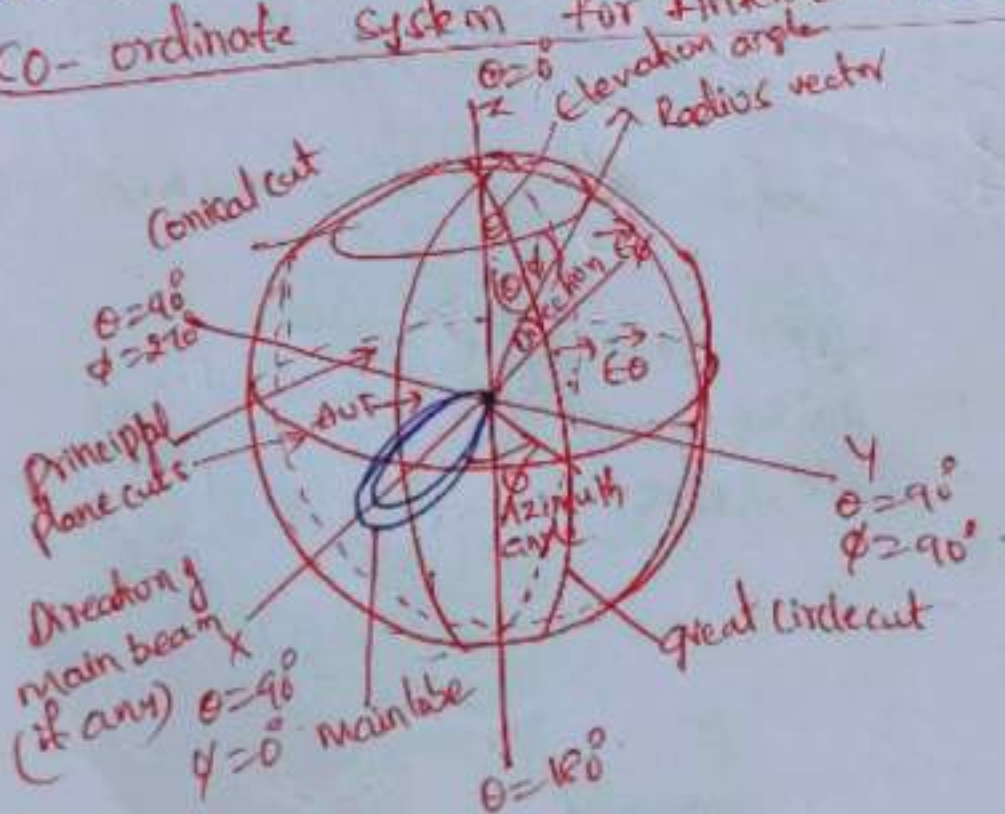
The advantages of the measurement in the far field are

- 1) Coupling and multiple reflections are least significant in the far field region.
- 2) In the far field region, only power measurement serves the purpose for obtaining power pattern.
- 3) At any pt in the rest far field region, the field pattern measured is valid.

→ But the major drawback of the farfield measurement is that it requires large distance b/w Tx & Rx antennas. Distance increases far field region suffers by atmospheric attenuation.

⇒ Near field region is located very close to the Ant. Also due to the mutual impedance as a result of the reactive coupling b/w two antennas, the measurement becomes complicated hence practically the reactive nearfield region is also not used for antenna measurement.

Co-ordinate system for Antenna Measurement.



- angle measured from the z -axis is called elevation angle and it is denoted by θ .
- The angle measured from the projection of the radio vector to the horizontal $x-y$ plane is called azimuth angle which is denoted by ϕ .
- ⇒ Depending upon the mechanical structure of the antenna the coordinate system is defined such that the peak radiation takes place along z -axis in general.
- ⇒ when the source antenna is moved along lines of const θ , the cuts obtained are called Conical cuts or θ -cuts.
- ⇒ when source antenna is moved along lines of const ϕ , the cuts obtained are called great circle cuts or ϕ -cuts. If the cut is taken along the equator with $\theta = \pi/2$ then such a cut is called θ -cut as well as ϕ -cut.
- ⇒ The two principle plane cuts are the orthogonal great circle cuts through the axis of the main lobe.



Fringing effect in MSA

→ Fringing effect is main role in designing of antenna.

→ Because of fringing effect it Radiation goes in air.

→ when fed antenna w/o ground plane EM waves coupling from dielectric to ground plane when it comes from edges to ground it goes into air.

→ Because of fringing MSA radiates in the space. & thus can increase the fringing effect by 3 ways.

1) To increase the fringing by 3 ways

1) Reducing the ϵ_r increase fringing

2) Increase height of substrate we can ↑ fringing

3) By increasing w we can ↑ fringing

Designing of MSA (Rectangular)

⇒ Effective length $L_{eff} = \frac{L + 2\Delta L}{2}$

⇒ operating freq $f = \frac{c}{2L\sqrt{\epsilon_r}} = \frac{c}{2L_{eff}\sqrt{\epsilon_{eff}}}$

$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[1 + \frac{h}{w} \right]^{-1/2}$

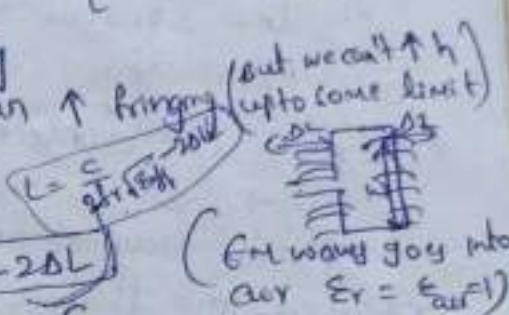
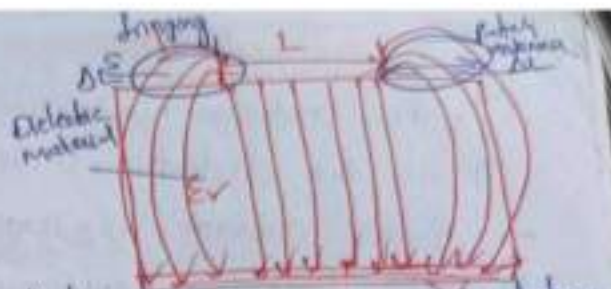
⇒ $\Delta L = h(0.412) \left(\frac{\epsilon_{eff} + 0.3}{\epsilon_{eff} - 0.258} \right) \left(\frac{\frac{w}{h} + 0.264}{\frac{w}{h} + 0.8} \right)$

⇒ Determine width it doesn't depend on the fringing

$w = \frac{c}{2fr} \sqrt{\frac{2}{\epsilon_r + 1}}$

⇒ Find Actual length L
 $L_{eff} = L + 2\Delta L \Rightarrow L = L_{eff} - 2\Delta L$

$L = \frac{c}{2fr\sqrt{\epsilon_{eff}}} - 2\Delta L$



2) Probe Feed MSA

→ outer conductor is connected to ground plane below the dielectric material.

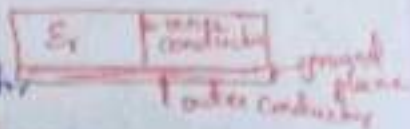
→ Inner conductor is connected to patch

→ to provide impedance matching inner conductor moving from center

TOP view



side view



Advantages

- 1) Easy to fabricate
- 2) Easy to match
- 3) Simple to model
- 4) Low spurious radiation.

disadvantages

- 1) Low Bandwidth (2%)
- 2) cross polarization (for impedance matching shift inner conductor from center)

3) Proximity Coupled Feed MSA

ϵ_{r1} → one dielectric material

ϵ_{r2} → 2nd "

→ patch mounted on top of dielectric material

→ feed line that is send which is 2 dielectric material

→ below 2nd dielectric material there is ground plane

→ Impedance matching provide by change length & width of feed line.

→ Design will be Symmetrical about axis.

Advantages

- 1) Low cross polarization
- 2) High Bandwidth (13%)
- 3) Low spurious radiation
- 4) Easy to model

disadvantages

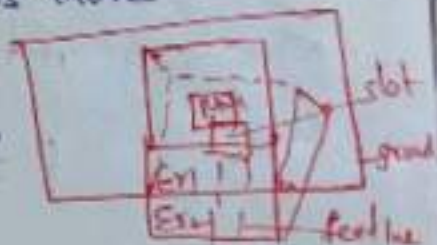
- 1) Difficult to fabricate (because two $\epsilon_{r1}, \epsilon_{r2}$)
- 2) cost is more



4) Aperture Coupled Feed MSA

→ There will be slot coupled to line ground dw

→ Symmetrical w.r.t to axis of line



Disadvantages

- 1) It is most difficult to fabricate
- 2) It has narrow bandwidth.

- 1) Low cross polarization
- 2) Easy to model
- 3) moderate spurious radiation

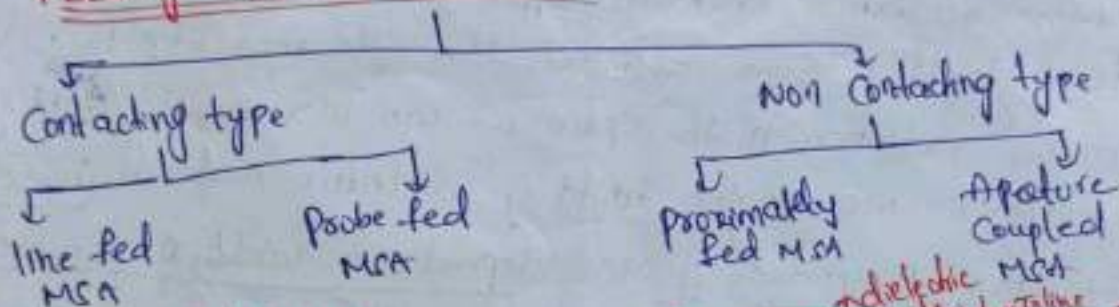
$$E_\theta = \frac{\sin\left[\frac{k w \sin\theta \sin\phi}{2}\right]}{\frac{k w \sin\theta \sin\phi}{2}} \cos\left(\frac{k L}{2} \sin\theta \cos\phi\right) \cos\phi$$

$$E_\phi = -\frac{\sin\left[\frac{k w \sin\theta \sin\phi}{2}\right]}{\frac{k w \sin\theta \sin\phi}{2}} \cos\left(\frac{k L}{2} \sin\theta \cos\phi\right) \sin\phi$$

width controls

- Based on width electric field value changed.
- length value l is fixed $L = d/2$
- w controls I/P impedance
- $\text{I/P impedance} \propto \frac{1}{w}$ $w \uparrow \text{imp} \downarrow$, $w \downarrow \text{imp} \uparrow$
- w controls the bandwidth of antenna (Bandwidth $\propto w$)
- L controls Radiation pattern.

Feeding methods in MSA

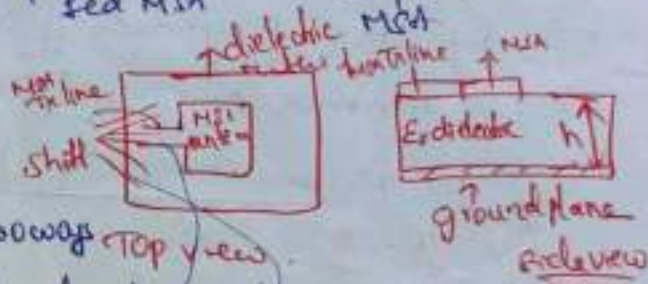


① line fed MSA

- Microstrip mounted on the dielectric material
- For impedance matching there are two ways
- 1) By shifting line position in this direction $\approx$
- 2) Insect feeding (not symmetrical to axis of MSA)
- Line Connected directly to the patch antenna.

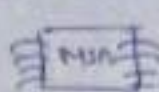
Advantages

- 1) easy to fabricate
- 2) It is simple to match
- 3) Simple to model



- #### Disadvantages
- 1) It has low Bandwidth (2-4%)
 - 2) It has cross polarization.

Rectangular microstrip Antenna

- It is having microstrip patch placed on dielectric material support with ground plane.
- Microstrip TX line feed to this microstrip antenna
- Impedance matching of MSA by varying position of microstrip line. not connected with center line it would be somewhere
- To provide impedance matching matching by shifting microstrip line like  w.r.t to center.
- dimension L (length) of a patch is $L = d/2$ d is the width of MSA. radiation happens from this side 
- Height of dielectric material justify the value of magnitude which is the radiation higher the height of dielectric substrate
- More radiation which will happen with increase the dielectric material height & some extended then after MSA stop to radiate
- more radiation in the space we can increase the height of dielectric
- we can't increase the height of dielectric material ($h < 0.05\lambda$)
- Radiation magnitude of MSA depends on width of the MSA
- If you have more width radiation will increase
- Impedance matching depends on width of MSA.

Analysis of MSA

operating freq $f_0 = \frac{c}{2L\sqrt{\epsilon_r}}$

Instead of ϵ_r we use ϵ_{eff}
electric field of MSA $E_0 = \sin$

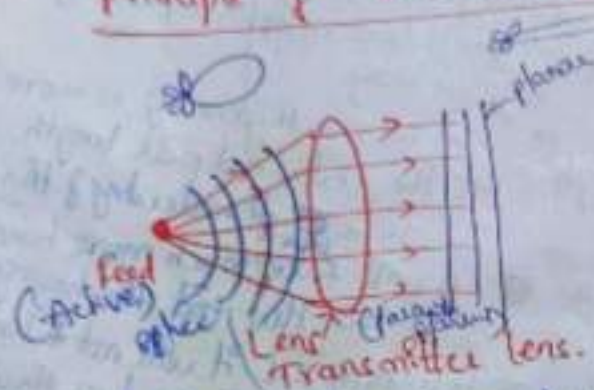
Lens Antennas

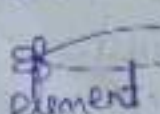
Basics of Lens antenna.

- It is an antenna which consists an electromagnetic lens with feed
- It converges spherical wavefront to planar wavefront and planar wavefront to focus at feed.
- It is typically thicker, heavier and more difficult to construct
- It has one advantage over Reflector antenna, blockage is not happening.

Principle of Lens Antenna.

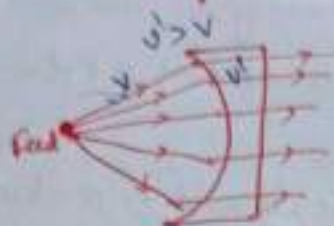
By using lens antenna improve the radiation characteristics



- 1) This lens antenna is used as Tx. This feed provide spherical wavefronts. going through lens antenna. This lens convert to planar waves.
 - 2) If you see R. char like this  If we use after lens using R. Converter is like  element we don't provide feed to lens.
 - 3) lens is parasitic element. It converts spherical to planar at the Tx lens.
 - 4) After using lens the directivity of beam increases.
- " This lens antenna is used as Rx when signal is coming from very long distance. signal will be a planar wave passing through a lens antenna it will be converting planar wave to spherical wave front. ^{But this} feed aperture is less ^{even though highly power} but ^{is it} Receiver signal getting.
- 3) lens antenna
 - Here the operation of Rx lens it converts planar waves to spherical waves even the aperture size is less. wave to be focus to feed at feed.

Types of Lens Antenna

Conducting type & Type.



1) This lens working as accelerated when EM waves propagating through conducting type material. If v' say velocity v' propagating through space the velocity is v .

→ If propagating through this material the velocity increases i.e. $v' > v$.

So the wave propagating (passing) through material is getting acceleration. The need of acceleration why the need of acceleration? The total path length which is imaging the path length is high. ^{because there is low frequency waves} We need to accelerated need to spherical wavefront to planar wavefronts.

→ velocity of wave increases when passing through this material.

→ There is higher path length ^{there is} wave need to provide ^{more} acceleration. It will ^(higher acceleration) convert spherical wavefronts to planar wavefronts.

Dielectric type



2) If v' say velocity inside is v' outside is v .

3) If it is slow $v' < v$.

② In that case we observe the path length of this wave is low compare to path length of this wave.

③ If we see thickness of is more at the center and path length is less ^{then} deceleration of this wave.

④ It will take much more time while at this end its having less thickness it will not taking much time to propagating through material. Ultimately the

⑤ spherical wavefront converts to planar wavefronts.

⑥ $v' < v$ means

thickness is more in middle and path length is less.

Zoned lens antenna. when we use lens-antenna it is very bulky. So it reduce width of lens (is called) zoning. So we cutting of width of lens we can reduce size.

→ In this case this lens are looking similar to the lens which we see in last topics.

→ Now



Advantages of Lens Antenna

- No blockage due to feed and feed support (because feeding is not coming from secondary wave lens)
- more EM can be received with respect to parabolic reflector
- Low noise
- Higher gain compared to Reflector Antenna.

Disadvantages

- Lens are heavy.
- Complex to construct
- Costlier compared to Reflector.

Applications

- 1) For narrow beam width
- 2) microwave transmission.

Lens Antenna

Reflector Antennas

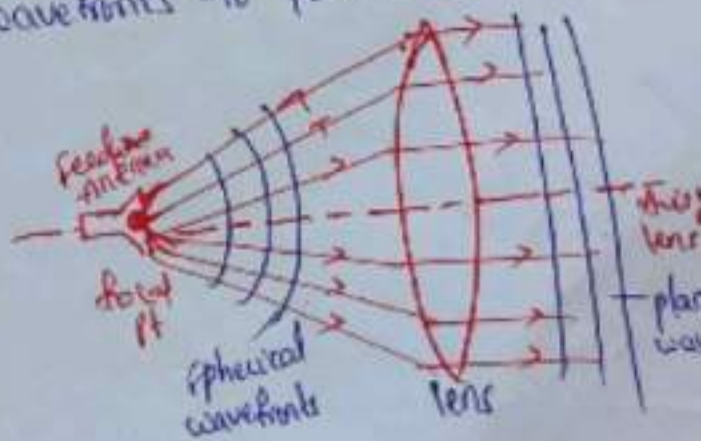
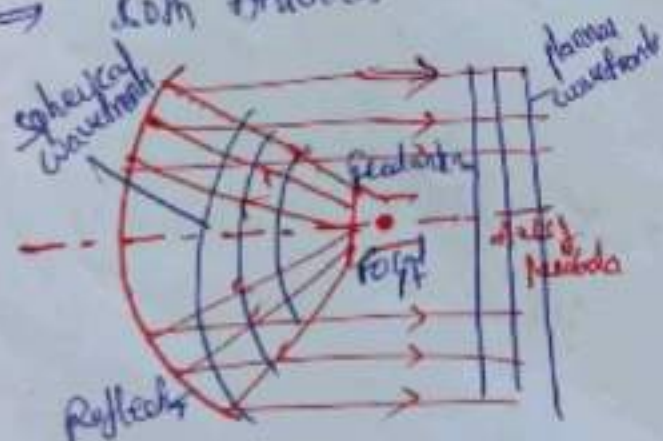
- 1) It operates based on Reflector
- It have blockage issue.
- If we remove blockage by offset parabolic Reflector, then cross-polarization increases.
- It has issue of Isolation
- It is easy to construct
- weight is lower than lens.
- cost is low.

→ It operates based on Refraction.
→ It doesn't have blockage issue.

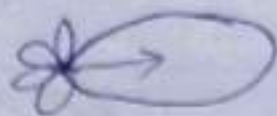
→ less cross-polarization.

- It doesn't have isolation.
- It is complex structure.
- It's weight is high
- cost is high.

⇒ Both Antennas convert spherical wavefronts to planar wavefronts.



Parabolic antenna. Radiation pattern.



→ beam with very narrow.

Applications

- 1) Radio astronomy, microwave freq, Satellitecom
- Deep space communication.

Examples on Reflector antenna

For parabolic reflector with uniform illumination

$$1) \text{ FNBW} \approx \frac{140\lambda}{D} \text{ (deg)}$$

$$2) \text{ HPBW} \approx \frac{58\lambda}{D} \text{ (deg)}$$

$$3) \text{ Power gain } G_p \approx 6 \left(\frac{D}{\lambda} \right)^2$$

$$4) \text{ Directivity } D = \frac{G}{K}$$

Calculate gain, FNBW and HPBW of a parabolic reflector of 2m diameter at 5GHz.

for $D = 2\text{m}$, $f = 5\text{GHz}$, $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{5 \times 10^9} = 0.06\text{m}$

$$1) \text{ gain } G = 6 \left(\frac{D}{\lambda} \right)^2 \Rightarrow 6 \left(\frac{2}{0.06} \right)^2 = 6666.67$$

$$2) \text{ FNBW} \approx \frac{140\lambda}{D} \Rightarrow \frac{140 \times 0.06}{2} = 4.2 \text{ deg}$$

$$3) \text{ HPBW} \approx \frac{58\lambda}{D} \Rightarrow \frac{58 \times 0.06}{2} = 1.74 \text{ deg}$$

② Calculate gain, FNBW, HPBW of parabolic reflector of 10m diameter at 10GHz.

$D = 10\text{m}$, $f = 10\text{GHz}$. $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10 \times 10^9} = 0.03\text{m}$

$$1) G = 6 \left(\frac{D}{\lambda} \right)^2 \Rightarrow 6 \left(\frac{10}{0.03} \right)^2 = 6.67 \times 10^5$$

$$2) \text{ FNBW} \approx \frac{140\lambda}{D} \Rightarrow \frac{140 \times 0.03}{10} = 0.42 \text{ deg}$$

$$4) \text{ HPBW} \approx \frac{58\lambda}{D} \Rightarrow \frac{58 \times 0.03}{10} = 0.174 \text{ deg}$$





- It is highly directional Antenna
- It is used to very long distance communication, such as satellite communications
- It is applicable to microwave frequency (1-100GHz) and beyond that
- It consists two types of elements

1) Active elements (Feed Antenna)

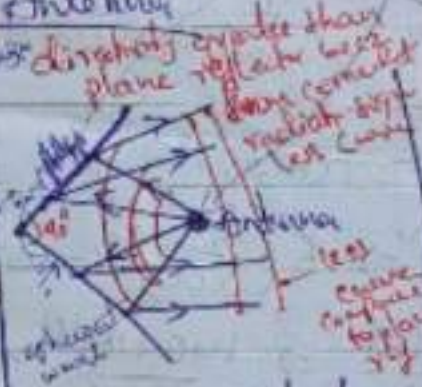
2) Parasitic element (Reflector)

Types of Reflector Antenna

Antenna
Radiates
Spherical
Wave

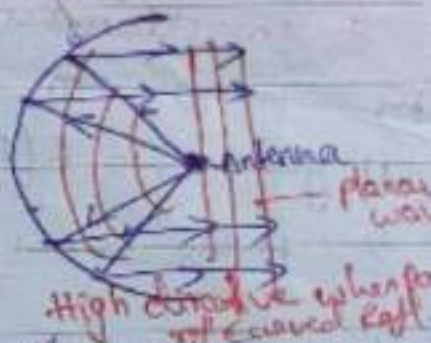


Plane Reflector



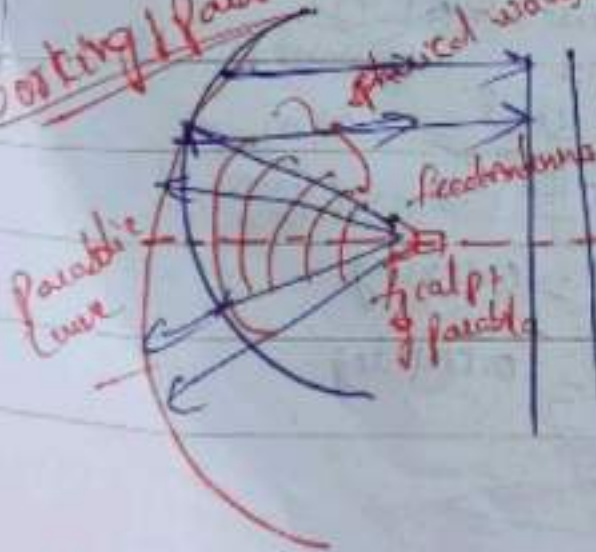
Corner Reflector

Curved Reflector



Based on Reflector shapes there are 2 types

Working of Parabolic



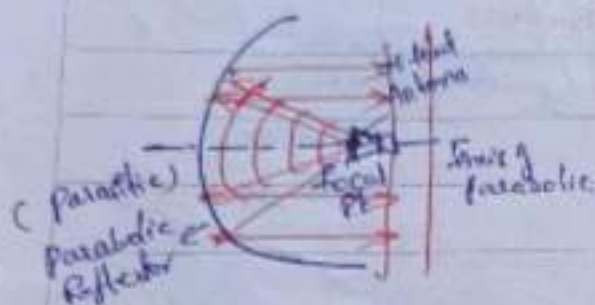
→ Parabolic Antenna converts spherical wavefronts into planar wavefronts
→ Due to that it is highly directional antenna.





Feeding mechanism of Parabolic Reflector.

1) Center fed parabolic Reflector Disadvantages



1) It is difficult to use for low noise application due to isolation.

2) Blockage due to feed.

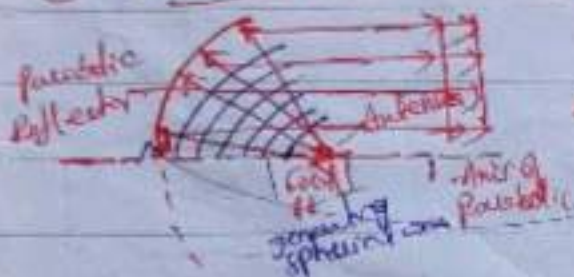
Advantages, it has less cross polarization.

2)

To overcome these two disadvantages by using offset feed.

② Offset feed P.R.

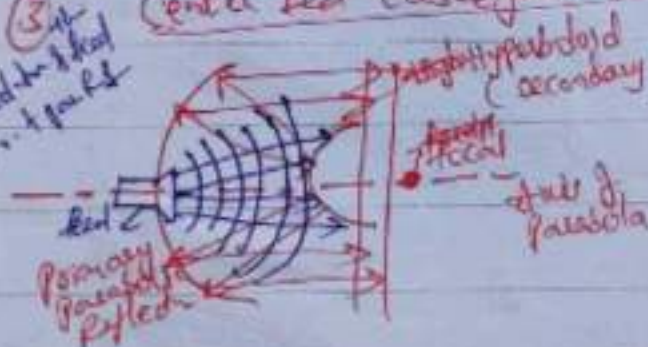
Advantages 1) No blockage due to feed.



Disadvantages 1) Cross polarization
(Antenna design not symmetrical to axis)

③ Center feed Cassegrain P.R.

Feed at focal point
w.r.t. parabola



Advantages

1) Isolation - feed leads it's use in low noise applications

2) Directivity is high.

3) Low cross polarization.

Disadvantages

1) Blockage due to secondary reflector

2)

$$\frac{6 \text{ dB}}{100 \text{ dB}} = \frac{1}{100}$$

$$\frac{1}{100} = \frac{1}{100}$$

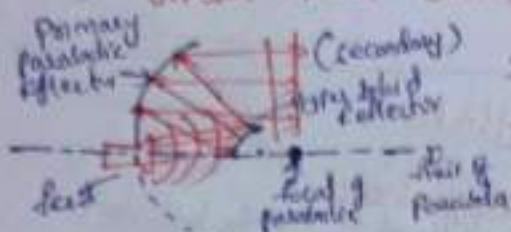
$$\frac{1}{100} = \frac{1}{100}$$

$$\frac{1}{100} = \frac{1}{100}$$



Name: _____

v) Offset feed Cassegrain parabolic reflector



- Advantages
- 1) It provides isolation.
 - 2) no blockage due to secondary reflector.

- Disadvantages
- 1) Cross polarization due to not symmetrical to axis of parabola.



Ex Design a microstrip patch with dimensions w & L over a single substrate whose center freq is 10GHz the dielectric constant of the substrate is 10.2 and the height of the substrate is 0.127 cm.

Ans given data $f_r = 10\text{GHz}$, $\epsilon_r = 10.2$, $h = 1.27 \times 10^{-3}\text{m}$
 $w = ?$, $L = ?$

$$1) \quad w = \frac{c}{2f_r \sqrt{\frac{\epsilon_r + 1}{2}}} = \frac{3 \times 10^8}{2 \times 10 \times 10^9 \sqrt{\frac{10.2 + 1}{2}}} = 6.83 \times 10^{-3}\text{m} \Rightarrow \underline{0.683\text{cm}}$$

$$\begin{aligned} 2) \quad \epsilon_{eff} &= \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[1 + 12 \left(\frac{h}{w} \right)^3 \right]^{-1/2} \\ &= \frac{10.2 + 1}{2} + \frac{10.2 - 1}{2} \left[1 + 12 \left(\frac{1.27 \times 10^{-3}}{6.83 \times 10^{-3}} \right)^3 \right]^{-1/2} \\ &= 5.6 + 4.6 (1 + 2.4)^{-1/2} = \underline{8.09} \end{aligned}$$

$$\begin{aligned} 3) \quad \Delta L &= h (0.412) \left(\frac{\epsilon_{eff} + 0.5}{\epsilon_{eff} - 0.258} \right) \left(\frac{\frac{w}{h} + 0.264}{\frac{w}{h} + 0.8} \right) \\ &= 1.27 \times 10^{-3} (0.412) \left(\frac{8.09 + 0.5}{8.09 - 0.258} \right) \left(\frac{\frac{6.83 \times 10^{-3}}{1.27 \times 10^{-3}} + 0.264}{\frac{6.83 \times 10^{-3}}{1.27 \times 10^{-3}} + 0.8} \right) \\ &= 1.27 \times \left(0.412 \times \frac{8.39}{7.832} \times \frac{5.25}{5.78} \right) \times 10^{-3} \end{aligned}$$

$$\Delta L = 0.5 \times 10^{-3} \Rightarrow \underline{5 \times 10^{-4}\text{m}} \text{ Ans.}$$

$$4) \quad L_{eff} = \frac{c}{2f_r \sqrt{\epsilon_{eff}}} = \frac{3 \times 10^8}{2 \times 10 \times 10^9 \sqrt{8.09}} = 0.52 \times 10^{-2} = 5.2 \times 10^{-3}\text{m}$$

$$\begin{aligned} 5) \quad \text{Actual length } L &= L_{eff} - 2\Delta L \\ &= 5.2 \times 10^{-3} - 2(5 \times 10^{-4}) \\ &= 5.2 \times 10^{-3} - 1 \times 10^{-3} \Rightarrow \underline{4.2 \times 10^{-3}\text{m}} \end{aligned}$$

$$L = \underline{0.42\text{cm}} \text{ Ans.}$$

Ant TV Unit

There are 3 antenna field zone

- 1) Reactive near field region
- 2) Radiating near field region
- 3) Far field region.

Reactive near field Region

→ It is that portion of the near field region $R < \frac{2L^2}{\lambda}$ immediately surrounding the antenna where in the reactive field predominates.

→ For most of the antennas, the outer boundary of this region is

$$R < 0.62 \sqrt{\frac{L^3}{\lambda}}$$

→ But for a very short dipole radiator the outer boundary

$$R < \lambda / (2\pi)$$

→ In general objects within this region will result in coupling with the antenna and distortion of the ultimately field antenna pattern.

→ In general objects large conductor within this distance will couple with the antenna and 'disturb' it. The result can be an altered resonant freq, radiation resistance and Radiation pattern.

2) It is that region of the field of antenna b/w the reactive near field region and the far field region.

→ For this region, the distance from the antenna R is

$$0.62 \sqrt{\frac{L^3}{\lambda}} < R < \frac{2L^2}{\lambda}$$

This region is also called transition region.



Properties of this region are

- 1) The antenna pattern is taking shape but is not truly ^{completely} developed.
- 2) The radiation field predominates the reactive field.
- 3) The radiated wave front is still fairly curved.
- 4) $\vec{E}$ field & magnetic field ^{vectors} are not orthogonal.

⑤ Far-field

→ It is that region of the field of an antenna where the angular field distribution is essentially independent of the distance from the antenna.

→ For this region, the distance from the antenna R is

$$R > \frac{2L^2}{\lambda}$$

Properties of this region are

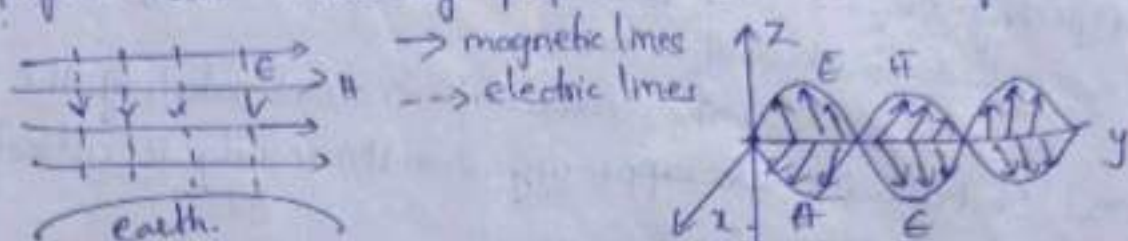
- 1) The wavefront becomes approximately planar.
- 2) The radiation pattern is completely formed and does ~~vary~~ vary with distance.
- 3) $\vec{E}$ & magnetic field vectors are orthogonal each & other.

UNIT-V Wave Propagation-I

①

Introduction.

- Electromagnetic waves are nothing but oscillations which propagate with the velocity of light ($c = 3 \times 10^8 \text{ m/s}$) in free space.
- Electromagnetic waves consists of moving fields of electric and magnetic ^{fields}.
- The direction of the electric field, the magnetic field and propagation are mutually perpendicular in electromagnetic waves.



- The orientation of electric field w.r.t earth gives the polarization.
- Electromagnetic waves are transverse in nature i.e. oscillations are perpendicular to the direction of wave propagation.

General classification of waves (categorization)

- Guided waves The waves guided by man made structures like parallel wire (or) coaxial transmission line, optical fibre, microstrip line, wave guides etc.
- Unguided waves waves propagating in terrestrial atmosphere along earth (or) outer space.
 - plane wave A wave for which the equiphase surface is a plane is called plane wave.
 - Uniform plane wave A wave for which the equi phase surface is also equi amplitude then it is called uniform plane wave.

- 1.1 with dimensions will give

- 3) Non Uniform plane wave - A wave for which the equiphase and equiamplitude surfaces are not parallel to each other.
- 4) Slow wave - A wave whose phase velocity v_p is less than the velocity of light is termed as slow wave.
- 5) Forward wave - A wave travelling in required direction is called forward wave.
- 6) Backward wave - When a forward travelling wave strikes a reflecting surface and returns back then such reflected wave is called backward wave.
- 7) Standing wave - The wave with a combination of the forward and backward wave appearing simultaneously is called standing wave.

Electromagnetic spectrum

freq range	Symbol	Application
1) 3-30 kHz	VLF	aeronautical
2) 30-300 kHz	LF	
3) 300-3000 kHz	MF	Long Casting
4) 3 MHz - 30 MHz	HF	long distance broadcasting
5) 30 - 300 MHz	VHF	mobile, AM & FM services, TV
6) 300 - 3000 MHz	UHF	"
7) 3 - 30 GHz	SHF	Radar, Satellite communication
8) 30 - 300 GHz	EHF	Research Radio astronomy

Fundamental eqn for free space propagation (or) Friis free space eqn

$$\frac{P_{rev}}{P_{rad}} = G_T G_R \left(\frac{\lambda}{4\pi R} \right)^2$$

P_{rad} - radiated power
 P_{rev} - received power
 $G_T \rightarrow$ Tx antenna gain
 $G_R \rightarrow$ Rx antenna gain

is known as fundamental eqn for free space propagation (or)

Friis-free space eqn.

If maximum directivity of transmitter is 1.

then $G = \sqrt{\frac{30 P_{rad}}{P_{rev}}}$

- a) ① A free space microwave link consisting transmitter and Receiver each of 30dB gain operates at 10GHz . The distance b/w transmitter and Receiver is 20km . The transmitter radiates 15W power. calculate the power received by the receiver.

Sol

given $G_T = G_R = 30\text{dB}$, $f = 10 \times 10^9 \text{ Hz}$

$r = 20 \times 10^3$, $P_{rad} = 15$

$$P_{rev} = P_{rad} \frac{G_T G_R}{\left(\frac{4\pi R}{\lambda} \right)^2}$$

$$= 15 \cdot 1000 \cdot 1000 \left(\frac{0.03}{4\pi \times 20 \times 10^3} \right)^2$$

$$\boxed{P_{rev} = 0.2137 \text{ } \mu\text{W}}$$

Modes of prop of ground wave

Disadvantages

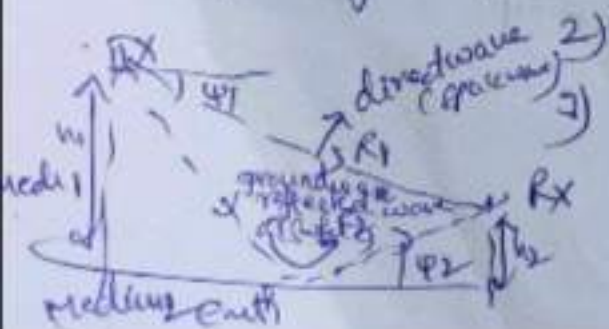
- 1) Requires relatively high transmission power.
- 2) Frequencies up to 2MHz .
- ③ Require large antennas
- ④ attenuation increases with frequency.

Modes of propagation

The radio waves from the transmitting antenna may reach the receiving antenna following any one of the modes of propagation depending upon several factors like 1) frequency of operation
2) Distance b/w TX & RX antenna.

① Ground wave (or) Surface wave propagation (up to 3 MHz)

- The ground wave is a wave that is guided along the surface of the earth just like an electromagnetic wave is guided by a waveguide (or) transmission line.
- This mode of propagation exists when the transmitting and receiving antennas are very close to the surface of earth.
- In general wave vertical antennas are used i.e. electric field vectors are vertical w.r.t to ground.
- Any horizontal component of the electric field in contact with the earth is short circuited.
- When ground wave glides over the surface of earth, the surface wave loses some of its energy by absorption.
- ground wave therefore suffers varying amount of attenuation while propagating along the curvature of the earth depending upon 1) frequency



2) Surface irregularities
3) Permittivity and conductivity.

→ Earth's attenuation increase as the frequency increases and hence the mode of propagation is suitable for low and medium frequency is up to 2 MHz only.

→ As the wave progress over the curvature of the earth, the wavefronts start gradually tilting more and more.

→ Field strength of groundwave at a distance.

→ To increase the in the tilt of wave causes more short cut of the electric field component and hence the field strength goes on reducing.

→ Field strength of the signal at a distance from the TX antenna due to the ground wave can be obtained by solving the Maxwell's eqn and it is given by.

$$E = \frac{120\pi h_r h_t I_s}{\lambda d} \text{ volt/meter.}$$

where $120\pi = 277 \Omega$ = intrinsic impedance of free space.

h_r, h_t → effective heights of Receiving & TX antenna.

I_s → Antenna current

λ - wavelength, d → Distance at Pt from the TX.

Attenuation characteristics for ground wave propagation.

According to Sommerfeld for a flat earth, the field strength for a ground wave propagation is given by.

$$E_G = \frac{E_0 A}{d}$$

where E_0 → ground wave field strength at the earth's surface at unit distance without considering earth losses.

E_G → ground wave field strength

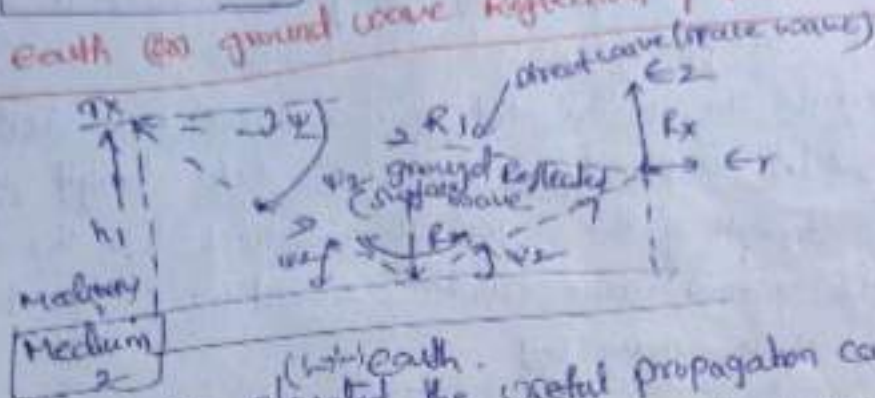
A → Attenuation factor accounting for earth losses

d → distance from the TX antenna.

- E_0 depends on
- ① power radiation of transmitting antenna.
 - ② Directivity in vertical and horizontal planes.

$$E_0 = \frac{\sqrt{300 P}}{d} \text{ V/m.}$$

Plane Earth (or) ground wave Reflection of Radio waves



Tx & Rx antennas are elevated (in air) above the earth. by means of the space wave propagation as the two antennas are within the line of sight of each other, the propagation of such space waves is also called line of sight propagation.

→ Basically for the line of sight propagation, the resultant signal obtained is the combination of space wave & surface wave.

→ According to Rayleigh Criterion, if the reflecting surface is rough the reflection is similar to that due to the smooth surface provided that the angle of incidence is very large.

Rayleigh criterion is given by $R = \frac{4\pi G \sin^2 \theta}{\lambda}$

R = measure of roughness

θ = angle of incidence

λ = w.v.

G = standard deviation of surface irregularities from mean surface height.

Space and Surface wave

Add strength of ground wave can be divided into two parts namely Space wave and Surface wave.

- Space wave dominates at larger heights from the earth.
- Surface " near the surface of earth.

Space wave expression

$$E_{\text{total (space)}} = \int 30 \mu E \, dL \cos \phi \left(\frac{j \mu R_1}{R_1} + R_v \frac{j \mu R_2}{R_2} \right)$$

Surface wave. $E_{\text{total surface}} = \int 30 \mu E \, dL (1 - R_v) \frac{j \mu R_2}{R_2} \left(1 - 2 \sin^2(\phi/2) \right) \left(\frac{j \mu R_2}{R_2} \right)$
Wave tilt (or) Angle of tilt [decreases in the amplitude of the wave with distance due to the 1/r voltage maintaining constant level]

⇒ wave tilt is defined as the change of orientation of the vertically polarized ground wave at the surface of the earth.

This occurs due to diffraction (bending)

- Due to tilt, both horizontal and vertical components of the electric field exists and are not in phase. This tilt changes the originally vertically polarized wave into an elliptically polarized wave.
- As the wave progresses it tilts over the surface of the earth. This results in power dissipation and ultimately waves lie down and die.

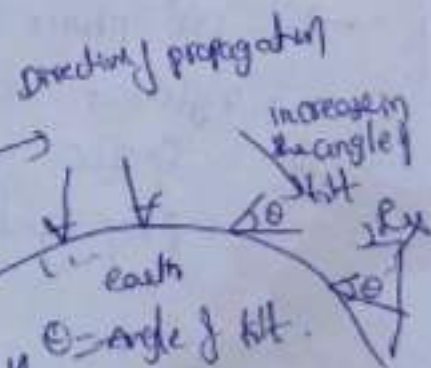
$$Z_s = \frac{\omega \mu}{\sqrt{(G + j\omega \epsilon)^2}} \frac{1}{2} \left\{ \tan^{-1} \frac{G}{\omega \epsilon} \right\} \quad \text{earth} \quad \theta = \text{angle of tilt}$$


μ = permeability of the earth

G = conductivity of the earth.

ϵ = permittivity of the earth.

- Surface wave induces currents in the ground while travelling over the earth's surface while and thus losses some energy by absorption.
- Due to diffraction, the wavefronts will tilt over the surface. This angle of tilt goes on increasing due to the curvature of the earth's surface.



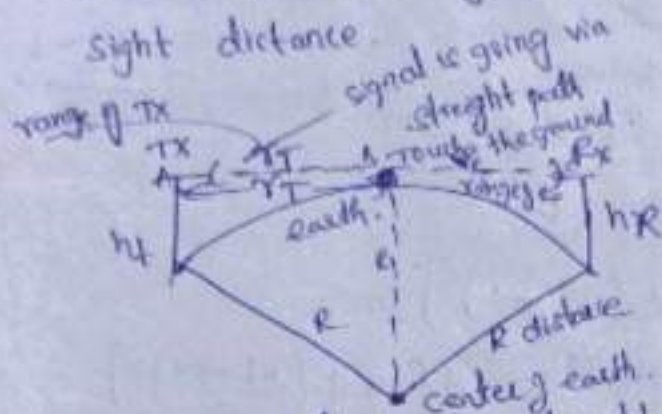
Sky wave (or) Ionosphere wave propagation (2-30 MHz)

② Space wave (or) Troposphere wave propagation (30-200 MHz)

- In this mode of propagation, Electromagnetic waves from the Tx antenna reach the receiving antenna either directly or after reflection from the ground in earth's tropospheric region.
- Troposphere is that portion of the atmosphere which extends up to 16 km from the earth surface.
- Space wave propagation consists two components.
 - ① Direct component
 - ② Indirect component (ground reflected component)
- ⇒ Ground waves above 30 MHz undergo high attenuation with considerable reduction in the amplitude within very short distance of the order of few 100 meters. Moreover at these frequencies the sky wave propagation fails because the ionosphere cannot refract these frequencies back to the earth. Under such conditions, the space wave propagation is the best option above 30 MHz.
- The space wave propagation is through troposphere hence such propagation is limited to few hundred of kilometers.
- 180° phase change will be introduced for the ground reflected wave.
- Space wave propagation is also called as Tropospheric propagation.

Line of sight propagation

Space wave propagation is also called line of sight propagation because this mode of propagation is limited to the line of sight distance.



Tx, Rx antennas are both on earth. See each other is (no hills, buildings etc.)

$\Rightarrow$ Here signal travels straight from Tx to Rx antenna.

$\rightarrow$ From Δ path $d_T^2 + R^2 = (h_t + R)^2$

$$d_T^2 + R^2 = h_t^2 + R^2 + 2h_t R$$

By neglecting h_t^2 then $d_T^2 = 2h_t R$

$$d_T = \sqrt{2h_t R} \quad \text{--- (1)}$$

By

$$d_R = \sqrt{2h_r R} \quad \text{--- (2)}$$

Total Range for LOS $d = d_T + d_R$

$$= \sqrt{2h_t R} + \sqrt{2h_r R}$$

$$= \sqrt{2R} [\sqrt{h_t} + \sqrt{h_r}]$$

$$d = 3.57 [\sqrt{h_t} + \sqrt{h_r}] \text{ km.} \quad R = 6370 \text{ km.}$$

Field strength of space (or) tropospheric propagation.

$\rightarrow$ Energy is received at the receiving point by two ways i.e. one by the direct rays (TR') and other by the indirect rays after reflection from the ground ($T'OR'$)

space wave propagation and field strength determination at receiving pt R.

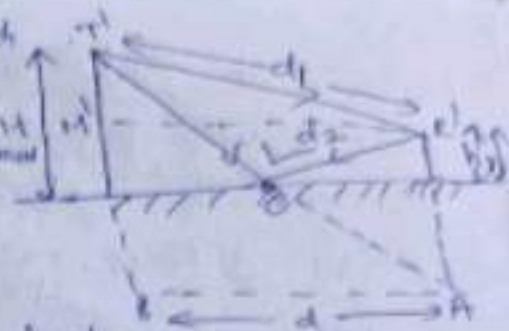
where $h_t, h_r \rightarrow$ heights of Tx, Rx antennae above ground

$d \rightarrow$ distance b/w Tx & Rx

$d_1 \rightarrow$ Direct path distance

$d_2 \rightarrow$ indirect path distance

$E_0 \rightarrow$ field strength at R' due to direct



from $\Delta TQR'$

$$(h_t - h_r)^2 + d^2 = d_1^2$$

$$d_1^2 = d^2 \left[1 + \left(\frac{h_t - h_r}{d} \right)^2 \right]$$

$$d_1 = d \left[1 + \left(\frac{h_t - h_r}{d} \right)^2 \right]^{1/2} = d \left[1 + \frac{1}{2} \left(\frac{h_t - h_r}{d} \right)^2 \right]$$

$$\boxed{d_1 = d + \frac{1}{2d} [h_t - h_r]^2}$$

from ΔTAB , $OR' = OA$

$$(h_t + h_r)^2 + d^2 = d_2^2$$

$$d_2 = d \left[1 + \left(\frac{h_t + h_r}{d} \right)^2 \right]^{1/2}$$

$$d_2 = d \left[1 + \frac{1}{2} \left(\frac{h_t + h_r}{d} \right)^2 \right]$$

$$\boxed{d_2 = d + \frac{1}{2d} [h_t + h_r]^2}$$

path difference b/w direct & indirect ray is $= d_2 - d_1$

$$= d + \frac{1}{2d} [h_t + h_r]^2 - d - \frac{1}{2d} [h_t - h_r]^2$$

$$= \frac{1}{2d} [(h_t + h_r)^2 - (h_t - h_r)^2] = \frac{1}{2d} 4h_t h_r$$

$$\boxed{P.D. = \frac{2h_t h_r}{d}}$$

$$\text{path difference} = \frac{2\pi}{\lambda} \cdot P.D.$$

$$\alpha = \frac{2\pi}{\lambda} \cdot \frac{2h_t h_r}{d} = \frac{4\pi h_t h_r}{\lambda d}$$

Besides this there is another phase difference of 180° due to reflection from the ground.

So total phase difference $\theta = \alpha + \beta$.

where $\alpha \rightarrow$ phase diff due to path difference.

$\beta \rightarrow$ phase diff due to reflection at the ground and is equal to 180° .

Resultant field strength $E_R = E_0(1 + K e^{j\theta})$

$$E_R = E_0 [1 + K(\cos\theta - j\sin\theta)]$$

$$|E_R| = E_0 \sqrt{(1 + K\cos\theta)^2 + (K\sin\theta)^2}$$

$$= E_0 \sqrt{1 + K^2 + 2K\cos\theta}$$

2f K=1
and $\beta = 180^\circ$

$$|E_R| = E_0 \sqrt{2 + 2\cos\theta} = E_0 \sqrt{2(1 + \cos\theta)}$$

$$= E_0 \sqrt{2 \cdot 2\cos^2\frac{\theta}{2}} = 2E_0 \cos\frac{\theta}{2}$$

$$= 2E_0 \cos\left(\frac{\alpha + \beta}{2}\right) = 2E_0 \cos\left(\frac{\alpha + \pi}{2}\right)$$

$$= 2E_0 \cos\left[\frac{\alpha}{2} + \frac{\pi}{2}\right] = 2E_0 \sin\frac{\alpha}{2}$$

$$\begin{cases} \theta = \alpha + \beta \\ \beta = \pi \end{cases}$$

$$|E_R| = 2E_0 \sin\frac{\alpha}{2}$$

$$= 2E_0 \sin \frac{\pi h_1 h_2}{Ad} = 2E_0 \sin \frac{2\pi h_1 h_2}{Ad} \quad [Ad \gg 2\pi h_1 h_2]$$

$$= 2E_0 \frac{2\pi h_1 h_2}{Ad} = 4E_0 \frac{\pi h_1 h_2}{Ad}$$

$$|E_R| = E_0 \frac{4\pi h_1 h_2}{Ad}$$

Standard value of $E_0 = \frac{\sqrt{P}}{d}$

[P $\rightarrow$ power]

$$|E_R| = \frac{8\pi \sqrt{P} h_1 h_2}{Ad}$$

$E_R \propto \sqrt{P}$ $\rightarrow$ effective power radiation
 $\propto \sqrt{h_1 h_2}$ $\rightarrow$ height of TX

$\propto \sqrt{h_r}$ $\rightarrow$ " R_x

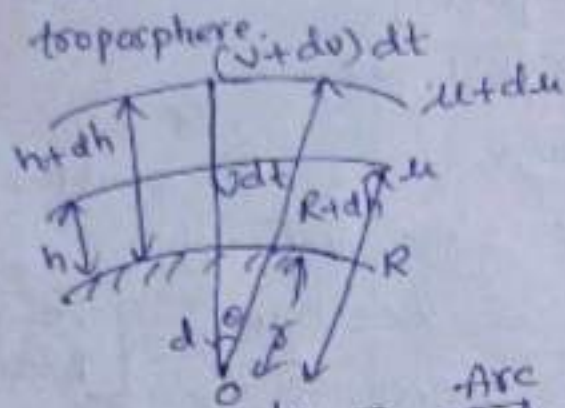
$\propto \frac{1}{\lambda}$ $\rightarrow$ Inversely with λ

$\propto \frac{1}{d^2}$ $\rightarrow$ " with square of distance from Tx.

Effective Earth's ^{Radius} Curvature

→ Decrease in refractive index with height causes refraction of the radio waves which results in the bending of radio wave towards the region of higher dielectric constant or refractive index i.e. towards the earth.

→ Consider a radio wave which is travelling horizontally in troposphere.



$v \rightarrow$ velocity of propagation
 $h \rightarrow$ height above the earth
 $R \rightarrow$ radius of curvature of path
 $r \rightarrow$ Actual radius of earth.

w.R.T Angle = $\frac{\text{Arc}}{\text{Radius}}$

$$d\theta = \frac{v dt}{R}$$

$$R d\theta = v dt \quad \text{--- (1)}$$

lly $(R+dh) d\theta = (v+dv) dt$
 $R d\theta + dh d\theta = v dt + dv dt \Rightarrow dh d\theta = dv dt$

$$\frac{d\theta}{dt} = \frac{dv}{dh} \quad \text{--- (2)}$$

w.R.T $v = \frac{c}{u} \Rightarrow \frac{dv}{dh} = -\frac{c}{u^2} \frac{du}{dh}$

$$= -\left[\frac{c}{u}\right] \left[\frac{1}{u}\right] \frac{du}{dh}$$

$$\frac{dv}{dh} = -\frac{v}{u} \frac{du}{dh}$$

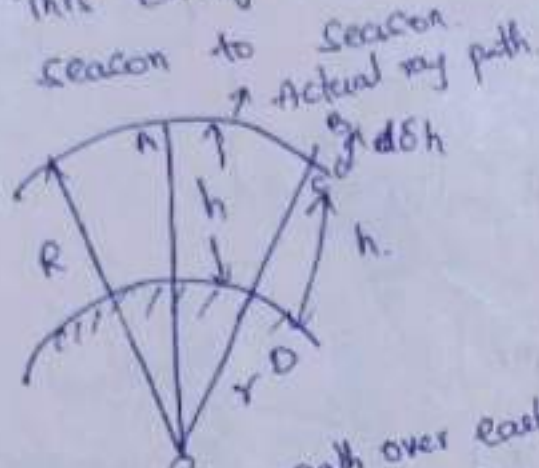
by (1) $R = \frac{v \cdot dt}{d\theta} = \frac{v}{\frac{d\theta}{dt}} = \frac{v}{\frac{dv}{dh}} \quad [\text{by (2)}]$

$$R = \frac{v}{-\frac{v}{u} \frac{du}{dh}} = -\frac{dh}{du} \quad [u=1]$$

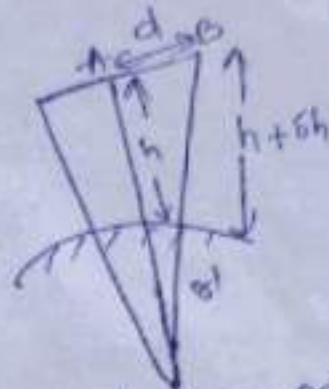
$$R = -\frac{dh}{du}$$

This shows that radius of curvature of the wave path is a function of the rate of change of dielectric constant refractive index with height.

→ This changes from hour to hour, day to day and season to season.



(a) equivalent ray path over earth with actual earth's radius r



(b) equivalent ray path over earth with effective earth's radius r' , $r' = kr$

From fig (b) Δabc .

$$d^2 + [h + r']^2 = [r' + h + \delta h]^2$$

$$d^2 + h^2 + 2hr' + 2h\delta h = r'^2 + h^2 + (\delta h)^2 + 2hr' + 2h\delta h + 2\delta h r'$$

$$d^2 = 2\delta h [h + r'] \quad [(\delta h)^2 \ll 2\delta h [h + r']]$$

$$\delta h = \frac{d^2}{2[h + r']} \Rightarrow \boxed{\delta h = \frac{d^2}{2r'}} \quad [h \ll r']$$

Fig (a) $DC = \frac{d^2}{2R}, BD = \frac{d^2}{2r}$

$$BC = \delta h = BD - DC$$

$$\delta h = \frac{d^2}{2R} - \frac{d^2}{2r} \Rightarrow \frac{d^2}{2R} \left[\frac{1}{R} - \frac{1}{r} \right] \quad \text{--- (2)}$$

equatn ① & ②.

$$\frac{d^2}{2r'} = \frac{d^2}{2} \left[\frac{1}{r} - \frac{1}{R} \right]$$

$$\frac{1}{r'} = \left[\frac{1}{r} - \frac{1}{R} \right]$$

$$\frac{1}{kr} = \frac{1}{r} - \frac{1}{R}$$

$$[r' = kr]$$

$$\frac{1}{x'} = \frac{R-y}{yR}$$

$$x' = \frac{yR}{R-y} = \frac{y}{1-\frac{y}{R}}$$

$$x' = \frac{y}{1-\frac{y}{R}} \Rightarrow \omega \cdot R \cdot \epsilon = \frac{x'}{y} = \frac{1}{1-\frac{y}{R}}$$

$$= \frac{1}{1-\frac{y}{R}} \Rightarrow K = \frac{1}{1-\frac{y}{R}}$$

$$K = \frac{4}{3} \text{ standard value.}$$

$$x' = \frac{4}{3} y$$

$$d = 2x' [\sqrt{h} + \sqrt{h_r}]$$

$$= \left[2 \frac{4}{3} y [\sqrt{h} + \sqrt{h_r}] \right]$$

$$d = 4.12 [\sqrt{h} + \sqrt{h_r}] \quad \left[\text{Sub. } y = 6370 \text{ km} \right]$$

$d \rightarrow$ line of sight distance.

$h_t, h_r \rightarrow$ heights of TX & Rx - Antenna.

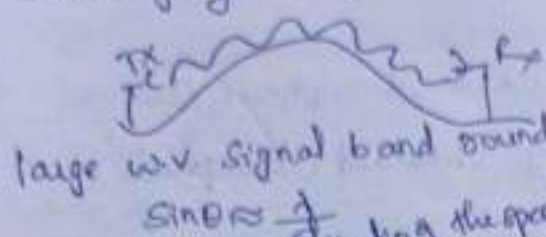
Effect of earth's curvature on space

Following two effects are introduced by the curvature of the earth.

① The diff in path lengths b/w direct & ground reflected wave is reduced as the point of reflection on the ground is raised

② Reflection at the ground takes place at a spherical point rather than a flat point and hence the reflected ray become more divergent which results in weaker at receiving pt.

⇒ Advantage of large wavelength.



M-curves.

large w.v. signal bend around the obstruction more efficiently.
 $\sin \theta \approx \frac{h}{r}$
 (1) Ratio of the speed of light in vacuum to the speed of light in the medium.
 (2) θ is the angle of incidence and ϕ is the angle of refraction.
 (3) n is the refractive index and n is the angle of refraction.
 For standard atmospheric propagation to be studied the normal refractive index will be sufficient but to understand non-standard propagation, we require a modified refractive index.

It is given by $N = n + \frac{h}{r}$

where n = actual refractive index, h = height above ground
 r = radius of earth.

→ Here value of N is unity and it is depending on h .
 hence we will calculate Excess modified refractive index
 $m = (N-1) \times 10^6$ so for that

$$N-1 = n-1 + \frac{h}{r}$$

$$(N-1) \times 10^6 = (n-1 + \frac{h}{r}) \times 10^6$$

$$m = (N-1) \times 10^6 = (n-1 + \frac{h}{r}) \times 10^6$$

→ when we plot m against h , it is called m -curves
 → note that in non-standard atmospheric, simple refraction doesn't occur, but when m curves are available it possible to predict roughly.
 → The gradient $\frac{dm}{dh}$ and its sign is depending on the tropospheric conditions.

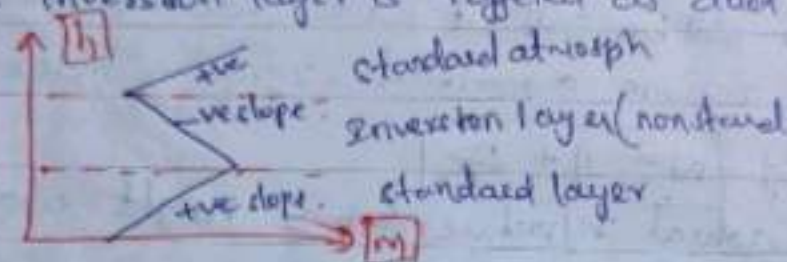
Inversion layer

— Inversion layer is a region where atmospheric condition are exactly opposite to that of standard atmosphere

→ For standard atmosphere slope of n is +ve

For inversion layer slope of n is -ve

This inversion layer is referred as duct layer.



W.D.P. Duct propagation (Super Refraction)

→ In the duct propagation radio signal follows a particular channel (or) duct in the atmosphere

→ This duct can be near to earth surface to predict the path on curves are veges

Formation duct

→ When temperature increases with height over a certain range of heights, it is known as temperature inversion.

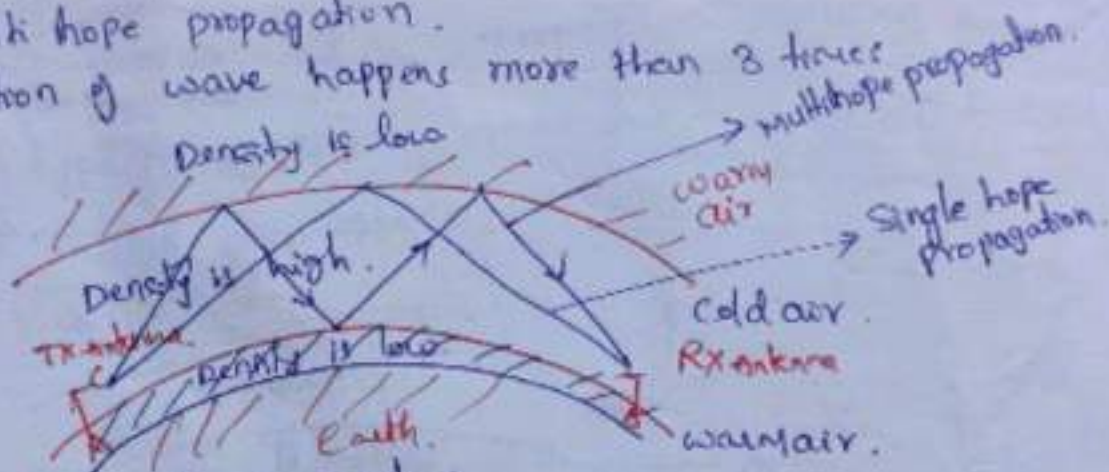
→ When inversion layer is sandwiched b/w the earth surface and standard atmosphere, then the cool air



• Tropospheric scatter propagation (or) Forward Scatter propagation

Trapped b/w the earth surface and the warm air. The region of cool air behaves like duct for radio waves.

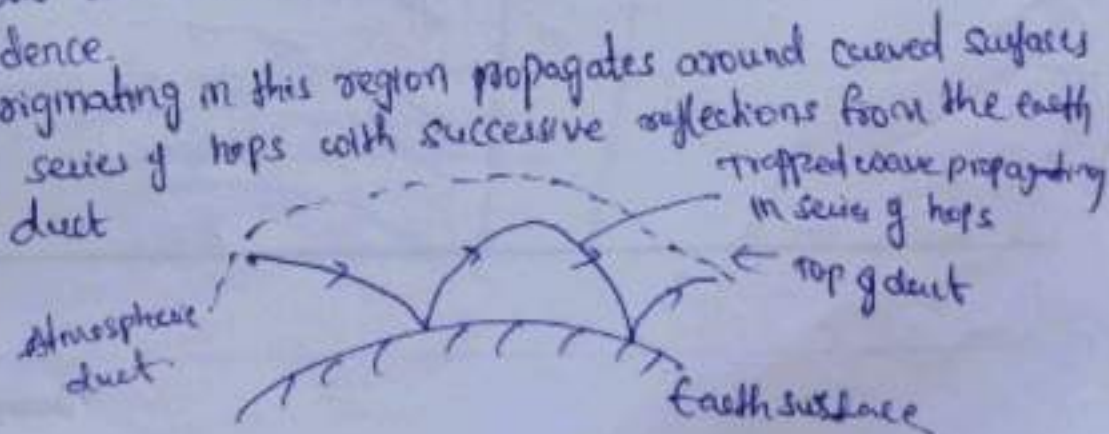
- There is elevated inversion layer is present, in such a case the cool air is trapped b/w the warm air above and below it.
- Warm air and cool air has different density. So when signal goes from cool air to warm air it phases reflection back to earth.
- That reflected signal may again reflect from transition of cool air to warm air.
- For single hop propagation.
 - Reflection of wave happens one time.
- For multi hop propagation.
 - Reflection of wave happens more than 3 times.



→ Condition for Duct propagation

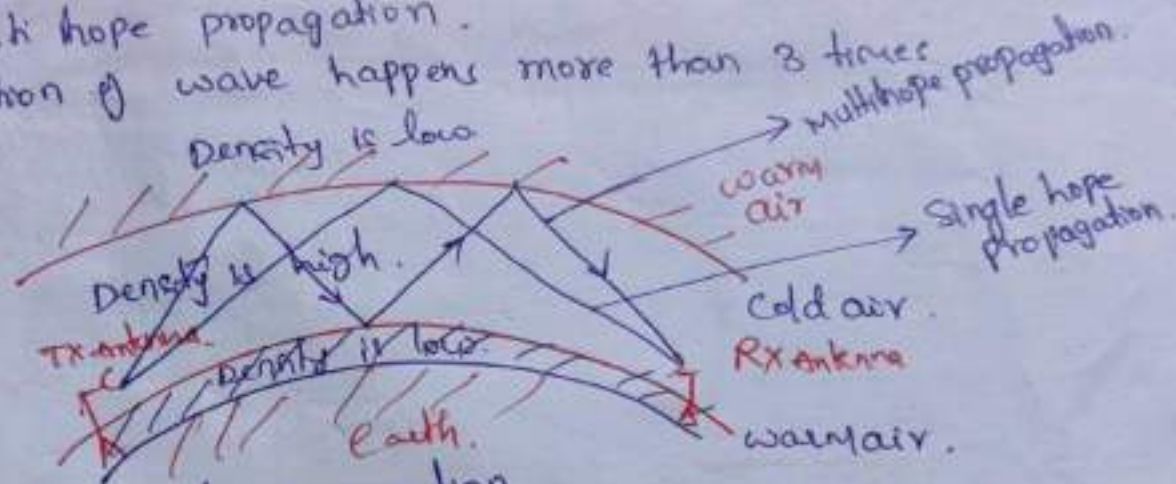
- ① The transmitting antenna is inside duct
- ② The radio wave enters the duct at very low angle of incidence.

⇒ The energy originating in this region propagates around curved surfaces in the form of series of hops with successive reflections from the earth. This is called duct.



Trapped b/w the earth surface and the warm air. The region of cool air behaves like duct for radio waves.

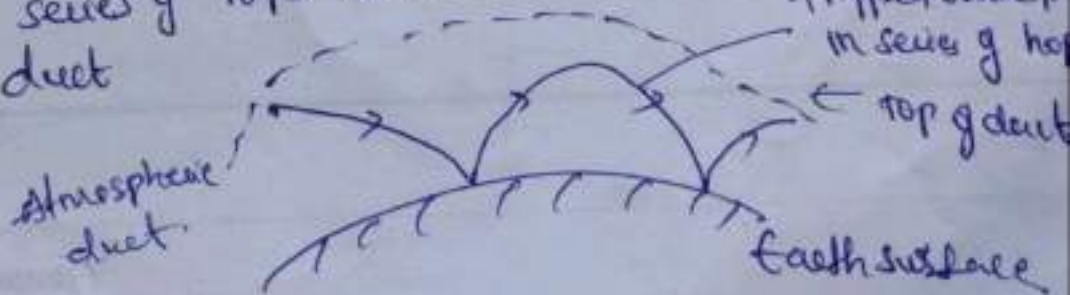
- There is elevated inversion layer is present, in such a case the cool air is trapped b/w the warm air above and below it.
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- For single hop propagation.
 - Reflection of wave happens one time.
- For multi hop propagation.
 - Reflection of wave happens more than 3 times.



→ Condition for Duct propagation

- ① The transmitting antenna is inside duct
- ② The radio wave enters the duct at very low angle of incidence.

Duct of energy originating in this region propagates around curved surface in the form of series of hops with successive reflections from the trapped wave. This is called duct.



Scattering definition separating and going in various directions

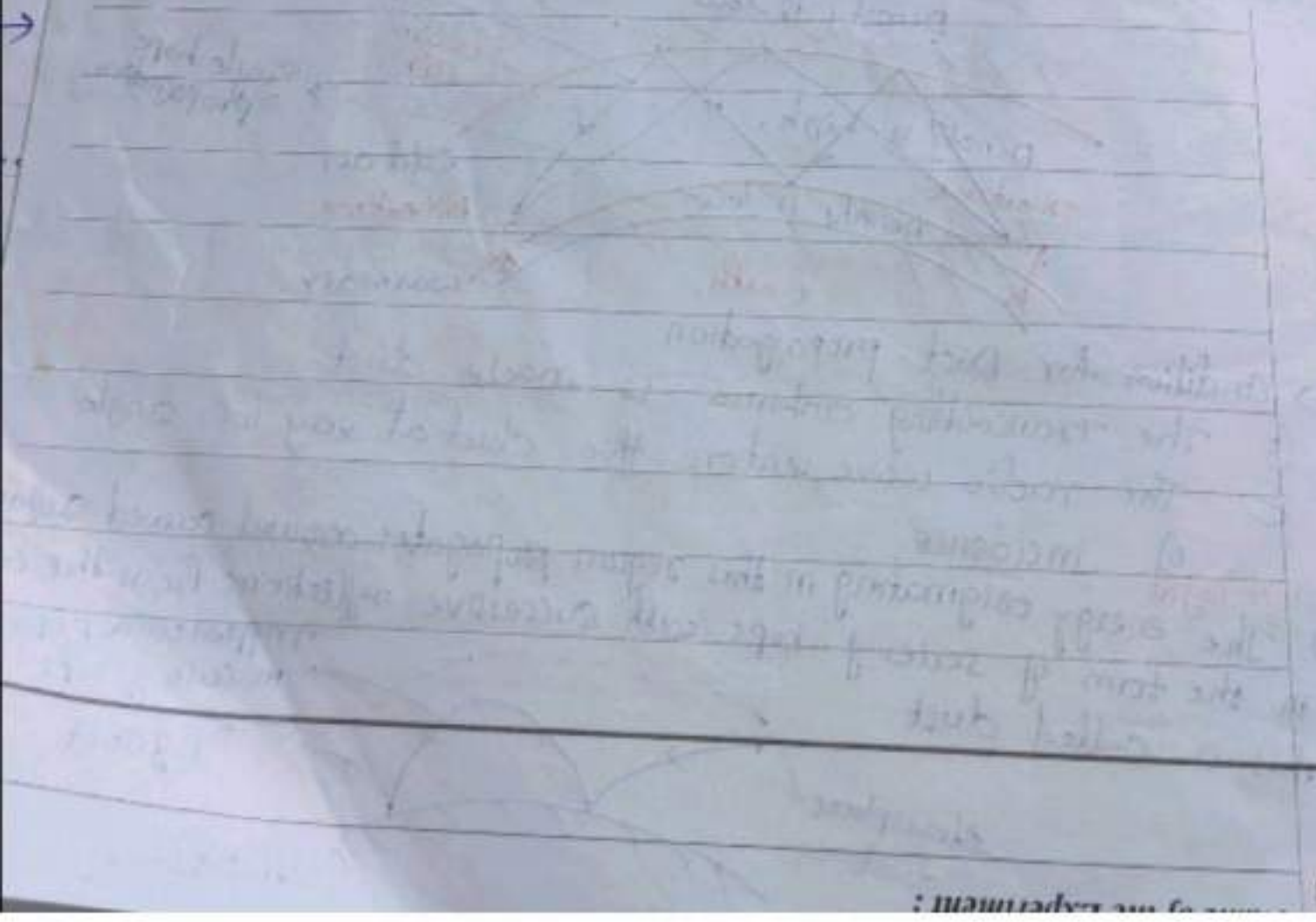
① The process in which electromagnetic radiation (particles) are deflected or diffused.

→ Reflection is the procedure of diversion of a path of a particle or wave owing to a non-interacting collision.

→ Scattering is a procedure where interaction b/w the two colliding particles occurs. where ⁱⁿ Reflection the light bounces off (gets reflected) from surface into a particular direction.

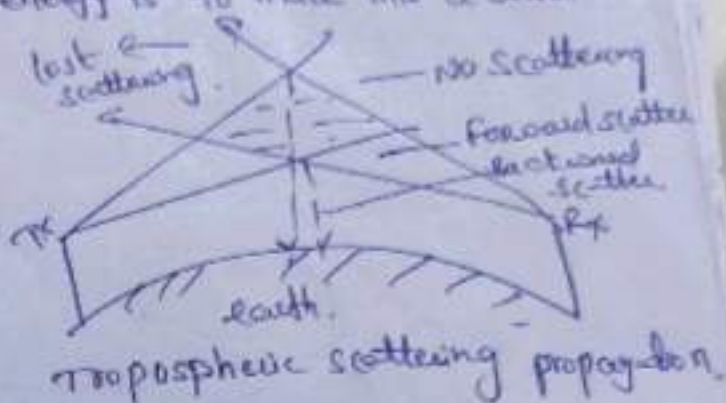
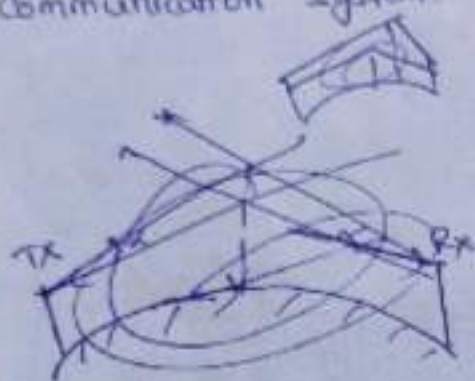
v.8

→

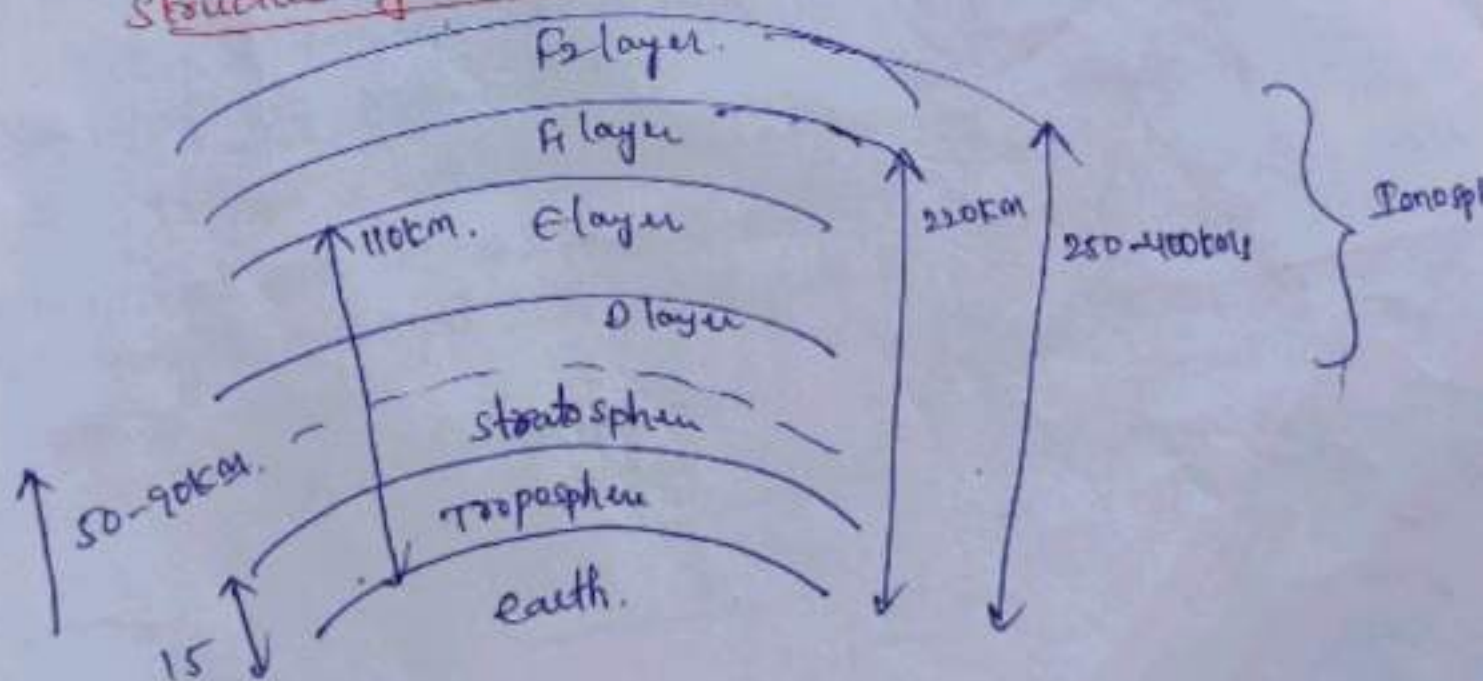


Tropospheric scatter propagation (or) Forward scatter propagation (above 3000ft)

- Two directional (transmitting and receiving) antennas are pointed such that their beams intersect at midway b/w them above the horizon.
- If one is UHF TX antenna and the other is UHF Rx antenna then sufficient radio energy is directed towards the receiving antenna then sufficient radio energy is to make this a useful communication system.



- This type of scattering arises due to blobs (or) fine layers in the troposphere
- ### Structure of Atmosphere



Skip distance is the shortest distance from a transmitter measured along earth's surface at which sky wave of fixed freq ($> f_c$) will be returned to earth.

→ by changing angle or we can observe increase the distance from TX to RX.



→ Sky wave propagation is possible for greater than skip distance d_{skip} .

→ eq of f_{muf} can and f_c is

$$f_{muf} = f_c \sqrt{1 + \left(\frac{d}{2H}\right)^2}$$

$$\Rightarrow \frac{f_{muf}}{f_c} = \sqrt{1 + \left(\frac{d}{2H}\right)^2} \Rightarrow \left(\frac{f_{muf}}{f_c}\right)^2 = 1 + \left(\frac{d}{2H}\right)^2$$

$$\Rightarrow \left(\frac{d}{2H}\right)^2 = \left(\frac{f_{muf}}{f_c}\right)^2 - 1 \Rightarrow \frac{d}{2H} = \sqrt{\left(\frac{f_{muf}}{f_c}\right)^2 - 1}$$

$$d_{skip} = 2H \sqrt{\left(\frac{f_{muf}}{f_c}\right)^2 - 1}$$

Virtual Height The virtual height is that height from which a wave sent up at an angle appears to be reflected due to gradual change in refractive index actual path is

→ TX - P - X' - Q - RX actual path

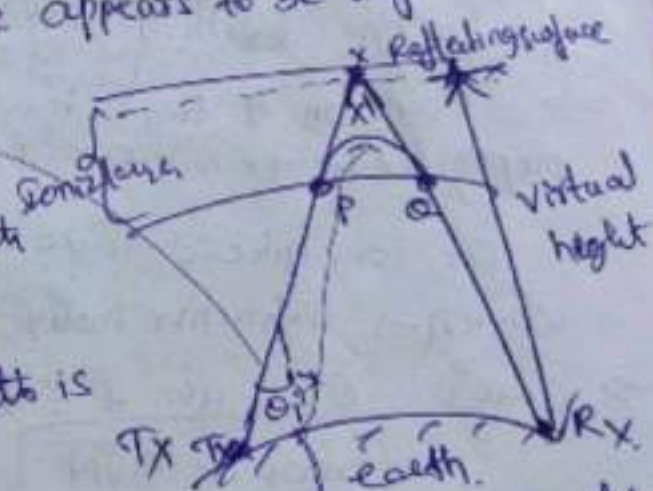
→ virtual path is TX - P - X - Q - RX

→ height associated with virtual path is virtual height

$$diff = \text{velocity} \times \text{time}$$

$$2H = c \times T$$

$$H = \frac{1}{2} cT$$



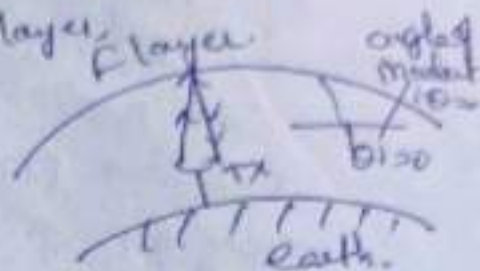
(due to a signal goes back due to refractive index change)

Critical Frequency (f_c) For any given time each ionospheric layer has a max freq at which radio waves can be transmitted vertically and reflected back to earth. This freq is known as critical freq.

For ionosphere $n = \sqrt{1 - \frac{e^2 N}{f^2}}$
 where n is refractive index of ionospheric layer,
 N is no density of electron.

Snell's law. we can say no density is

$$n = \frac{\sin i}{\sin \theta} = \sqrt{1 - \frac{e^2 N}{f^2}}$$



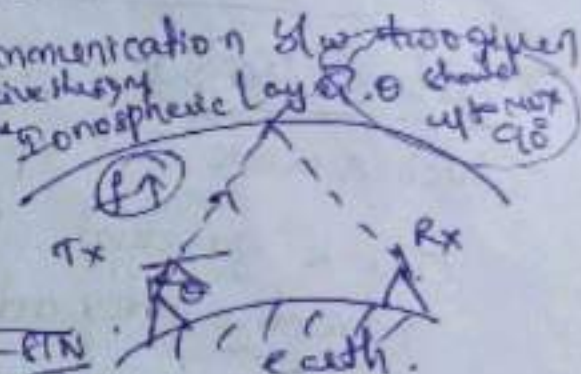
→ For $f = f_c$ (Critical freq) $i = 0$.
 $n = 0 = \sqrt{1 - \frac{e^2 N}{f_c^2}} \Rightarrow \boxed{f_c = 9\sqrt{N}}$ [N →]

D layer	Critical freq → 100 kHz
E layer	3.5 MHz
F ₁ layer	5.7 MHz
F ₂ layer	10 MHz

Maximum Usable Freq. (MUF) It is defined as the highest

freq that can be used for sky wave communication b/w two given points on earth.
 (Note: If the angle of incidence is greater than 90°, the signal will not be reflected back to earth.)

→ keeps on ↑ freq at which signal propagated by Tx ant & MUF



→ For ionospheric layer $n = \frac{\sin \theta}{\sin \theta_i} = \sqrt{1 - \frac{e^2 N}{f^2}}$

where n → refractive index, θ_i → angle of incident, N → no. density

→ Case I $\theta_i = 90^\circ$ $f = f_c = f_{MUF}$

$$\boxed{f_{MUF} = 9\sqrt{N}}$$

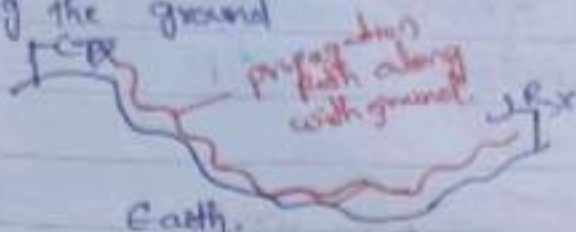
Case II $\theta_i < 90^\circ$, θ can be max by 90° then after signal goes outside.

$n = \sin \theta = \sqrt{1 - \frac{e^2 N}{f^2}}$

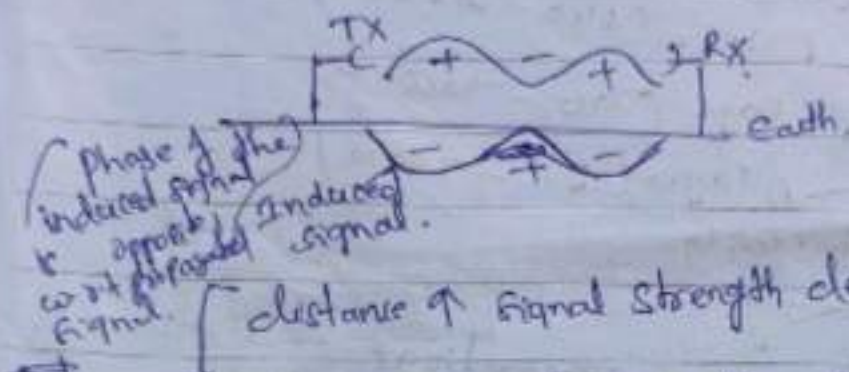
$$f_{MUF} = f_c \sqrt{\left(\frac{1}{\sin \theta_i}\right)^2 + 1}$$

Ground wave propagation (used up to 2 MHz)

→ In this propagation signal (em waves) propagates along the ground



→ It is utilized for short range communication.

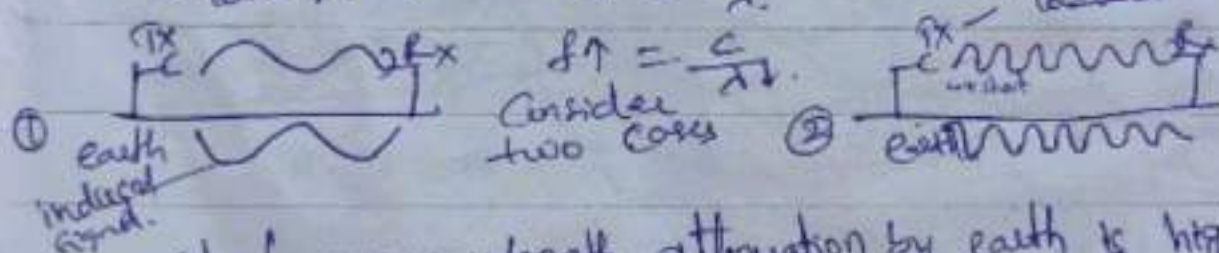


→ Induced wave by ground attenuate signal
 ⇒ As distance increases magnitude of propagating signal decreases by large amount due to attenuation by earth.

✓ lower frequency is high wave.

$$f = \frac{c}{\lambda}$$

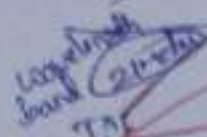
high frequency is low wave.



⇒ At lower wavelength attenuation by earth is high.

Fidd sh

②



Distance



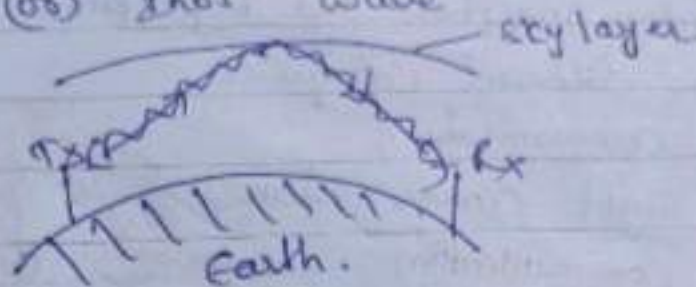
✓

- 2

Angle K is 90 degrees

Sky wave propagation (or) ionospheric wave propagation

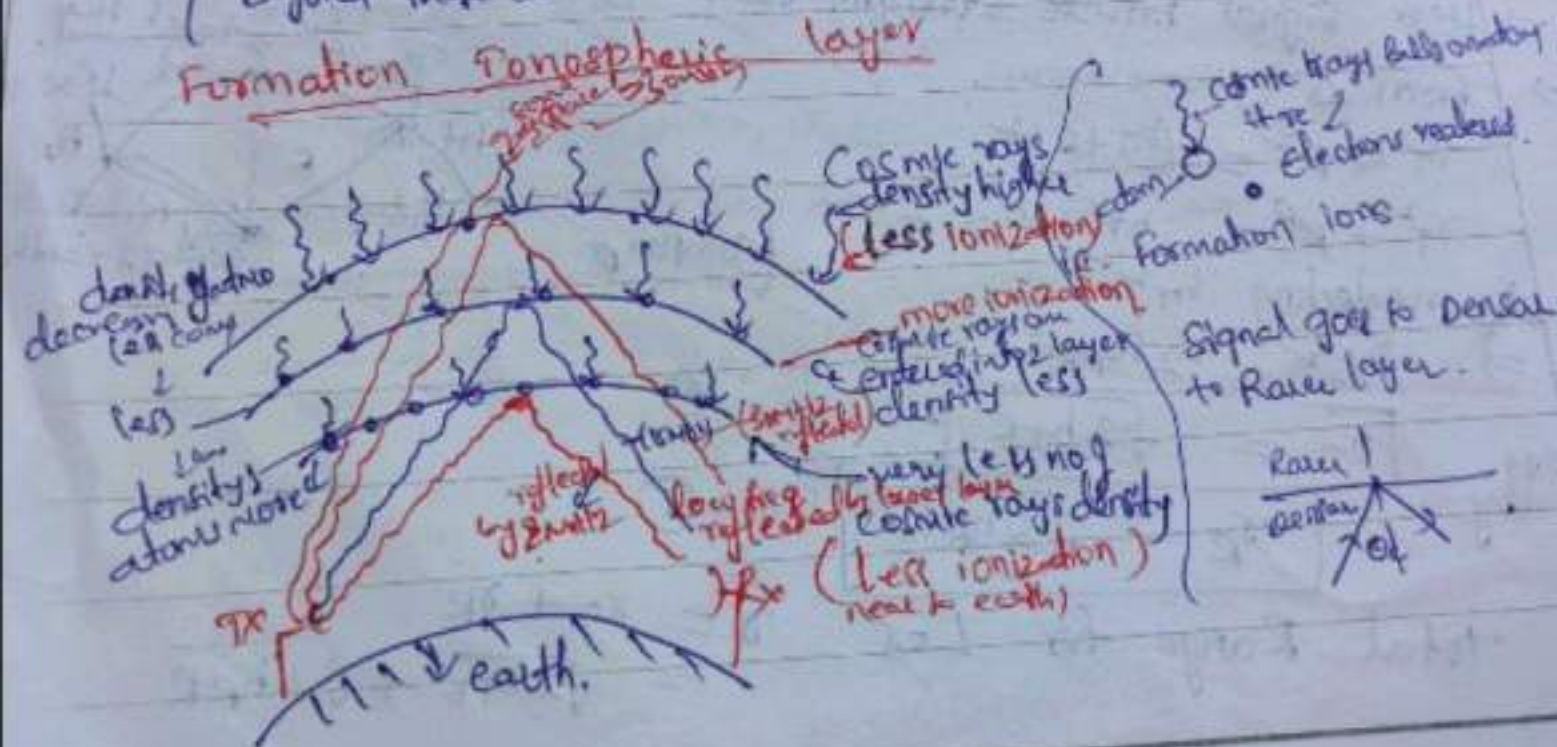
→ Transmitted signal by TX antenna Reflected by ionosphere layer (sky) and received by RX antenna i.e. sky wave propagation (or) 2nd wave sky layer



⇒ for ground wave propagation range was limited so for long range we should go for sky wave propagation

⇒ we use 3MHz to 30MHz for sky wave propagation.
(beyond these zone, penetrate use space)

Formation of phosphatic layer

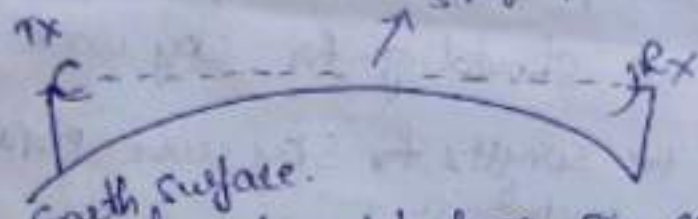


Space wave propagation

- Space wave propagation that happens at $f > 30 \text{ MHz}$
- due to freq is high wavelength is very narrow that results propagation of wave by straight path.
- At these frequencies there is no reflection & scattering will happen by ionosphere and troposphere
- It has two different categories.
 - 1) Satellite Communication
 - 2) Line of sight Communication.

Line of sight Communication

→ signal is going via straight path i.e. LOS



→ Here signal travels straight from Tx antenna to Rx antenna

→ From Δ Pythagorean theorem.

$$x_T^2 + R^2 = (h_T + R)^2$$

$$x_T^2 + R^2 = h_T^2 + R^2 + 2h_TR$$

By neglecting h_T^2

$$\Rightarrow x_T^2 = 2h_TR$$

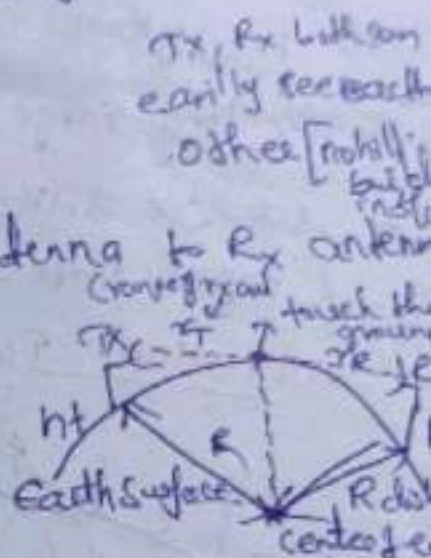
$$x_T = \sqrt{2h_TR}$$

Similarly

$$x_R = \sqrt{2h_R R}$$

total Range for LOS

$$\begin{aligned} \gamma &= x_T + x_R \\ &= \sqrt{2h_TR} + \sqrt{2h_R R} \end{aligned}$$

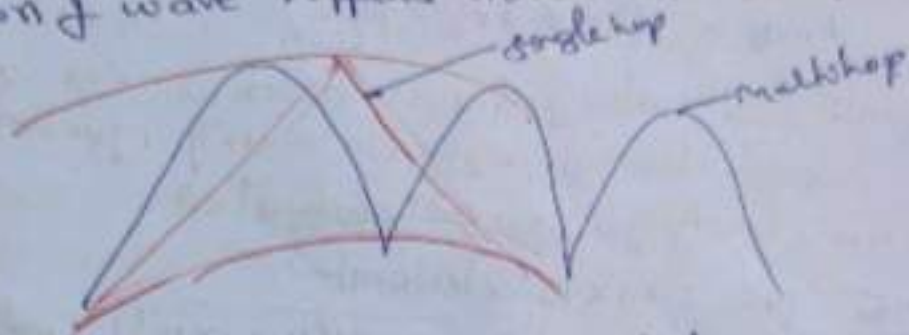


Single hop propagation

Reflection of wave happens one time.

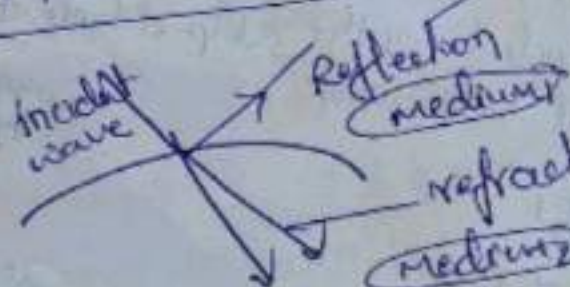
Multi hop propagation

Reflection of wave happens more than 2 times



Ray path def : Ray path is a line that is perpendicular to successive wave fronts as an acoustic pulse moves outward from the source.

Refraction & Reflection



[In Reflection doesn't change the direction & medium of wave]

Refraction [change of the direction of wave & moving from one medium to other medium]
Relation b/w MUF & skip distance is called Refraction

$$d_{skip} = 2h \sqrt{\left(\frac{f_{MUF}}{f_c}\right)^2 - 1}$$

(2) Determine the maximum usable freq for a critical freq of 20 MHz and an angle of incidence of 31° .

→ Critical freq = 20 MHz = 20×10^6 Hz

Angle of incidence = 31°

$f_{muf} = (\sec \phi_i) f_c = \frac{f_{cr}}{\cos \phi_i} = \frac{20 \times 10^6}{\cos 31^\circ} = 24.41 \times 10^6$ Hz.

(12) Calculate the value of freq at which an EM wave must propagate through the D-region an index of refraction of 0.5 and an electron density of 3.24×10^{24} electrons/m³.

$n = 0.5$, $N = 3.24 \times 10^{24}$ electrons/m³.

$n = \sqrt{1 - \frac{81N}{f^2}} \Rightarrow 0.5 = \sqrt{1 - \frac{81(3.24 \times 10^{24})}{f^2}} \Rightarrow f = 1.87061 \text{ kHz}$

14) Find the maximum distance that can be covered if the virtual height of the ionospheric layer is 250 km?

Skip distance d is obtained as $d = 2R\theta$

R = Radius of curved earth = 6370 km.

2θ = Angle subtended by the Skip distance at centre of earth expressed in radian.

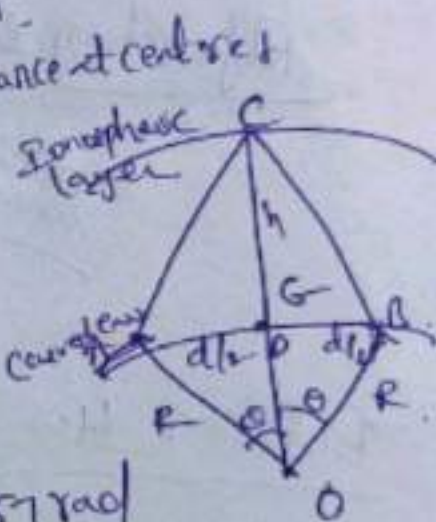
$\cos \theta = \frac{OA}{OC} = \frac{R}{R+h}$

where $OC = OE + EC$

$\cos \theta = \frac{6370}{6370 + 250} = 0.9622$

$\theta = \cos^{-1}(0.9622) = 15.8^\circ = 0.2757 \text{ rad}$

$d = 2R\theta = 2 \times (6370)(0.2757) = 3512.418 \text{ km}$



- ⑧ At which freq waves propagate in the D-region to have index of refraction 0.5 if the electron density for the D-region is $400 \text{ electrons/cm}^2$.

$$\eta = 0.5, N = 400 \text{ electrons/cm}^2 = 400 \times 10^6 \text{ elu}^3$$

$$\eta (\text{refractive index}) = \sqrt{1 - \frac{81N}{f^2}}$$

$$0.5 = \sqrt{1 - \frac{81 \times 400 \times 10^6}{f^2}} \Rightarrow f = 0.2018 \text{ MHz}$$

- ⑨ calculate maximum usable freq MUF for a high freq radio link b/w two points at a distance of 2500km on the surface of earth. Consider the height of ionosphere is 200km and the critical freq is 5MHz.

$$D = \text{propagation distance} = 2500 \text{ km}, h = \text{height of ionosphere} = 200 \text{ km}$$

$$f_{cr} = \text{critical freq} = 5 \text{ MHz}$$

Maximum usable freq (MUF) is given by

$$MUF = f_{cr} \sqrt{1 + \left(\frac{d}{2h}\right)^2} = 5 \times 10^6 \sqrt{1 + \left(\frac{2500}{2 \times 200}\right)^2} = 31.64 \text{ MHz}$$

- ⑩ A pulse of a given freq transmitted upward is received back after a period of 5 milli seconds. Find the virtual height of the reflecting ionosphere.

$$T = 5 \text{ msec}$$

$$\text{virtual height is given by } h' = \frac{CT}{2} \quad \left[\begin{array}{l} \text{velocity of light} \\ C = 3 \times 10^8 \text{ m/sec} \end{array} \right]$$

$$h = \frac{3 \times 10^8 \left(\frac{5 \times 10^{-3}}{2} \right)}{2} = 750 \times 10^3 \text{ m} \Rightarrow \boxed{750 \text{ km} = h}$$

- ⑪ For a flat earth assume that at 400km reflection takes place. The maximum density of ionosphere corresponds to a refractive index of 0.9 at 10MHz. calculate range of which $f_{muf} = 10 \text{ MHz}$.

$$\eta = 0.9, f_{muf} = 10 \text{ MHz} = 10 \times 10^6 \text{ Hz}, h = 400 \text{ km}$$

$$\eta = \sqrt{1 - \frac{81N_{max}}{f^2}} \Rightarrow 0.9 = \sqrt{1 - \frac{81N_{max}}{(10 \times 10^6)^2}} \Rightarrow N = 4.8432 \times 10^5$$

$$f_{cr} = \sqrt{81N_{max}} = \sqrt{81 \times 4.8432 \times 10^5} = 4.3588 \times 10^6 \text{ Hz} = 4.3588 \text{ MHz}$$

$$\text{skip distance } d_{skip} = \frac{2h}{\sqrt{\left(\frac{f_{muf}}{f_{cr}}\right)^2 - 1}} \Rightarrow 2 \times 400 \sqrt{\frac{(10 \times 10^6)^2}{(4.3588 \times 10^6)^2} - 1}$$

$$\boxed{D_{skip} = 1651.8 \text{ km}}$$

5) Determine the height of the tx antenna to obtain a maximum distance of transmission upto 30km from a 24 meter high receiving antenna

Height of Rx ant = $h_r = 24\text{m}$

maximum distance of transmission = $L_{os} = 30\text{km}$

$L_{os} = 4.12 [\sqrt{h_t} + \sqrt{h_r}]$

$30 = 4.12 [\sqrt{h_t} + \sqrt{24}] \Rightarrow \sqrt{h_t} = \left(\frac{30}{4.12} \right) - \sqrt{24}$

$h_t = 18.699\text{m} \approx 18.7\text{m}$

6) At a particular day time, the critical frequencies observed for the the E & F layers are 2.5 MHz and 8.5 MHz respectively. calculate the maximum electron density of both the layers in per cubic meter.

given f_{cr} for E-layer = 2.5 MHz

f_{cr} for F-layer = 8.5 MHz

$f_{cr} = \sqrt{81 N_{max}} \text{ @ } (\infty) \quad N_{max} = \frac{f_{cr}^2}{81}$

i) for E-layer maximum electron density per cubic meter

$N_{max} = \frac{f_{cr}^2}{81} = \frac{(2.5 \times 10^6)^2}{81} = 0.077 \times 10^{12} / \text{m}^3$

ii) for F-layer $N_{max} = \frac{(f_{cr})^2}{81} = \frac{(8.5 \times 10^6)^2}{81} = 0.8919 \times 10^{12}$

7) The maximum electron density for the F₁, F₂, & E-layers are $2.5 \times 10^6 \text{ cm}^{-3}$, $3.5 \times 10^6 \text{ cm}^{-3}$ & $1.5 \times 10^6 \text{ cm}^{-3}$ respectively. calculate the critical frequencies for each layer. (1 cm = 10^{-2} m)

for F₁-layer, $N_{max} = 2.5 \times 10^6 \text{ cm}^{-3} = 2.5 \times 10^6 \times 10^{-6} \text{ m}^{-3} = 2.5 \text{ m}^{-3}$

F₂ layer $N_{max} = 3.5 \times 10^6 \times 10^{-6} \text{ m}^{-3} = 3.5 \text{ m}^{-3}$

E₃-layer $N_{max} = 1.5 \times 10^6 \text{ cm}^{-3} = 1.5 \times 10^6 \times 10^{-6} \text{ m}^{-3} = 1.5 \text{ m}^{-3}$

for F₁-layer $f_{cr} = \sqrt{81 N_{max}} = \sqrt{81 \times 2.5} = 14.22 \text{ MHz}$

for F₂-layer $f_{cr} = \sqrt{81 N_{max}} = \sqrt{81 \times 3.5} = 16.83 \text{ MHz}$

for E₃-layer $f_{cr} = \sqrt{81 N_{max}} = \sqrt{81 \times 1.5} = 11.02 \text{ MHz}$

$h_t = 120\text{m}$, $h_r = 16\text{m}$, $f = 50\text{MHz} = 50 \times 10^6\text{Hz}$,
 $P = 15\text{kW} = 15 \times 10^3\text{W}$, $d = \text{distance b/w Tx \& Rx} = 12\text{km}$

i) Loc range = $4.12[\sqrt{h_t} + \sqrt{h_r}] \text{ km} = 4.12[\sqrt{120} + \sqrt{16}] = 61.612\text{ km}$

ii) Field strength at a receiving pt is given by

$$E_R = \frac{88\sqrt{P} h_t h_r}{\lambda d^2} \Rightarrow \lambda = \frac{c}{f} = \frac{3 \times 10^8}{50 \times 10^6}$$

$$E_R = \frac{88\sqrt{15 \times 10^3} (120)(16)}{\left(\frac{3 \times 10^8}{50 \times 10^6}\right) (12 \times 10^3)^2} = 23.9125 \times 10^3 \text{ V/m} = 23.9125 \text{ mV/m}$$

iii) Let the distance at which the field strength reduce to 1 mV/m be denoted by d_1 . Then at $d = d_1$

$$\begin{aligned}
 E_R &= 1 \times 10^{-3} = \frac{88\sqrt{P} h_t h_r}{\lambda d_1^2} \Rightarrow d_1^2 = \frac{88\sqrt{P} h_t h_r}{\left(\frac{3 \times 10^8}{50 \times 10^6}\right) (1 \times 10^{-3})} \\
 &= \frac{88\sqrt{15 \times 10^3} (120)(16)}{\left(\frac{3 \times 10^8}{50 \times 10^6}\right) (1 \times 10^{-3})} = 3.44 \times 10^9 \Rightarrow \boxed{58.72 \text{ km} = d_1}
 \end{aligned}$$

4) A 150m antenna transmitting at 1.2 MHz by ground wave has an antenna current of 8 A . what voltage is received by the Rx antenna 40 km away with a height of 2 m ?

$h_t = 150\text{m}$, $h_r = 2\text{m}$, $f = \text{Tx signal freq} = 1.2 \times 10^6\text{ Hz}$

$I = 8\text{ A}$, $d = \text{dist b/w Tx \& Rx} = 40\text{ km} = 40 \times 10^3\text{ m}$

Expression for field strength at a distance b/w Tx \& Rx is given by

$$E = \frac{120\pi h_t h_r I}{\lambda d} = \frac{(120\pi)(150)(2)(8)}{250(40 \times 10^3)} \quad \left[\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1.2 \times 10^6} = 250\text{ m} \right]$$

$$E = 6.09048 \text{ V/m}$$

voltage received by antenna $V = E(h_r) = (6.09048)(2) = 0.1809 \text{ V}$

VHF-2 problems

① Two aircraft are flying at altitudes of 2000m & 5000m respectively. What is the minimum possible distance along the surface of the earth over which they can have effective point-to-point microwave communication? Radius of earth is 6.37×10^6 m.

Let $h_t \rightarrow$ altitude of one aircraft = 2000m
 $h_r \rightarrow$ " other " = 5000m

Maximum distance along surface of earth is given by

$$d = \sqrt{2R} [\sqrt{h_t} + \sqrt{h_r}]$$

$$= \sqrt{2 \times 6.37 \times 10^6} [\sqrt{2000} + \sqrt{5000}] = 447.881 \text{ km}$$

② VHF communication is to be established with 50W transmitter at 100MHz. Calculate the LOS distance if the heights of transmitting and receiving antennas are respectively 50m & 10m. Assuming the capture area of transmitting antenna is 2m^2 . Calculate the field strength at the receiving end neglecting ground reflected wave.

$P =$ power transmitted = 50W, $h_t \rightarrow$ Height of Tx ant = 50m

$h_r =$ height of Rx ant = 10m, $f = \text{freq} = 100\text{MHz} = 100 \times 10^6 \text{ Hz}$

$a =$ capture area = 2m^2

Line of sight (LOS) distance is given by

$$d = 4.12 [\sqrt{h_t} + \sqrt{h_r}] = 4.12 [\sqrt{50} + \sqrt{10}] = 42.164 \text{ km}$$

Field strength at the receiving end is given by

$$E_r = \frac{88\sqrt{P} h_t h_r}{\lambda d^2} = \frac{88\sqrt{P} h_t h_r}{\left(\frac{c}{f}\right) d^2} = \frac{88\sqrt{50}(50)(10)}{\left(\frac{3 \times 10^8}{100 \times 10^6}\right) (42.163 \times 10^3)^2} = 58.34 \text{ mV/m}$$

③ A television transmitting antenna mounted at height of 120m radiates 15kW power equally in all directions in azimuth at a freq of 50 MHz. Calculate 1) The maximum LOS range 2) The field strength at a Rx ant mounted at a height of 16m at a distance of 12km.

④ The distance at which the field strength reduces to 1mV/m