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Sri Indu
College of Engineering & Technology
UGC Autonomous Institution
Recognized under 2(f) & 12(B) of UGC Act 1956,
NAAC, Approved by AICTE &
Permanently Affiliated to JNTUH

NAAC
NATIONAL ASSESSMENT AND ACCREDITATION COUNCIL

Course Material

II Year I Semester

DEPARTMENT OF INFORMATION TECHNOLOGY

ACADEMIC YEAR 2024-2025

COMPUTER ORIENTED STATISTICAL METHODS

B.Tech. II Year I Sem.

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Pre-requisites: Mathematics courses of first year of study.

Course Objectives: To learn

- The theory of Probability, Probability distributions of single and multiple random variables
- The sampling theory, testing of hypothesis and making statistical inferences
- Stochastic process and Markov chains.

Course outcomes: After learning the contents of this paper the student must be able to

- Apply the concepts of probability and distributions to case studies.
- Formulate and solve problems involving random variables and apply statistical methods for analyzing experimental data.
- Apply concept of estimation and testing of hypothesis to case studies.
- Correlate the concepts of one unit to the concepts in other units.

UNIT - I: Probability **14 L**

Sample Space, Events, Counting Sample Points, Probability of an Event, Additive Rules, Conditional Probability, Independence, and the Product Rule, Baye's Rule,

Random Variables and Probability Distributions: Concept of a Random Variable, Discrete Probability Distributions, Continuous Probability Distributions.

UNIT - II: Expectation and discrete distributions **12 L**

Mean of a Random Variable, Variance and Covariance of Random Variables, Means and Variances of Linear Combinations of Random Variables, Chebyshev's Theorem.

Discrete Probability Distributions: Binomial Distribution, Poisson distribution.

UNIT - III: Continuous and Sampling Distributions **14 L**

Uniform Distribution, Normal Distribution, Areas under the Normal Curve, Applications of the Normal Distribution, Normal Approximation to the Binomial Distributions.

Fundamental Sampling Distributions: Random Sampling, Some Important Statistics, Sampling Distributions, Sampling Distribution of Means and the Central Limit Theorem, t - Distribution,
F- Distribution.

UNIT - IV: Sample Estimation & Tests of Hypotheses **15 L**

ntroduction, Statistical Inference, Classical Methods of Estimation, Single Sample: Estimating the mean, standard error of a point estimate, prediction interval. Two sample: Estimating the difference between two means, Single sample: Estimating a proportion, Two samples: Estimating the difference between two proportions, Two samples: Estimating the ratio of two variances.

Statistical Hypotheses: General Concepts, Testing a Statistical Hypothesis, Single sample: Tests concerning a single mean, Two samples: tests on two means, One sample: test on a single proportion. Two samples: tests on two proportions, Two-sample tests concerning variances.

UNIT-V: Stochastic Processes and Markov Chains 9 L

Introduction to Stochastic processes- Markov process. Transition Probability, Transition Probability Matrix, First order and Higher order Markov process, n-step transition probabilities, Markov chain, Steady state condition, Markov analysis.

TEXT BOOKS:

1. Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, Keying Ye, Probability & Statistics For Engineers & Scientists, 9th Ed. Pearson Publishers.

S C Gupta and V K Kapoor, Fundamentals of Mathematical statistics, Khanna publications.

2. S.D.Sharma, Operations Research, Kedarnath and Ramnath Publishers, Meerut, Delhi.

REFERENCE BOOKS:

1. T.T. Soong, Fundamentals of Probability and Statistics For Engineers, John Wiley & Sons, Ltd, 2004.
2. Sheldon M Ross, Probability and statistics for Engineers and scientists, academic press.
3. Miller and Freund’s, Probability and Statistics for Engineers, 8th Edition, Pearson Educations.

UNIT – I

Introduction:– Probability

Variable (or Variate):–

A quantity which can vary from one individual to another is called a variable or variate.

eg:- heights, weights, ages, wages of persons, rainfall records of cities etc.

→ Quantities which can take any numerical value within a certain range are called continuous variables. eg:– as the child grows, his/her height takes all possible values from 50 cm to 100 cm.

→ Quantities which are incapable of taking all possible values are called discrete (or) discontinuous variables eg:- the number of children a man can have are positive integers 1, 2, 3 etc. (no values between any two consecutive integers).

Frequency distributions:–

Consider the marks obtained by 60 Students of B.Tech, M_2 Class in a college test in mathematics according to their roll numbers

26, 15, 5, 45, 7, 32, 2, 18, 42, 8, 31, 27, 4, 38, 11, 40, 0, 34, 10, 7, 5, 1, 17, 22, 35, 8, 36, 12, 35, 15, 0, 7, 28, 46, 9, 16, 29, 41, 38, 2, 17, 42, 45, 48, 47, 11, 30, 19, 0, 16, 14, 16, 18, 28, 7, 21, 8, 28, 5, 20.

The data doesnot give any useful information. It is (2) rather confusing to mind. These are called raw data (or) ungrouped data.

<u>Introduction</u>:- Probability (or) Chance is a word which ~~we~~ encounter in our daily life. We often predict that it will rain tonight. One may also say that it <u>is</u> more likely to have a good downpour (or) rain at one place than at other place. The above predictions are purely based on beliefs, past experiences and present knowledge. ~~Eg:~~

→ Tossing of a fair coin, we are not sure as to whether we shall get a head (or) a tail.

→ Throwing of a die, we cannot expect a particular number.

Introduction to Probability:-

Experiment:- An experiment is any physical action (or) process which is observed and whose result is noted.

Examples:

(i) Tossing of a coin

(ii) Rolling a dice

(iii) Picking up a card from a packet

Experiments could be divided into two types.

(i) Deterministic experiment (or) Predictable experiment

(ii) Non-deterministic (or) probabilistic (or) Random experiment.

Deterministic Experiment:- A deterministic experiment is an experiment whose outcome (or) result is known with certainty prior to the outcome of its result.

(or)

* It could be said as an experiment whose result is predictable i.e result is unique.

Ex:- (i) Throwing a stone upwards, It is known that the stone will surely fall on ground.

(ii) Ohm's law $I = \frac{V}{R}$ determines current uniquely.

Random Experiment:-

A random experiment is an experiment whose outcome (or) result is not unique and therefore cannot be predicted with certainty.

Ex:- (i) In tossing a coin one is not sure whether a head or a tail will occur.

(ii) In throwing a die one is not sure whether a 1, 2, 3, 4, 5, 6 will be obtained

(iii) Lifetime of a human being.

(iv) Lifetime of a Computer System.

Note:- (i) Probability is the study of random experiments.

(ii) Probability is a measure of Certainty.

Trial:- Trial is a Single Performance of an experiment.

Outcome:- The result of a trial in a random experiment is called an outcome.

Sample space:- The set of all possible outcomes of a random experiment is known as sample space and is denoted by S which may be finite (or) infinite.

Each element in the Sample space is called Sample point.

Ex:-1. for tossing a coin $S = \{H, T\}$ for rolling a die $S = \{1, 2, 3, 4, 5, 6\}$.

2. In tossing two coins (or) a coin tossed twice, the possible outcomes are HH, HT, TH, TT

Sample space = {HH, HT, TH, TT}

3. In tossing three coins (or) a coin is tossed thrice, the possible outcomes are 8.

Sample space = {HHH, HTH, HHT, THH, HTT, THT, TTH, TTT}

If a coin is tossed "n" times, the number of possible outcomes are "2^n" elements.

4. If a die is thrown, the possible outcomes are 1 or 2 or 3 or 4 or 5 or 6 then

Sample space S = {1, 2, 3, 4, 5, 6}

5. If a die is thrown two times, the number of outcomes are 36.

The required sample space is

(1,1), (1,2), (1,3), (1,4), (1,5), (1,6).
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6).

If a die is thrown for "n" times (or) n dice are thrown, the sample space consists of "6^n" elements.

Sample (or) Sample point:-

Sample (or) Sample point is a outcome (particular element) of S.

Event:- Result (outcome) of any experiment is considered as an Event.

Note:- Event is a subset of a sample space.

Events can further be divided into two:→

(i) Elementary Event.
(ii) Compound Event.

Elementary Event:- It can be defined as an event whose further divison into smaller events is impossible.

Eg:- In ~~toss~~ing a coin each of the events getting head (or) tail is an elementary event.

Compound Event:- This event is obtained through the combination of several elementary events thus the compound event could be further broken into elementary events.

Ex:- In throwing a die the event that an even number turns up is combination of 3 elementary events 2, 4, 6 turns up.

Independent Events:-

The Occurence (or) non-Occurence of event A has no influence (or) impact on the occurence (or) non-occurence of B.

Other wise, they are Said to be dependent.

Ex:-
In tossing an unbiased (fair) coin the event of getting a head in the first toss is independent of getting a tail in the Second, third and subsequent throws.

Universal Event ~~(or) Sure Event~~:-

The entire Sample Space S is Called an universal ~~(or) Certain (or) Sure event.~~

An event is Called a Sure event when all possible outcomes of an experiment are favourable to the event.

Impossible Event:- It is denoted by ϕ. An event is Called impossible when there is no Outcome favourable to the event. i.e $P(\phi) = 0$.

Exhaustive events :-

All possible events in any trial are known as exhaustive events.

Eg :- ① In tossing a coin, there are two exhaustive elementary events. i.e head and tail

② In throwing a die, there are six exhaustive elementary i.e getting 1 or 2 or 3 or 4 or 5 or 6.

③ In drawing 3 balls out of 9 balls in a box, there are $^9C_3$ exhaustive elementary events.

(OR)

A list of events A_1, A_2, A_3 ------- A_n are Said to be collectively exhaustive ~~(or)~~ exhaustive events if $\bigcup_{i=1}^{n} A_i = S$ (i.e $A_1 \cup A_2 \cup A_3 \cup --- \cup A_n = S$)

④ In rolling a die two times, there are $6^2 = 36$ exhaustive events.

⑤ In tossing a coin two times, there are $2^2 = 4$ exhaustive events i.e HH, HT, TH, TT.

(9)

Mutually exclusive events :— Events are said to be mutually exclusive if the happening of anyone of them prevents the happening of all ~~other~~ others.

i.e no two (or) more events can happen simultaneously in the same trial.

(or)

Two events A and B are mutually exclusive if A and B cannot happen simultaneously

i.e A and B are disjoint.

i.e $A \cap B = \phi$.

Eg :- 1. In tossing a coin, the events head and tail are mutually exclusive

2. In throwing a die, all the 6 faces are mutually exclusive.

Equally likely events :— Outcomes of a trial are said to be equally likely, if one cannot be expected in preference to the others.

Eg :- 1. In tossing an unbiased (fair) coin, H or T are equally likely events.

2. In throwing an unbiased die, all the 6 faces are equally likely to come.

3. When a card is drawn from a pack, any card may be obtained. In this trial all the 52 elementary events are equally likely.

Favourable cases:-

The number of cases favourable to an event in a trial is the number of outcomes which entail the happening of an event.

Eg:-

① In tossing a coin two times, the favourable cases of getting two heads is one. i.e HH

Sample Space = {HH, HT, TH, TT}

② In throwing of two dice, the number of cases favourable to getting the sum 6 is.

(1,5), (2,4), (3,3), (4,2), (5,1) i.e 5

Mathematical (or) Classical (or) a priori probability:-

In a random experiment, let there be 'n' mutually exclusive and collectively exhaustive and equally likely elementary events. Let E be an event of the experiment. If "m" elementary events from event E (are favourable to E) then the probability of E (probability of happening of E or chance of E) is defined as

$$P(E) = \frac{m}{n} = \frac{\text{Number of elementary events favourable to } E}{\text{Total number of elementary events in the random experiment}}$$

If $\bar{E}$ denotes the event of non-occurence of E, (11)
then the number of elementary events in $\bar{E}$ (favourable to $\bar{E}$) is n-m
and hence the probability of $\bar{E}$ (non-occurence of E) is

$$q = P(\bar{E}) = \frac{n-m}{n} = \frac{n}{n} - \frac{m}{n}$$

$$= 1 - \frac{m}{n}$$

$$P(\bar{E}) = 1 - P(E)$$

$$P(E) + P(\bar{E}) = 1 \quad \text{(or)} \quad p+q=1$$

and $0 \leq p \leq 1$
$0 \leq q \leq 1$

Since m is a non-negative integer,
n is a positive integer and $m \leq n$,
we have $0 \leq \frac{m}{n} \leq 1$.

Hence $0 \leq P(E) \leq 1$ and $0 \leq P(\bar{E}) \leq 1$.

Note:-
① Suppose we say that Occurence of an event E as success and non-occurence $\bar{E}$ as failure. probability P(E) of the happening of the event E is known as the probability of success and the probability $P(\bar{E})$ of non-happening of the event is known as the probability of failure.

② If $P(E)=1$, the event E is called certain event (12) and if $P(E)=0$, the event E is called an impossible event.

Axioms on probability

(i) Axiom of positivity:—

$$P(E) \geq 0$$

for all the events in the sample space.

(ii) Axiom of Certainity:—

$$P(S)=1$$

(iii) Axiom of union :—

if E_1, E_2 are the events of sample space S then

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

Similarly, for "n" number of events in the Sample Space S we have

$$P(E_1 \cup E_2 \cup E_3 \cup \cdots \cup E_n) = P(E_1) + P(E_2) + P(E_3) + \cdots + P(E_n)$$

Complementary events:— Two events of a Sample space whose union is entire Sample space and whose intersection is ϕ are called complementary events

$$E \cup \bar{E} = S, \quad E \cap \bar{E} = \phi$$

Here, E and $\bar{E}$ are complementary events to ~~the~~ each other.

A Class consists of 6 girls and 10 boys. If a Committee of 3 is chosen at random from the class, find the probability that (i) 3 boys are selected (13)

(ii) Exactly 2 girls are selected.

Sol:- Total number of students = 16

n = No. of ways of choosing 3 from 16 $= {}^{16}C_3 = {}^{16}C_{13} = \frac{\cancel{14}\times\cancel{15}\times 16}{\cancel{3}\times\cancel{2}\times 1} = 560$ (i)

	girls	boys	Total
	6	10	16
(ii)	${}^{6}C_2$	${}^{10}C_9$	

(i) Suppose 3 boys are selected.

This can be done in ${}^{10}C_3$ ways

$$m = {}^{10}C_3 = {}^{10}C_7 = \frac{\cancel{8}\times\cancel{9}\times 10}{\cancel{3}\times\cancel{2}\times 1} = 120$$

$P(E)$ = The probability that 3 boys are selected $= \frac{m}{n}$

$$= \frac{{}^{10}C_7}{{}^{16}C_3} = \frac{8\times 9\times 10}{{}^{16}C_{13}} = \frac{\cancel{8}\times\cancel{9}\times\cancel{10}}{\cancel{14}\times\cancel{15}\times\cancel{16}} = \frac{\cancel{12}}{\cancel{56}} \quad \frac{3}{14}$$

$$= \frac{3}{14} = 0.214285$$

(ii) Suppose exactly 2 girls are selected

Then $m = {}^{6}C_2 \times {}^{10}C_1$

$P(E)$ = The probability that exactly 2 girls and 1 boy are selected $= \frac{m}{n}$

$$P(E) = \frac{{}^{6}C_2 \times {}^{10}C_9}{{}^{16}C_3} = \frac{{}^{6}C_4 \times {}^{10}C_9}{{}^{16}C_{13}} = \frac{(\cancel{5}\times\cancel{6})\times(\cancel{10})}{\cancel{14}\times\cancel{15}\times\cancel{16}} = \frac{15}{56}$$

$= 0.2678$

*) What is the probability that a card drawn at random from the pack of playing cards may be either a queen or a King. (14)

Sol:- Let S be the sample space associated with the drawing of a card

$$n = {}^{52}C_1 = 52$$

Let E_1 be the event of the card drawn being a queen

$$n(E_1) = {}^4C_1 = 4$$

Let E_2 be the event of the card drawn being a King

$$n(E_2) = {}^4C_1 = 4$$

But E_1, E_2 are mutually exclusive events

Since $E_1 \cup E_2$ is the event of drawing either a queen or a king, we have

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

$$= \frac{n(E_1)}{n(S)} + \frac{n(E_2)}{n(S)}$$

$$= \frac{4}{52} + \frac{4}{52} = \frac{\cancel{8}^2}{\cancel{52}_{13}} = \frac{2}{13}$$ Ans

Note:- Total 52 — 26 black (B), 26 Red (R)

Diamond — 13
Heart — 13
Spade — 13
Kalavar — 13

(King 4 (R 2 + B 2), Jack 4 (R 2 + B 2), Queen 4 (2 R + 2 B), Ace 4 (2 R + 2 B))

* In a group there are 3 men and 2 women. (15)
Three persons are selected ~~at~~ random from this group. Find the probability that one man and two women or two men and one women are selected.

Sol:- Let S be the sample space associated with the selection of 3 persons out of 5. in ~~way~~

men	women	Total
3	2	5

$n(S) = {}^5C_3 = 10$ ways.

Let E_1 be the event of selecting 1 man and 2 women

$n(E_1) = {}^3C_1 \times {}^2C_2 = 3 \times 1 = 3.$

Let E_2 be the event of selecting 2 men and a women

$n(E_2) = {}^3C_2 \times {}^2C_1 = 6$

But $E_1 \cap E_2 = \phi$ i.e E_1, E_2 are mutually exclusive

Now $E_1 \cup E_2$ is the event of selecting 1 man and 2 women or 2 men and 1 women

$P(E_1 \text{ or } E_2)$
$P(E_1 \cup E_2) = P(E_1) + P(E_2)$

$$= \frac{n(E_1)}{n(S)} + \frac{n(E_2)}{n(S)}$$

$$= \frac{3}{10} + \frac{6}{10} = \frac{9}{10} = 0.9$$

Ans

(16)

* Out of 15 items 4 are not in good Condition 4 are Selected at random. Find the probability that

(i) All are not good (ii) Two are not good.

Total	Not good	good
15	4	11

Sol:- Total number of items = 15

n = Number of ways of picking 4 items out of 15 in = $^{15}C_4$ ways.

(i) Suppose 4 items are chosen which are not good

m = Number of ways of selecting 4 items which are not good = $4_{C_4} = 1$

Let E_1 be the ~~event of the getting~~ all are not ~~good~~ good

Let E_1 = The probability all are not good = $\frac{m}{n} = \frac{4C_4}{15C_4}$

$= \frac{1}{1365}$

(ii) Suppose two items are not good

Number of ways of selecting 2 bad items = 4_{C_2}

Number of ways selecting 2 good items = 11_{C_2}

Let E_2 be the ~~event of~~ the getting two items which are not good and Two items are good

Required probability = Probability of getting two items which are not good

= P(2 good and 2 bad items)

$= \frac{4_{C_2} \times 11_{C_2}}{15_{C_4}} = \frac{22}{91}$ Ans

* What is the probability that 4S's appear consecutively in the word MISSISSIPPI assuming that the letters are arranged at random.

Sol:- The word MISSISSIPPI contains 11 letters and it consists of M-1, I-4, S-4, P-2

n = No. of all possible arrangements with all the letters $= \frac{11!}{1!4!4!2!}$

Let E be the event of getting arrangements in which all 4S's appear consecutively.

In this type of arrangement we have M-1, (4S's-1), I-4, P-2

Total = 8

$$n(E) = m = \frac{8!}{1!1!4!2!}$$

$$P(E) = \frac{m}{n} = \frac{8!}{4!2!} * \frac{4!4!2!}{11!} = \frac{4}{165}$$

$$P(E) = 0.02424 \text{ Ans}$$

** A ten digit number is formed using the digits from 0 to 9 and every digit being used only once. Find the probability that the number is divisible by 4.

Sol:- Ten digits in a ten digit number can be arranged in 10! ways.

Out of which 9! will begin with the digit zero

Thus the total number of 10 digit numbers formed is $10! - 9!$

$$n = 32,65,920$$

A number will be divisible by 4 if the two extreme right digits are divisible by 4.

Thus the two extreme right digits could be

04, 08, 12, 16, 20, 24, 28, 32, 36, 40, ~~44~~, 48, 52, 56

60, 64, 68, 72, 76, 80, 84, ~~88~~, 92, 96.

The number of 10 digit number ending

with 04 are $8! = 40320$

with 12 are $8! - 7! = 35280$

with 20 are $8! = 40320$

with 24 are $8! - 7! = 35280$

with 28 are $8! - 7! = 35280$

with 32 are $8! - 7! = 35280$

with 40 are $8! = 40320$

with 48 are $8! - 7! = 35280$ etc

Thus the total number of 10 digit number divisible by 4 are $= 6(8!) + 16(8! - 7!)$

$= 6(40320) + 16(35280)$

$m = 806400$

The required probability $= \frac{806400}{3265920} = 0.24691$ Ans

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* The students in a class are selected at random, one after another, for an examination. Find the probability that the boys and girls in the class arranged alternatively if (i) The class consists of 4 boys and 3 girls

(ii) The class consists of 3 boys and 3 girls

Sol:-

(i) Suppose the class consists of 4 boys and 3 girls

Total number of students = 7

Number of ways of arranging themselves = $^{7}P_{7} = 7!$

Suppose the boys and girls are arranged alternatively

One arrangement is BGBGBGB.

The 4 boys can be arranged in $^{4}P_{4} = 4!$ ways

The 3 girls can be arranged in $^{3}P_{3} = 3!$ ways

Total number of ways = $4! \times 3!$

∴ Required probability = $\frac{4!\,3!}{7!} = \frac{1}{35}$

(ii) Suppose the class consists of 3 boys and 3 girls

Total no. of students = 6

→ Number of ways of arranging themselves = 6!

Two different arrangements are possible

(a) BGBGBG (b) GBGBGB

The 3 boys can be arranged in $^{3}P_{3} = 3!$ ways

The 3 girls can be arranged in $^{3}P_{3} = 3!$ ways

∴ Required probability = $\frac{2 \times 3! \times 3!}{6!} = \frac{1}{10} = 0.1$ Ans

* Two cards are selected at random from 10 each numbered (20)

1 to 10. Find the probability that the sum is odd if

(i) Two cards are drawn.

(ii) Find the probability that the sum is even if Two cards are drawn one after another with replacement.

(iii) Two cards are drawn one after another without replacement.

Sol:-

(i) Two cards can be drawn at a time from 10 cards in $^{10}C_2 = 45$ ways

Let E_1 denote the event of the two cards are such that the sum is odd.

We must have one card even and another odd

Number of ways of doing it $= {}^5C_1 \times {}^5C_1 = 25$

Required probability $= \frac{25}{45} = \frac{5}{9}$

(ii) Let E_2 = The sum is even when two cards are drawn one after another with replacement.

The no. of favourable cases = 50

∴ The no. of ways in which two cards can be drawn one after another with replacement $= {}^{10}C_1 \times {}^{10}C_1 = 100$

∴ Required probability $= \frac{50}{100} = \frac{1}{2}$

(iii) The no. of favorable cases = 50 (sum is even)

The number of cases that the two cards can be drawn one after another without replacement $= {}^{10}C_1 \times {}^9C_1 = 90$

∴ Required probability $= \frac{50}{90} = \frac{5}{9}$

Ans

* **Simple event:-** An event in a trial that cannot be further split is called a simple event (or) an elementary event.

Sample space:- The set of all possible simple events in a trial is called a sample space for the trial.
Each element of a sample space is called a sample point.

Any subset of a sample space is an event. It is generally denoted by E. Thus a simple event is a sample point.
Sample space is denoted by S.

Eg:- Two coins are tossed
Then the possible simple events of the trial are HH, HT, TH, TT
∴ The sample space of the trial $S = \{HH, HT, TT, TH\}$
If an event (E) is that either two heads (or) two tails appear then $E = \{HH, TT\}$
clearly the elements of E, are sample points and $E \subseteq S$.

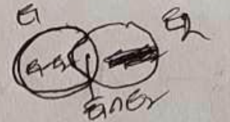
* If $E_1 = \{a_1, a_3, a_5\}$ and $E_2 = \{a_2, a_3, a_7, a_9\}$ are the subsets of S where $S = \{a_1, a_2, a_3 \text{---} a_n\}$ then

(i) $E_1 \cup E_2$ is the event that occurs "if E_1 occurs or E_2 occurs or both"

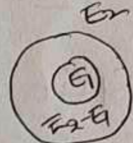
(ii) $E_1 \cap E_2$ is the event that occurs "if E_1 occurs and E_2 occurs"

(iii) $E_1 - E_2$ is the event that occurs "if E_1 occurs and E_2 doesn't occur".

* $E_1 - E_2 = E_1 - (E_1 \cap E_2)$



* $E_1 \subseteq E_2$ then $E_2 = E_1 \cup (E_2 - E_1)$



An event E of a sample space S is an impossible event if it contains no sample point. It is denoted by ϕ.
The event S is called certain event.

Mutually Exclusive events:— Two events E_1, E_2 of a sample space S are said to be mutually exclusive if they have no sample points in common i.e $E_1 \cap E_2 = \phi$.
Mutually exclusive events are sometimes called as Disjoint Events.

Complementary events:— Two events of a sample space whose intersection is ϕ and whose union is the entire sample space are called complementary events.
Thus if E is an event of a sample space S, its complement is denoted by $\overline{E}$ or E' and

$$E \cap \overline{E} = \phi, \quad E \cup \overline{E} = S, \quad \overline{E} = S - E$$

Probability – Axiomatic approach :–

Def:– Let S be a finite sample space. A real valued function P from the power set of S into R is called a probability function on S if the following three axioms are satisfied.

Axioms of probability:–

(i) Axiom of positivity :– $P(E) \geq 0$ for every subset E of S.

(ii) Axiom of certainity :– $P(S) = 1$

(iii) Axiom of union :– If E_1, E_2 are disjoint subsets of S then $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

The image $P(E)$ of E is called the probability of the event E.

Note:– If $E_1, E_2, E_3 \text{ --- } E_n$ are disjoint subsets of S, then $P(E_1 \cup E_2 \cup E_3 \text{ --- } \cup E_n)$

$$= P(E_1) + P(E_2) + P(E_3) + \text{ --- } + P(E_n).$$

Theorem:—
* P is a probability function defined on the power set of a sample space S. then

(i) $P(\phi) = 0$ (ii) $P(\bar{E}) = 1 - P(E) \leq 1$

Proof:– We know that $E \cup \phi = E$ and $E \cap \phi = \phi$

$P(E) = P(E \cup \phi)$

$P(E) = P(E) + P(\phi)$ $(E, \phi$ are disjoint sets$)$

$\cancel{P(E)} = \cancel{P(E)} + P(\phi)$

$\therefore P(\phi) = 0$

(ii) we know that $E \cup \bar{E} = S$ and $E \cap \bar{E} = \phi$

$P(E \cup \bar{E}) = P(S)$ $(E, \bar{E}$ are disjoint sets$)$

$P(E) + P(\bar{E}) = P(S)$ $P(S) = 1$

$P(E) + P(\bar{E}) = 1$

$P(\bar{E}) = 1 - P(E)$

$P(E) = 1 - P(\bar{E}) \leq 1$ $(\because P(\bar{E}) \geq 0)$

* If E is any event of a sample space S, then $0 \leq P(E) \leq 1$

proof:- $E \subseteq S$

$P(E) \leq P(S)$

$P(E) \leq 1$ $(\because P(S) = 1)$

~~But~~ by definition of probability $P(E) \geq 0$

$\therefore 0 \leq P(E) \leq 1$

* $P(E) = \frac{m}{n} = \frac{n(E)}{n(S)} = \frac{\text{Number of simple events (sample points) favourable to } E}{\text{Total no of simple events (sample points) in } S}$ 25

Note:- While calculating $P(E)$ with formula $P(E) = \frac{n(E)}{n(S)}$,

We must be sure that S consists of only simple events which are equally likely.

Notation:-

* A, B are events of a Sample space S.

(i) B occurs whenever A occur — $A \subseteq B$

*(ii) Neither A nor B occurs — $\overline{A} \cap \overline{B}$
$= \overline{A \cup B}$
$= S - (A \cup B)$

*(iii) A occurs and B doesnot occur — $A \cap \overline{B}$ or $A - B$
or $A - (A \cap B)$

*(iv) Exactly one of A and B occurs — $(A \cap \overline{B})$ or $(\overline{A} \cap B)$
(or) $(A \cup B) - (A \cap B)$

(v) Not more than one of A or B occurs
— $(A \cap B) \cup (\overline{A} \cap \overline{B}) = S - (A \cap B)$

* A, B, C are events of a Sample space S.

*(i) Among them only A occurs — $A \cap \overline{B} \cap \overline{C}$

*(ii) Both A and B occur and C doesnot occur — $A \cap B \cap \overline{C}$

(iii) All three of A, B, C occur — $A \cap B \cap C$

(iv) At least one of A, B, C occurs — $A \cup B \cup C$

v) None of A, B, C occurs — $\bar{A} \cap \bar{B} \cap \bar{C}$

vi) At least two events occur $= (A \cap B) \cup (B \cap C) \cup (C \cap A)$

(vii) one and no more occurs — $(A \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap B \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C)$

(viii) Two and no more occur — $(A \cap B \cap \bar{C}) \cup (C \cap \bar{A} \cap B) \cup (\bar{B} \cap A \cap C)$

* A box contains 3 white, 4 red and 5 black balls. A ball is drawn at random. Find the probability that it is (a) Red (b) not Black (c) Black or white

Sol:- Let W, R and B be the event of drawing a white, a Red and a Black ball respectively.

W	R	B	Total
3	4	5	12

Total number of events $= 3+4+5 = 12$

(a) Let probability of drawing a red ball $= P(R)$

$$P(R) = \frac{4C_1}{12C_1} = \frac{4}{12} = \frac{1}{3}$$

$$P(R) = \frac{1}{3}$$

(b) Let $P(B)$ = Probability of drawing a Black ball

$$= \frac{5C_1}{12C_1} = \frac{5}{12}$$

(c) Let $P(\bar{B})$ = probability of drawing a ball which is not Black

$$\boxed{P(\bar{B}) = 1 - P(B)}$$

$$= \frac{1}{1} - \frac{5}{12} = \frac{12-5}{12} = \frac{7}{12} \quad \therefore P(\bar{B}) = \frac{7}{12}$$

(i) B and W are mutually exclusive events

So $P(B\cap W)$ = probability of drawing a ball which is black and white $= 0$

$P(\text{Black or White}) = P(B\cup W)$

$= P(B) + P(W)$

$= \frac{5y}{12y} + \frac{3y}{12y}$

$= \frac{5+3}{12} = \frac{\cancel{8}^2}{\cancel{12}_3}$

$$\boxed{P(B\cup W) = 2/3}$$

* Three Students A, B and C are in swimming race. A and B have the same probability of winning and each is twice as likely to win as C. Find the probability that B or C wins.

Sol:- Given that $P(A) = P(B) = 2P(C)$

A, B, C are in a swimming race.

$A\cup B\cup C = S$

$P(A\cup B\cup C) = P(S)$

$P(A\cup B\cup C) = 1$

$P(A) + P(B) + P(C) = 1$

$P(A) + P(B) + P(C) = 1$

$2P(C) + 2P(C) + P(C) = 1$

$5P(C) = 1$

$\boxed{P(C) = 1/5}$

$\therefore P(A) = P(B) = 2/5$

Probability that B or C wins $= P(B \cup C)$

$P(B \cup C) = P(B) + P(C) \quad (B \cap C = \phi)$

$= \frac{2}{5} + \frac{1}{5}$

$P(B \cup C) = \frac{3}{5}$

* Find the probability of getting sum 10 if two dice are thrown at a time.

Sol:- The total number of exhaustive cases $= 6^2 = 36$

They are

{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

Let E be the event of getting the sum 10. (22)

The total number of cases favourable to E = 3

They are {(4,6), (5,5), (6,4)}

$$P(E) = \frac{\cancel{3}}{\cancel{36}} = \frac{1}{12}$$

$$P(E) = \frac{1}{12} \text{ Ans}$$

* Two dice are tossed once. Find the probability of getting an even number on the first die or a total of 8.

Sol:- Let S be the sample space

$n(S) = 36$.

The total number of exhaustive cases $= 6^2 = 36$

The total number of the first die.

A = Event of getting an even number on the first die.

B = Event of getting a total of 8.

$A \cap B$ = ~~Event~~ of getting an even number on the first die and a total of 8.

~~Sample~~ Space of A = {(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(4,1) (4,2), (4,3), (4,4), (4,5), (4,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

$\therefore n(A) = 18$

Sample Space of B = {(2,6), (3,5), (4,4), (5,3), (6,2)} $\therefore n(B) = 5$

Sample Space of $A \cap B$ = {(2,6), (4,4), (6,2)}, $n(A \cap B) = 3$

$P(A) = \frac{n(A)}{n(S)} = \frac{\cancel{18}^{1}}{\cancel{36}_{2}} = \frac{1}{2}$

$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$

$P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{\cancel{3}^{1}}{\cancel{36}_{12}} = 1/12$

P(even number on the first die or total of 8)

$= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{1}{2} + \frac{5}{36} - \frac{1}{12} = \frac{18 + 5 - 3}{36}$

$= \frac{\cancel{20}^{5}}{\cancel{36}_{9}} = \frac{5}{9}$ Ans

$12 \mid 12, 36$
$1, 3$
$= 12 \times 1 \times 3 = 36$

$2 \mid 36, 2$
$18, 1$
$= 2 \times 18 \times 1 = 36$

*** Two cards are selected at random from 10 cards numbered from 1 to 10.
Find the probability that the sum is even
(i) If the 2 cards are drawn together.
(ii) If the 2 cards are drawn one after another with replacement.
(iii) If the 2 cards are drawn one after another without replacement.

Sol:- Suppose two cards are drawn at a time (together) (31)

(i) i.e no. of ways of drawing 2 cards at a time from ten cards

even + even = even
even + odd = odd
odd + even = odd
odd + odd = even

$n(S) = {}^{10}C_2 = 45$ ways

even number cards are 2, 4, 6, 8, 10

odd number cards are 1, 3, 5, 7, 9.

Let E_1 denote the event of the two cards are such that the sum is even.

Both the cards should be even numbered

Both the cards should be odd numbered

(or) Both the cards can be choosen from

Two even numbered cards in ${}^5C_2$ ways

5 even numbered cards

(or) from 5 odd numbered cards in ${}^5C_2$ ways

The Number ways (favourable cases to E1) of doing it $= {}^5C_2 + {}^5C_2$

$= 10 + 10$

$m = 20$

Required probability $P(E_1) = \frac{m}{n} = \frac{20}{45} = \frac{4}{9}$ Ans

(ii) Suppose the two cards are drawn one after another with replacement.

this can be done ${}^{10}C_1 \times {}^{10}C_1 = 10 \times 10 = 100$

Let E_2 = The sum is even when two cards are drawn one after another with replacement

Sol: For the sum to be even, both the cards must be even (or) both the cards must be odd. (32)

No. of ways of selecting two even cards ~~one~~ after another with replacement (5 cards)

$= {}^5C_1 \times {}^5C_1$

$= 5 \times 5 = 25$

Similarly number of ways of selecting two odd cards (from 5 odd cards) one after another with replacement $= {}^5C_1 \times {}^5C_1$

$= 5 \times 5 = 25$

No. of favourable cases to $E_2 = 25 + 25 = 50$

Required probability $= \cancel{25+25}$ $\frac{50}{100} = \frac{1}{2}$ Ans

(ii) without replacement :-

$n = {}^{10}C_1 \times {}^9C_1$, $m = {}^5C_1 \times {}^4C_1 + {}^5C_1 \times {}^4C_1$

$P(E) = \frac{5\times4 + 5\times4}{10\times9} = \frac{40}{90} = \frac{4}{9}$ Ans

H.W.

* Two cards are selected at random from 10 each numbered 1 to 10

(i) Find the probability that the sum is odd if two card are drawn.

(ii) Find the probability that the sum is even (a) if two cards are drawn ~~one after another~~ one after another with replacement.

(b) Two cards are drawn one after another without replacement

Sol:- (i) Suppose two cards are drawn at a time from 10 cards (33)

in $^{10}C_2$ ways = 45 ways

Let E_1 denote the event of the two cards are such that the sum is odd

even + odd = odd
(or) odd + even = odd

we must have one card even (from 5 even cards) and another odd (from 5 odd cards)

The number of favourable cases to $E_1 = {}^5C_1 \times {}^5C_1$

$= 5 \times 5 = 25$

Required probability $P(E_1) = \dfrac{\text{No. of favourable cases to } E_1}{\text{Total no. of possible outcomes}}$

$= \dfrac{\cancel{25}^{5}}{\cancel{45}_{9}} = \dfrac{5}{9}$ Ans

(ii) The sum is even when two cards are drawn one after another with replacement. It can be done $^{10}C_1 \times {}^{10}C_1 = 100$

For the sum to be even, both the cards must be even (or) both the cards must be odd.

Let E_2 = the sum is even when two cards are drawn one after another with replacement.

No. of ways selecting two even cards from 5 even cards (or) two odd cards from 5 odd cards with replacement.

$= {}^5C_1 \times {}^5C_1 + {}^5C_1 \times {}^5C_1 = 5\times5 + 5\times5$

$= 25 + 25 = 50$

$\therefore$ No. of favourable cases to $E_2 = 50$

Required probability $= \frac{\cancel{50}^{1}}{\cancel{100}_{2}} = \frac{1}{2}$

(iii) ~~The number of favourable cases = 50 (25 and 25 evens)~~

Total number of cases that the two cards can be drawn one after another with out replacement

$= 10_{9} \times 9_{9}$

$= 10 \times 9 = \underline{90}$

Let E_3 = The sum is even when two cards are drawn one after another without replacement.

No. of favourable cases Selecting two even cards from 5 even cards (or) two odd cards from 5 odd cards without replacement

$= 5_{9} \times 4_{9} + 5_{9} \times 4_{9}$

$= (5 \times 4) + (5 \times 4)$

$= 20 + 20$

$= \underline{\underline{40}}$

Required probability

$P(E_3) = \frac{40}{90} = \frac{4}{9}$ Ans

A and B throw alternatively with a pair of ordinary dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, show that his chance of winning is $\frac{30}{61}$.

(35)

Sol:- When two dice are thrown, we have $n(S) = 36$.

The probability of A throwing "6" is $P(A) = \frac{5}{36}$

(i.e (1,5), (2,4), (3,3), (4,2), (5,1))

The probability A not throwing "6" is given by

$$P(\bar{A}) = 1 - P(A) = 1 - \frac{5}{36} = \frac{31}{36}$$

The probability of B throwing "7" is $P(B) = \frac{6}{36} = \frac{1}{6}$

(i.e (1,6), (2,5), (3,4), (4,3), (5,2), (6,1))

The probability of B not throwing 7 is

$$P(\bar{B}) = 1 - P(B) = 1 - \frac{1}{6} = \frac{5}{6}$$

Chances of winning of A is

$$= P(A) + P(\bar{A})\,P(\bar{B})\,P(A) + P(\bar{A})\,P(\bar{B})\,P(\bar{A})\,P(\bar{B})\,P(A) + \text{------}$$

$$= \frac{5}{36} + \left(\frac{31}{36} \times \frac{5}{6} \times \frac{5}{6}\right) + \left(\frac{31}{36} \times \frac{5}{6} \times \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36}\right) + \text{------}$$

$$= \frac{5}{36}\left[1 + \left(\frac{31}{36} \times \frac{5}{6}\right) + \left(\frac{31}{36} \times \frac{5}{6}\right)^2 + \text{------}\right]$$

$$= \frac{5}{36}\left[\frac{1}{1-\left(\frac{31}{36}\times\frac{5}{6}\right)}\right]$$

(Series ~~Bg~~ in G.P)
$1+r+r^2+ \dots + \infty$

$S_\infty = \frac{a}{1-r}$ if $r<1$

$S_\infty = \frac{a}{\cancel{r-1}}$ for $r>1$)

$$= \frac{30}{61}$$

Ans

$\left(\because r = \frac{31}{36}\times\frac{5}{6} = 0.7 < 1\right)$

~~* Two cards are selected at random from 10 cards numbered 1 to 10. Find the probability that the sum is even if (i) The two cards are drawn together (ii) The two cards drawn one after other with replacement. (iii) The two cards are drawn at a time.~~

~~Sol:- (i) Suppose two cards are drawn at a time from Number of ways drawing two cards at a time from 10 cards $= {}^{10}C_2 = 45$~~

Total Cards (52)

26 Red

26 Black

Diamond – 13

Heart – 13

Spade – 13

Kalavar – 13

King – 4 < 2 red, 2 black

Queen – 4 < 2 red, 2 black

Jackey – 4 < 2 red, 2 black

Ace – 4 < 2 red, 2 black

* What is the probability of picking an ace and a king from 52 cards deck?

Sol:- Number of ways picking 2 cards from 52 cards = $^{52}C_2$

Number of ways of picking one ace and a king

$= {}^4C_1 \times {}^4C_1 = 16$

Probability of picking one ace and a king $= \frac{16}{^{52}C_2}$

$= \frac{16}{26 \times 51} = \frac{8}{663}$ Ans

* Twelve balls are distributed at random among three boxes. What is the probability that the first box will contain 3 balls?

Sol:- Since each ball can go to any one of the three boxes there are three ways in which a ball can go to any one of the three boxes. Hence there are 3^{12} ways in which 12 balls can be placed in three boxes.

Number of ways in which 3 balls out of 12 can go to the first box is $^{12}C_3$ ways

Now the remaining 9 balls are to be placed in remaining 2 boxes and their can be done in 2^9 ways.

$\therefore$ Total number of favourable cases $= {}^{12}C_3 \times 2^9$

Required probability $= \dfrac{{}^{12}C_3 \times 2^9}{3^{12}} = \underline{\underline{0.212}}$

*HW

Five persons in a group of 20 are engineers. If three persons are selected at random, determine the probability that all are engineers and the probability that at least one being an engineer.

Sol:- Number of engineers in a group of 20 people $= 5$

Number of ways selecting 3 persons from 20 people $= {}^{20}C_3$

Number of ways in which all are engineers $= {}^{5}C_3$

(i) Number of ways in which all are engineers

$\therefore$ The probability that all the three are engineers

$$= \frac{{}^{5}C_3}{{}^{20}C_3} = \frac{1}{114}$$

Eng	Non-Eng	Total
5	15	20

(ii) Number of ways in which atleast one engineer is selected $= {}^{5}C_1 + {}^{5}C_2 + {}^{5}C_3$

$= 5 + 10 + 10 = 25$

$\therefore$ The probability that atleast 1 engineer is selected

$$= \frac{25}{{}^{20}C_3} = \frac{5}{228}$$

* What is the Chance that a leap year Selected at random will contain 53 Sundays?

Sol: A leap year consists of 366 days, So that there are 52 full weeks (and hence 52 Sundays) and two extra days.

These two days can be

(i) Monday, Tuesday
(ii) Tuesday, wednesday
(iii) wednesday, Thursday
(iv) Thursday, Friday
(v) Friday, Saturday
(vi) Saturday, Sunday
(vii) Sunday, monday

of these 7 cases, the last two are favourable and hence the required probability $= \frac{2}{7}$

* A committee consists of 9 students two of which are from 1^{st} year, three from 2^{nd} year and four from 3^{rd} year. Three students are to be removed at random. what is the chance that
(i) the three students belong to different classes
(ii) Two belong to the Same class and third to the different class
(iii) The three belong to the Same class?

(i) The total number of ways of choosing 3 students out of 9 is ${}^{9}C_{3} = 84$

A student can be ~~removed~~ from 1^{st} year students in ${}^{2}C_{1}$ ways, from 2^{nd} year in ${}^{3}C_{1}$ ways and from 3^{rd} year in ${}^{4}C_{1}$ ways,

So that the total number of ways of removing three students, one from each group

$${}^{2}C_{1} \times {}^{3}C_{1} \times {}^{4}C_{1} = 2 \times 3 \times 4 = 24$$

Hence the required chance $= \frac{24}{84} = \frac{2}{7}$

(ii) The number of ways of removing two from 1^{st} year students and one from others

$$= {}^{2}C_{2} \times {}^{7}C_{1}$$

The number of ways removing two from 2^{nd} year students and one from others $= {}^{3}C_{2} \times {}^{6}C_{1}$

The number of ways of removing 2 from 3^{rd} year students and one from others $= {}^{4}C_{2} \times {}^{5}C_{1}$

$\therefore$ The total number of ways in which two students of the same class and third from the others may be removed $= {}^{2}C_{2} \times {}^{7}C_{1} + {}^{3}C_{2} \times {}^{6}C_{1} + {}^{4}C_{2} \times {}^{5}C_{1}$

$$= 7 + 18 + 30$$

$$= 55$$

Hence the required chance $= \frac{55}{84}$ (41)

iii) Three students can be removed from 2^{nd} year group in $^3C_3 = 1$ way.

and from 3^{rd} year group in $^4C_3 = 4$ ways.

$\therefore$ The total number of ways in which three students belong to the same class $= 1+4 = 5$

Hence the required chance $= \frac{5}{84}$

* A bag contains 8 white and 6 red balls. Find the probability of drawing two balls of the same colour.

Sol:- Two balls out of 14 can be drawn in $^{14}C_2$ ways which is the total number of outcomes.

Two white balls out of 8 can be drawn in $^8C_2$ ways.

Thus the probability of drawing 2 white balls

White	Red	Total
8	6	14

$$= \frac{^8C_2}{^{14}C_2} = \frac{28}{91}$$

Similarly 2 red balls out of 6 can be drawn in $^6C_2$ ways.

Thus the probability of drawing 2 red balls

$$= \frac{^6C_2}{^{14}C_2} = \frac{15}{91}$$

Hence the probability of drawing 2 balls of the same colour. (either both white or both red) $= \frac{28}{91} + \frac{15}{91} = \frac{43}{91}$

Ans

* A card is drawn from a well shuffled of cards. (42)
What is the probability that it is either a spade or an ace?

Sol:- Let S is the sample space of all the simple events.

$n(S) = 52$.

Let A denote the event of getting a spade $= 13/4$

and B denote the event of getting an ace. $= 4/4$

Then $A \cup B$ = The event of getting a spade or an ace.

$A \cap B$ = The event of getting a spade and ace. $= \frac{1}{52}$

$$P(A) = \frac{13}{52},\ P(B) = \frac{4}{52},\ P(A \cap B) = \frac{1}{52}$$

By addition theorem,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

$$= \frac{16}{52}$$

$$= \frac{4}{13}$$

Ans

*) From a city 3 news papers A, B, C are being published. A is read by 20%, B is read by 16%, C is ready by 14%. both A and B are read by 8%, both A and C are read by 5%. both B and C are read by 4% and all three A, B, C are read by 2%. What is the percentage of the population that read at least one paper.

Sol:- Given $P(A) = \frac{20}{100}$ (20%), $P(B) = \frac{16}{100}$, $P(C) = \frac{14}{100}$ and

$P(A \cap B) = \frac{8}{100}$, $P(A \cap C) = \frac{5}{100}$, $P(B \cap C) = \frac{4}{100}$, $P(A \cap B \cap C) = \frac{2}{100}$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$= \frac{20}{100} + \frac{16}{100} + \frac{14}{100} - \frac{8}{100} - \frac{5}{100} - \frac{4}{100} + \frac{2}{100}$$

$$= \frac{35}{100}$$

percentage of the population that read at least one paper $= \frac{35}{100} \times 100 = \underline{35\%}$

HW

When two dice are rolled simultaneously, find the probability that their sum is a prime number. (493/1)

Sol:- When two dice are rolled simultaneously then sample space is

{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5) (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

When two dice are rolled, the sum of the numbers on the dice may be a number from 2 to 12 in which the prime numbers are 2, 3, 5, 7, 11.

Total number of favourable cases are 15. They are

{(1,1), (1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), (3,4)
(4,1), (4,3), (5,2), (5,6), (6,1), (6,5)}

Required probability $= \frac{\cancel{15}^{5}}{\cancel{36}_{12}} = \frac{5}{12}$ Ans

** If we toss a coin twice, the outcome of the second toss will in no way be affected by the outcome of the first toss. This is an example of independent events

** If two cards are drawn from a well-shuffled pack of 52 cards one after the other with replacement then getting an ace in the first draw and getting a King in the second draw are independent events.
But if the card drawn in the first draw is not replaced and then only 51 cards will remain in the pack, and the outcome of the second draw is dependent on the first draw.

* If two events A and B are independent

(i) $P(A \text{ and } B) = P(A) \times P(B)$

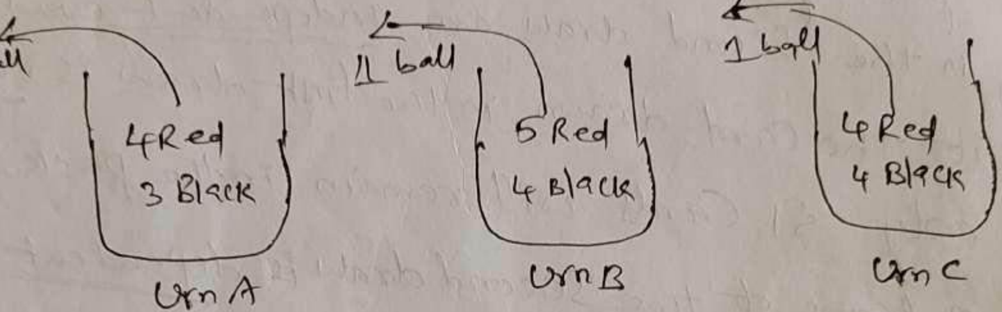
(ii) $P(A|B) = P(A)$ (iii) $P(B/A) = P(B)$

* Independent events:- first ball replaced before second ball is picked. (eg)

* Dependent events:- First ball not replaced before second ball is picked.

* There are 3 urns, A, B and C. A contains 4 red balls and 3 black balls, urn B contains 5 red balls and 4 black balls. Urn C contains 4 red balls and 4 black balls. One ball is drawn from each of these urns. What is the probability that the 3 balls drawn consists of 2 red balls and a black ball?

Sol:-



Out of 3 balls drawn, 2 red and one black ball can be obtained in the following mutually exclusive ways.

(i) Red, Red, Black (ii) Red, black, Red (iii) Black, Red, Red

The Required probability $= P[(RRB) \text{ or } (RBR) \text{ or } (BRR)]$

$$= P(RRB) + P(RBR) + P(BRR)$$

$$= \frac{^4C_1}{^7C_1} \times \frac{^5C_1}{^9C_1} \times \frac{^4C_1}{^8C_1} + \frac{^4C_1}{^7C_1} \times \frac{^4C_1}{^9C_1} \times \frac{^4C_1}{^8C_1} + \frac{^3C_1}{^7C_1} \times \frac{^5C_1}{^9C_1} \times \frac{^4C_1}{^8C_1}$$

$$= \frac{80+64+60}{7\times9\times8} = \frac{204}{504} = \frac{17}{42}$$

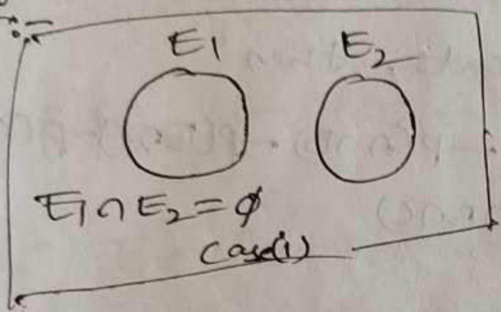
* A bag X contains 3 white balls and 2 black balls and another bag Y contains 2 white balls and 4 black balls. A bag and a ball out of it are picked at random. What is the probability that the ball is white.

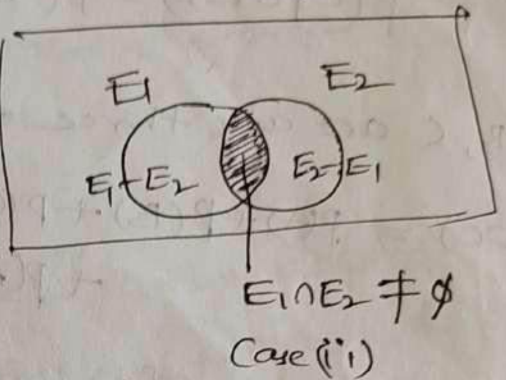
Addition theorem on Probability:-

*Theorem:- If S is a sample space, and E_1, E_2 are any events in S then

$$P(E_1 \text{ or } E_2) = P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

proof:-





case(i) Let $E_1 \cap E_2 = \phi$

If E_1, E_2 are independent then

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

$$= P(E_1) + P(E_2) - 0$$

$$= P(E_1) + P(E_2) - P(\phi)$$

$$\boxed{P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)}$$

case(ii) Let $E_1 \cap E_2 \neq \phi$ then

$$E_1 \cup E_2 = E_1 \cup (E_2 - E_1) \quad \text{and} \quad E_1 \cap (E_2 - E_1) = \phi$$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2 - E_1) \quad \text{--- (1)}$$

Also $E_2 = (E_2 - E_1) \cup (E_1 \cap E_2)$ and $(E_2 - E_1) \cap (E_1 \cap E_2) = \phi$

$$P(E_2) = P(E_2 - E_1) + P(E_1 \cap E_2)$$

$$\Rightarrow P(E_2 - E_1) = P(E_2) - P(E_1 \cap E_2) \quad \text{--- (2)}$$

from eqn ① and eqn ②

$$\boxed{P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)}$$

* Note:- If E_1, E_2 are two mutually exclusive events, (or) independent events

then $P(E_1 \cup E_2) = P(E_1) + P(E_2)$ since $P(E_1 \cap E_2) = \phi$

$P(\phi) = 0$

* If A, B, C are any three events, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

Proof:- LHS $= P(A \cup B \cup C) = P[(A \cup B) \cup C]$

$$= P(A \cup B) + P(C) - P[(A \cup B) \cap C]$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - P[(A \cap C) \cup (B \cap C)]$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - \{P(A \cap C) + P(B \cap C) - P[(A \cap C) \cap (B \cap C)]\}$$

$$\boxed{P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)}$$

* If ϕ is an impossible event then S.T $P(\phi) = 0$

Proof:- Let S be the sample space

$S = S \cup \phi$, $S \cap \phi = \phi$

$P(S \cap \phi) = P(\phi) = 0$

$P(S) = P(S \cup \phi)$

$$P(S) = P(S) + P(\phi) - P(S \cap \phi)$$

(∵ S and ϕ are disjoint sets $S \cap \phi = \phi$) 46

$$\cancel{P(S)} = \cancel{P(S)} + P(\phi) - 0$$

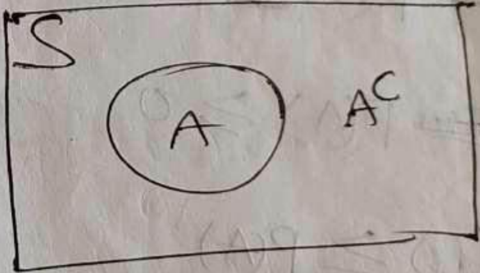
$$\boxed{P(\phi) = 0}$$

* Probability of complementary event

$$P(A^C) = 1 - P(A) \leq 1$$

Proof:- The sample space S can be divided into two mutually exclusive events A and A^C

$$S = A \cup A^C$$



$$P(S) = P(A \cup A^C)$$

$$P(S) = P(A) + P(A^C) - P(A \cap A^C)$$

$A \cap A^C = \phi$

$P(A \cap A^C) = \phi$

$$P(S) = P(A) + P(A^C) - 0$$

$$1 = P(A) + P(A^C)$$

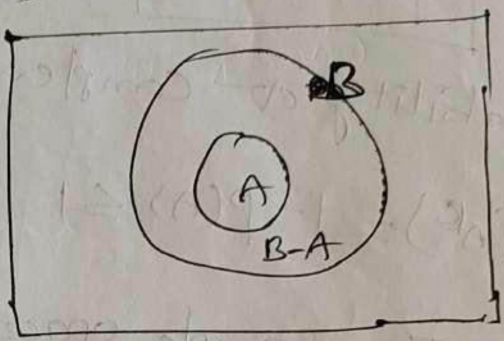
$$P(A^C) = 1 - P(A) \quad (\because P(A) \geq 0)$$

$$\boxed{\therefore P(A^C) = 1 - P(A) \leq 1}$$

* If A and B are two events of a sample space S such that $A \subseteq B$ then $P(A) \leq P(B)$. (47)

Proof:- Let $A \subseteq B$ then $B = A \cup (B-A)$

A and $B-A$ are disjoint events



$A \cap (B-A) = \phi$

$B = A \cup (B-A)$

$P(B) = P(A) + P(B-A)$

$P(B) - P(A) = P(B-A)$ $\quad (\because P(B-A) \geq 0)$

$P(B) - P(A) \geq 0$

$P(B) \geq P(A)$

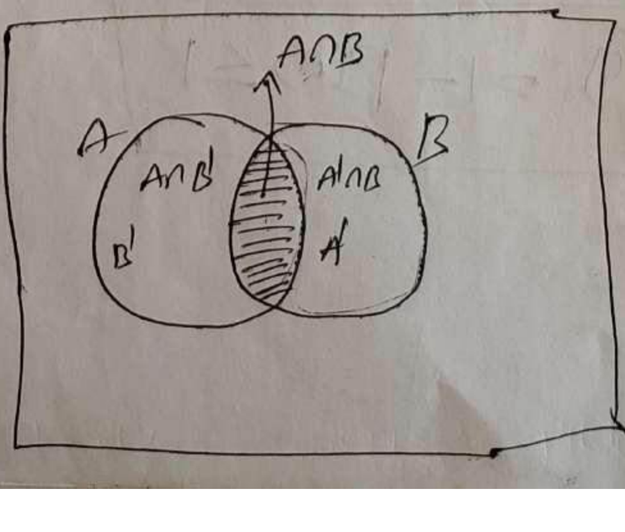
$\therefore P(A) \leq P(B)$

* For any two events A and B S.T

$P(A^c \cap B) = P(B) - P(A \cap B)$

Proof:-

$B = (A \cap B) \cup (A' \cap B)$



$P(B) = P(A \cap B) + P(A' \cap B)$

($A \cap B$ and $A' \cap B$ are disjoint sets)

$P(B) - P(A \cap B) = P(A^c \cap B)$

$\boxed{\therefore P(A^c \cap B) = P(B) - P(A \cap B)}$ (or) $P(A' \cap B) = P(B) - P(A \cap B)$

Similarly we can prove

$\boxed{P(A \cap B^c) = P(A) - P(A \cap B)}$

Note:-

$(A \cup B)^c = A^c \cap B^c$

$(A \cap B)' = A' \cup B'$

* ① $P(A' \cap B) = P(B) - P(A \cap B)$
* ② $P(A \cap B') = P(A) - P(A \cap B)$
* ③ $P(A^c \cap B^c) = P[(A \cup B)^c] = 1 - P(A \cup B)$
* ④ $P(A^c \cup B^c) = P[(A \cap B)^c] = 1 - P(A \cap B)$

Note:- Total cards = 52

- Red 26
  - Heart (13)
  - Diamond (13)
- Black 26
  - Club (13)
  - Spade (13)

Kings — 4 (2 red + 2 black)

Queen — 4 (2 red + 2 black)

Jockey — 4 (2 red + 2 black)

Ace — 4 (2 red + 2 black)

Conditional Probability: — For two dependent events A and B, the symbol $P(B/A)$ denotes the probability of occurence of B, when A has already occured. It is known as the conditional probability and is read as a probability of B given A.

(The prob. of B can be calculated where probability of A is given)

$$P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{n(A\cap B)}{n(A)}$$

Given

(OR)

$$P(E_2/E_1) = \frac{P(E_1\cap E_2)}{P(E_1)}$$

$$P(E_1/E_2) = \frac{P(E_2\cap E_1)}{P(E_2)}$$

Similarly we define

$$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{n(A\cap B)}{n(B)}$$

General Multiplication theorem of Probability: —

Statement: — In a random experiment if E_1, E_2 are two events such that $P(E_1)\neq 0$ and $P(E_2)\neq 0$

then $P(E_1\cap E_2) = P(E_1)\cdot P\left(\frac{E_2}{E_1}\right)$

$$P(E_2\cap E_1) = P(E_2)\cdot P\left(\frac{E_1}{E_2}\right)$$

Independent (with replacement) and dependent events (without replacement): — If the occurence of the event E_2 is not affected by the occurence (or) non-occurence of the event E_1, then the event E_2 is said to be independent of E_1 and $P\left(\frac{E_2}{E_1}\right) = P(E_2)$.

Similarly $P\left(\frac{E_1}{E_2}\right) = P(E_1)$

i.e. $P(E_2/E_1) = \frac{P(E_1\cap E_2)}{P(E_1)} = \frac{P(E_1)P(E_2)}{P(E_1)} = P(E_2)$

If $P(E_1) \neq 0$, $P(E_2) \neq 0$ and E_2 is independent of E_1, then E_1 is independent of E_2. In this case we say that E_1, E_2 are mutually independent (or) simply independent and $P\left(\frac{E_2}{E_1}\right) = P(E_2)$ or $P\left(\frac{E_1}{E_2}\right) = P(E_1)$. 50

* If the occurence of the event E_2 is effected by the occurence of E_1, then the events E_1, E_2 are dependent and $P\left(\frac{E_2}{E_1}\right) \neq P(E_2)$

Note:- The event E is independent of ϕ and S.

Pairwise independent events:- Three events E_1, E_2, E_3 are mutually independent (or) simply independent if (i) $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$,

$$P(E_2 \cap E_3) = P(E_2) \cdot P(E_3)$$

$$P(E_3 \cap E_1) = P(E_3) \cdot P(E_1)$$

(ii) $P(E_1 \cap E_2 \cap E_3) = P(E_1) \cdot P(E_2) \cdot P(E_3)$

* A bag contains 8 red and 6 blue balls. Two drawing of each, 2 balls are made. Find the probability that the first drawing gives two red balls and second drawing gives 2 blue balls, if the balls drawn first are replaced before the second draw.

Sol:- Let E_1 be the event of drawing 2 red balls in the first draw from the bag containing 8 red and 6 blue balls.

Red	blue	Total
8	6	14

Then $p(E_1) = \frac{8C_2}{14C_2}$. The two drawn balls are replaced

Let E_2 be the event of drawing 2 blue balls in the Second draw from the bag then

1st draw 2 red $8C_2$ | 8 red. 6 blue | 2nd draw 2 blue → $6C_2$

$$p(E_2) = \frac{6C_2}{14C_2}$$

Now $E_1 \cap E_2$ = Event of drawing 2 red balls in the ~~first~~ draw and another drawing of 2 blue balls in the Second draw after the balls are replaced.

Also E_1, E_2 are independent

$$p(E_1 \cap E_2) = p(E_1) \times p(E_2) = \frac{8C_2}{14C_2} \times \frac{6C_2}{14C_2}$$

$$= \frac{8 \times 7}{14 \times 13} \times \frac{6 \times 5}{14 \times 13}$$

$$= \frac{60}{1183}$$

* A bag contains 50 tickets numbered 1, 2, 3, 4 — — 50 of which five are drawn at random and arranged in ascending order of the magnitude $(x_1 < x_2 < x_3 < x_4 < x_5)$. What is the probability that $x_3 = 30$?

Sol:- If $x_3 = 30$, then the two tickets with numbers x_1 and x_2 must have come out of 29 tickets numbered from 1 to 29. This can be done in $^{29}C_2$ ways. (52)

The other two tickets with numbers x_4 and x_5 must have been drawn out of 20 tickets numbered from 31 to 50. This can be done in $^{20}C_2$ ways.

Number of favourable cases $= {}^{29}C_2 \times {}^{20}C_2 = m$

Total no. of cases $= {}^{50}C_5$ (n)

$$\text{Required probability} = \frac{{}^{29}C_2 \times {}^{20}C_2}{{}^{50}C_5}$$

$$= \frac{406 \times 190}{2118760}$$

$$= \frac{77140}{2118760} = \underline{0.036408}$$

$$= \frac{27550}{1567}$$

Ans

* **Addition theorem on probability:** (Addition rule for arbitary events)

Statement:- If S is a sample space and E_1, E_2 are any events in S then $P(E_1 \text{ or } E_2) = P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

Proof:- Case (i) $E_1 \cap E_2 \neq \phi$

Let E_1, E_2 contain the sample points $a_1, a_2, a_3, \text{---} a_k, a_{k+1}, \text{------} a_{k+l}$, and $a_{k+1}, a_{k+2}, \text{---}, a_{k+l}, a_{k+l+1}, \text{------} a_{k+l+m}$ respectively.

(53)

$\therefore E_1 = \{a_1, a_2, a_3, \text{------} a_k, a_{k+1}, a_{k+2} \text{------} a_{k+l}\}$

and $E_2 = \{a_{k+1}, a_{k+2}, a_{k+3}, \text{---------} a_{k+l}, a_{k+l+1}, \text{---} a_{k+l+m}\}$

$\therefore E_1 \cup E_2 = \{a_1, a_2, a_3 \text{------} a_k, a_{k+1} \text{------} a_{k+l}, a_{k+l+1} \text{------}$

$\text{---------------} a_{k+l+m}\}$

and $E_1 \cap E_2 = \{a_{k+1}, a_{k+2} \text{------} a_{k+l}\}$

$p(E_1) + p(E_2) - p(E_1 \cap E_2) = p(a_1) + p(a_2) + p(a_3) + \text{---------}$
$+ p(a_k) + p(a_{k+1}) + \text{------} + p(a_{k+l})$

$+ p(a_{k+1}) + p(a_{k+2}) + \text{------} + p(a_{k+l}) +$
$p(a_{k+l+1}) + \text{------} + p(a_{k+l+m})$

$- \{p(a_{k+1}) + p(a_{k+2}) + \text{------} + p(a_{k+l})\}$

$= p(a_1) + p(a_2) + \text{---} + p(a_k) + p(a_{k+1}) + \text{------} + p(a_{k+l})$
$+ \cancel{p(a_{k+1})} + \cancel{p(a_{k+2})} + \text{------} + \cancel{p(a_{k+l})} + p(a_{k+l+1})$
$+ \text{------} + p(a_{k+l+m}) - \cancel{p(a_{k+1})} - \cancel{p(a_{k+2})} \text{---} - \cancel{p(a_{k+l})}$

$= (p(a_1) + p(a_2) + \text{------} + p(a_k) + p(a_{k+1}) + \text{------}$
$+ p(a_{k+l}) + p(a_{k+l+1}) + \text{---} + p(a_{k+l+m}))$

$= p(E_1 \cup E_2) \quad \therefore p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$

Case(ii) $E_1 \cap E_2 = \phi$

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$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - \cancel{P(E_1 \cap E_2)}$$
$$= P(E_1) + P(E_2) - \cancel{\;\;} \; 0$$
$$= P(E_1) + P(E_2) - P(\phi)$$
$$\boxed{P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)}$$

* If E_1, E_2 are two mutually exclusive events,

then (i) $P(E_1 \cup E_2) = P(E_1) + P(E_2)$ Since $P(E_1 \cap E_2) = P(\phi) = 0$

(ii) $P(\overline{E_1 \cup E_2}) = P\left(S - (E_1 \cup E_2)\right)$
$$= P(S) - P(E_1 \cup E_2)$$
$$= 1 - \{P(E_1) + P(E_2)\}$$
$$= 1 - P(E_1) - P(E_2)$$
$$= P(\overline{E_1}) - P(E_2)$$

* Conditional event :— If E_1, E_2 are events of a sample space S and if E_2 occurs after the occurence of E_1, then the event of occurence of E_2 after the event E_1 is called conditional event of E_2 given E_1. It is denoted by $\frac{E_2}{E_1}$ (or) (E_2/E_1)

Similarly we define $\frac{E_1}{E_2}$ (or $E_1/\overset{\to \text{given}}{E_2}$)

Eg:- Two unbaised dice are thrown. If the sum of the numbers thrown on them is 7, the event of getting 1 on any one of them is a Conditional event.

Special multiplication Rule:- If A and B are independent events then $P(A \cap B) = P(A) \cdot P(B)$

i.e Probability of the product ~~is~~ the Product of the Probabilities.

Proof :- By the def. of general multiplication rule

$$P(A \cap B) = P(A) \cdot P(B/A)$$

But as A and B are independent

$$P(B/A) = P(B)$$

$$P(A \cap B) = P(A) \cdot P(B)$$ Ans

* If A and B are independent then A and B' are independent.

* If A and B are independent then A' and B' are also independent

* If A, B, C are mutually independent events then (i) A and $B \cup C$ are independent. (ii) $A \cup B$ and C are also independent.

* For any three events A, B and C

$$P\left(\frac{A \cup B}{C}\right) = P(A/C) + P(B/C) - P(A \cap B/C)$$

* Determine (i) $P(B/A)$ (ii) $P(A/B^c)$ (iii) $P(B/A^c)$ if A and B are event $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, $P(A \cup B) = \frac{1}{2}$.

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Sol:- $P(B/A) = \dfrac{P(A \cap B)}{P(A)}$

Now,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{1}{2} = \frac{1}{3} + \frac{1}{4} - P(A \cap B)$$

$$P(A \cap B) = \frac{7}{12} - \frac{1}{2}$$

$$\boxed{P(A \cap B) = \frac{1}{12}}$$

$$P(B/A) = \frac{(1/12)}{(1/3)} = \cancel{3} \times \frac{1}{\cancel{12}_4} = \frac{1}{4} \text{ Ans}$$

(ii) $P(A/B^c) = \dfrac{P(A \cap B^c)}{P(B^c)}$

we have, $\boxed{P(B^c) = 1 - P(B)} = 1 - \frac{1}{4} = \frac{3}{4}$

we have $\boxed{P(A \cap B^c) = P(A) - P(A \cap B)}$

$$P(A \cap B^c) = \frac{1}{3} - \frac{1}{12}$$

$$P(A \cap B^c) = \frac{4-1}{12} = \frac{\cancel{3}}{\cancel{12}_4} = \frac{1}{4}$$

$$P(A/B^c) = \frac{P(A \cap B^c)}{P(B^c)} = \frac{(\frac{1}{\cancel{4}})}{(\frac{3}{\cancel{4}})} = \frac{1}{3}$$

Ans

(iii) $P(B/A^c) = \frac{P(A^c \cap B)}{P(A^c)}$ (57)

we have, $P(A^c) = 1 - P(A) = 1 - \frac{1}{3} = 2/3$

$$\boxed{P(A^c \cap B) = P(B) - P(A \cap B)}$$

$$= \frac{1}{4} - \frac{1}{12}$$

$$= \frac{12-4}{12 \times 4} = \frac{\cancel{8}^{1}}{\cancel{12}_{6} \times \cancel{4}} = \frac{1}{6}$$

$$P(B/A^c) = \frac{P(A^c \cap B)}{P(A^c)} = \frac{(1/6)}{(2/3)} = \frac{\cancel{3}}{2} \times \frac{1}{\cancel{6}_2} = \frac{1}{4} \text{ Ans}$$

x* (Thrice) A bag contains 8 red and 6 blue balls. Two drawing of each 2 balls are made. Find the Probability that the first drawing gives two red balls and Second drawing gives 2 blue balls, if the balls drawn first are replaced before the second draw.

Proof:- Let E_1 be the event of drawing 2 red balls in the first draw from the bag containing 8 red and 6 blue balls.

Then $P(E_1) = \frac{8C_2}{14C_2} = \frac{8C_6}{14C_{12}} = \frac{\left(\frac{7 \times 8}{2!}\right)}{\left(\frac{13 \times 14}{2!}\right)} = \frac{7 \times \cancel{8}^4}{13 \times \cancel{14}_2} = \frac{4}{13}$

The two drawn balls are replaced.

Let E_2 be the event of drawing 2 blue balls in the Second draw from the bag.

Then $P(E_2) = \frac{6C_2}{14C_2} = \frac{6C_4}{14C_{12}} = \frac{5 \times 6/2!}{13 \times 14/2!} = \frac{5 \times \cancel{6}^3}{13 \times \cancel{14}_7} = \frac{15}{91}$

Now $E_1 \cap E_2$ = Event of drawing 2 red balls on the first draw and another drawing of 2 blue balls in the second draw after the balls are replaced. (58)

Also E_1, E_2 are ~~[illegible]~~ independent (with replacement)

$$\boxed{P(E_1 \cap E_2) = P(E_1) \times P(E_2)}$$

$$= \frac{4}{13} \times \frac{15}{91} = \frac{60}{1183}$$

Ans

* Find the probability of drawing 2 red balls in succession (one after another) from a bag containing 4 red and 5 black balls when the ball that is drawn first is (i) not replaced (ii) replaced.

Sol:- Let E_1 be the event of drawing a red ball on the first draw and E_2 be the event of drawing a red ball in second draw also.

Red	Black	Total
4	5	9

(i) After the first draw the ball is ~~not~~ replaced.

The first ball can be drawn in 9_{C_1} ways. and the second ball can be drawn in $8C_1$ ways since the ~~first~~ ball is not replaced.

Then both the balls can be drawn in 9×8 ways. i.e 72 ways.

There are $4_{C_1} = 4$ ways in which E_1 can occur and $3_{C_1} = 3$ ways in which E_2 can occur, so that E_1 and E_2 can occur in $4 \times 3 = 12$ ways.

$P\left(\frac{E_2}{E_1}\right)$ By Conditional probability

$= P(E_2$, given the probability of $E_1)$ (E_1, E_2 are dependent) (not replaced)

$= P$ (2^{nd} ball is red, given that first ball is red)

$= \frac{3}{8}$

$P(E_1 \cap E_2) = P(E_1) \times P\left(\frac{E_2}{E_1}\right)$ $\left(\because P(E_1) = \frac{4}{9}\right)$

$= \left(\frac{\cancel{4}}{\cancel{9}_3}\right)\left(\frac{\cancel{3}}{\cancel{8}_2}\right)$

$= \frac{1}{6}$ Ans

$P\left(\frac{E_2}{E_1}\right) \rightarrow$ given condition
E_1 is already occured
E_2 is occuring.
$\frac{E_2}{E_1}$

(ii) Suppose the ball is replaced after the first draw.

(E_1, E_2 are Independent) with replace

By multiplication theorem

Then $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$

$= \frac{4}{9} \cdot \frac{4}{9} = \frac{16}{81}$ Ans

* In a certain town 40% have brown hair, 25% have brown eyes and 15% have both brown hair and brown eyes. A person is selected at random from the town. What is the probability that

(i) If he has brown hair (A), what is the probability that he has brown eyes (B) also?

(ii) If he has brown eyes (B), determine the probability that he doesn't have brown hair (A')?

Sol:— Given $n(S) = 100$

Let A = The set of people who have brown hair, $n(A) = 40$

Then $P(A) = 40/100 = \frac{n(A)}{n(S)}$

Let B = The set of people who have brown eyes.

Then $n(B) = 25$ $\qquad P(B) = \frac{n(B)}{n(S)}$

$P(B) = \frac{25}{100}$

$A \cap B$ = The set of people who have both brown hair and brown eyes.

$n(A\cap B) = 15$, $P(A\cap B) = \frac{15}{100} = \frac{n(A\cap B)}{n(S)}$

By conditional probability

(i) $P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{\frac{15}{100}}{\frac{40}{100}} = \frac{15}{40} = \frac{3}{8} = 0.375$

By conditional probability

(ii) $P\left(\frac{\bar{A}}{B}\right) = \frac{P(\bar{A}\cap B)}{P(B)}$

$= \frac{P(B) - P(A\cap B)}{P(B)}$

$= \cancel{\frac{P(B)}{P(B)}} - \frac{P(A\cap B)}{P(B)}$

$= 1 - \left(\frac{15/100}{25/100}\right) = 1 - \frac{\cancel{15}^{3}}{\cancel{25}_{5}} = \frac{2}{5} = 0.4$ Ans

$P\left(\frac{\bar{A}}{B}\right) = \frac{2}{5}$ Ans

Note :- multiplication theorem

(i) For dependent event (without replacement) $P(A\cap B) = P(A)\cdot P(B/A)$

(ii) for Independent (with replacement) $P(A\cap B) = P(A)\cdot P(B)$.

* Two marbles are drawn in succession from (64)
a box containing 10 red, 30 white, 20 blue and 15 orange marbles, with replacement being made after each draw. Find the probability that
(i) Both are white (ii) First is red and Second is white.

Sol:- Total number of marbles in the box $= 75$ $= 10+30+20+15$

(i) Let E_1 be the event of the first drawn marble is white. Then $P(E_1) = \frac{30}{75} = \frac{30}{75}$

Let E_2 be the event of second drawn marble is also white. Then $P\left(\frac{E_2}{E_1}\right) = \frac{30}{75} = \frac{30}{75} = \frac{2}{5}$ (with replacement)

The probability that both marbles are white

$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2/E_1)$ (~~E_1, E_2 are independent~~)

(~~$P(\frac{E_2}{E_1}) = P(E_2)$~~)

$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2/E_1)$

$\boxed{P(E_1 \cap E_2) = P(E_1) \times P(E_2) = \frac{30}{75} \times \frac{30}{75} = \frac{4}{25}}$ (E_1, E_2 are independent)

(ii) Let E_3 be the event that the first drawn marble is ~~[illegible]~~ red. Then $P(E_3) = \frac{10}{75} = \frac{2}{15}$

Let E_4 be the event that the second drawn marble is white. Then $P\left(\frac{E_4}{E_3}\right) = P(E_4) = \frac{30}{75} = \frac{2}{5}$ (with replacement)

$\therefore$ The probability that the first marble is red and second marble is white $= P(E_3 \cap E_4)$

$$P(E_3 \cap E_4) = P(E_3) \cdot P(E_4/E_3)$$
$$= P(E_3) \cdot P(E_4/E_3)$$ (~~E_3, E_4 are dependent~~)

$$\boxed{P(E_3 \cap E_4) \Rightarrow P(E_3) \cdot P(E_4)}$$ (E_3, E_4 are independent events)

$$= \frac{2}{15} \times \frac{2}{5}$$
$$= \frac{4}{75} \text{ Ans}$$

68

* 40% of the population of a town are voters, 50% are educated, and 20% are educated voter. A person is chosen at random.

(a) If he is educated, What is the probability that he is a voter?

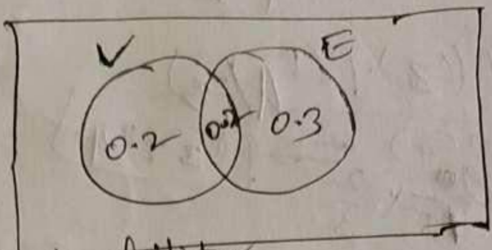
(b) If he is a voter, what is the probability that he is not educated?

(c) what is the probability that he is neither a voter nor educated?

Sol:- Let "V" and "E" be the events that the person is voter and educated respectively. And let $(E \cap V)$ be the event that the person is an educated voter.

Then $P(V) = 40\% = 0.4$
$P(E) = 50\% = 0.5 \Rightarrow P(E) = 0.5$
$P(E \cap V) = 20\% = 0.2$

This is clearly shown on the venn diagram given below



By conditional probability

(a) The required probability

$$P\left(\frac{V}{E}\right) = \frac{P(E\cap V)}{P(E)} = \frac{0.2}{0.5} = 0.4$$

$$P\left(\frac{V}{E}\right) = 0.4$$

By conditional probability

(b) The required probability

$$P\left(\frac{E}{V}\right) = \frac{P(E\cap V)}{P(V)} = \frac{P(E)\cdot P(V)}{P(V)}$$

$$= \frac{P(E\cap V)}{P(V)} = \frac{0.5\times 0.4}{0.4} = 0.5$$

(c) The probability that a person is neither a voter nor educated.

$$= P(\overline{V\cup E})$$

$$= 1 - P(V\cup E)$$

$$= 1 - \left(P(V) + P(E) - P(V\cap E)\right)$$

$$= 1 - \{0.4 + 0.5 - 0.2\}$$

$$P(\overline{V\cup E}) = \underline{\underline{0.3}}$$

Independent and dependent events:-

If the occurence of the event E_2 is not affected by the occurrence (or) non-occurence of the event E_1, then the event E_2 is said to be independent of E_1 and $p\left(\frac{E_2}{E_1}\right) = p(E_2)$.

If $p(E_1) \neq 0$, $p(E_2) \neq 0$ and E_2 is independent of E_1, then E_1 is independent of E_2.

In this case we say that E_1, E_2 are mutually independent (or) simply independent.

If the occurence of the Event E_2 is effected by the occurence of E_1, then the events E_1, E_2 are dependent and $p\left(\frac{E_2}{E_1}\right) \neq p(E_2)$.

Note:- 1. The event E is independent of ϕ and S.

2. If A and B are independent events of a sample space S, then $(\bar{A}, \bar{B})$, $(A, \bar{B})$, $(\bar{A}, B)$ are also independent.

Note:- Multiplication rule:- Multiplication rule is applied when two events doesn't occur. ~~together~~ It is applied for the following notations.

- $p(A \cap B)$
- $p(AB)$
- p(Both A, B)
- p(A and B)
- p(A as well as B)

(P.T.O)

Types of multiplication rules:—

(1) ~~Independent events (with replacement)~~:— (by)

One event doesn't effect on the other event, then A and B events are called independent events and it is given by $P(A \cap B) = P(A) \cdot P(B)$

(2) Dependent events (without replacement):—

one event effects on the other event, (Conditional probability), then A and B events are called dependent events and its given by

$$P(A \cap B) = P(A) \times P(B/A)$$

(or)

$$P(B \cap A) = P(B) \times P(A/B)$$

Note:—

(1) Addition theorem:— It is applied when one of the events occur & it is applied when both the events should occur together.

→ The notation of Addition theorem are applied as

- $P(A \cup B)$ (or) $P(A \text{ or } B)$ (or) $P(\text{either } A \text{ or } B)$ (or) $P(\text{At least})$

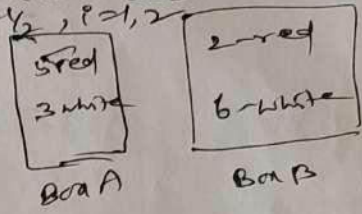
(2) Mutually exclusive events:— This theorem is applied when two events doesn't occur together $A \cap B = \phi$ (disjoint sets)

* Box A contains 5 red and 3 White marbles and box B contains 2 red and 6 white marbles. If a marble is drawn from each box, what is the probability that they are both of same colour.

$\frac{64}{3}$

Sol: → Let E_i denote the event that the box is choosen $i = 1, 2$ $P(E_i) = \frac{1}{2}, i = 1, 2$

Suppose E_1 be the event that the marble is drawn from box A and is red



$$P(E_1) = \frac{1}{2}\left(\frac{5}{8}\right) = \frac{5}{16}$$

and E_2 be the event that the marble is drawn from box B and is red

$$P(E_2) = \frac{1}{2}\left(\frac{2}{8}\right) = \frac{1}{8}$$

The probability that both the marbles are red is

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2) = \frac{5}{16} \cdot \frac{1}{8} = \frac{5}{128}$$ (E_1, E_2 are independent)

Let E_3 = The event that the marble is drawn from box A and is white

$$P(E_3) = \frac{1}{2}\left(\frac{3}{8}\right) = \frac{3}{16}$$

Let E_4 = The event of marble is drawn from box B is white

$$P(E_4) = \frac{1}{2}\left(\frac{6}{8}\right) = \frac{3}{8}$$

and $P(E_3 \cap E_4) = P(E_3) \cdot P(E_4) = \frac{3}{16} \cdot \frac{3}{8} = \frac{9}{128}$

$P(A/E_1) = \frac{1}{2}$ ($\because$ The total no. of coins in box 1 is 2) $\frac{64}{5}$

$P(A/E_2) = \frac{2}{2} = 1$ (There are two gold coins in box 2)

$P(A/E_3) = \frac{0}{2} = 0$ (There is no gold coin in box 3)

(i) The probability that the drawn coin is gold

$P(A) = P(E_1 \cap A) + P(E_2 \cap A) + P(E_3 \cap A)$

$$\boxed{P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + P(E_3) \cdot P(A/E_3)}$$

$\rightarrow$ By ~~Total~~ ~~probability~~ multiplication theorem / Total probability

$= \frac{1}{3}\left(\frac{1}{2}\right) + \frac{1}{3}(1) + \frac{1}{3}(0)$

$= \frac{1}{6} + \frac{1}{3} = \frac{3}{6} = \frac{1}{2}$

(ii) The probability of getting a silver coin

$P(A') = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$ Ans

* In a group consisting of equal number men and women, 10% of the men and 45% of the women are unemployed. If a person is selected randomly from the group then find the probability that the person is an employee.

Sol - We have

$P(\text{men}) = \frac{1}{2}$, $P(\text{women}) = P(W) = \frac{1}{2}$

$P(M)$

Let A be the event of unemployed

$P(A/M) = 10\% = \frac{10}{100} = \frac{1}{10}$

Let E be the Employed event.

$P(E/M) = 1 - \frac{1}{10} = \frac{9}{10} = 0.9$

64/6

$P(A/W) = 45\% = \frac{45}{100} = 4.5$

$P(E/W) = 1 - \cancel{P(A/W)} = 1 - \frac{45}{100} = \frac{55}{100} = 5.5$

$P(E) = P(M \cap E) + P(W \cap E)$ → probability that the person is employed

~~$P(E) = P(M \cap W) = P(M) \cdot P(W/M)$~~

$$\boxed{P(E) = P(M) \cdot P(E/M) + P(W) \cdot P(E/W)}$$ → By Total prob. (M, W are dependent)

$= \frac{1}{2}\left(\frac{9}{10}\right) + \frac{1}{2}\left(\frac{5.5}{10}\right)$

$= \frac{14.5}{20} = 0.725$ Ans

* Three men A, B and C hit a target with the respective probabilities $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ each one of them shoots once at the target.

(a) What is the probability that one of them misses the target?

(b) If only one misses the target, then what is the probability that it is the second man?

Sol:- Given that

$P(A)$ = probability that A can hit a target $= \frac{1}{3}$

$$P(A') = 1 - P(A) = 1 - \frac{1}{3} = 2/3$$

$P(B)$ = probability that B can hit a target $= \frac{1}{4}$

$$P(B') = 1 - P(B) = 1 - \frac{1}{4} = \frac{3}{4}$$

$P(C)$ = probability that C can hit a target $= \frac{1}{5}$

$$P(C') = 1 - P(C) = 1 - \frac{1}{5} = \frac{4}{5}$$

(a) The number of ways by which one of them can miss the target = 3 ways.

They are (i) A misses while B and C hit

(ii) B misses while A and C hit

(iii) C misses while A and B hit

Now the probability that one of them misses the target

$$P(M) = P(A' \cap B \cap C) + P(A \cap B' \cap C) + P(A \cap B \cap C')$$

Since A, B, C, A', B', C' are independent events.

$$P(M) = P(A') \cdot P(B) \cdot P(C) + P(A) P(B') \cdot P(C) + P(A) P(B) P(C')$$

$$= \left(\frac{2}{3} \times \frac{1}{4} \times \frac{1}{5}\right) \times \left(\frac{1}{3} \times \frac{3}{4} \times \frac{1}{5}\right) + \left(\frac{1}{3} \times \frac{1}{4} \times \frac{4}{5}\right)$$

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$$= \frac{3}{20}$$

$$\therefore p(M) = 3/20$$

(ii) If only one misses the target, then the probability that it is the second man.

$$p(B/M) = \frac{p(A \cap B)}{p(M)} = \frac{p(A \cap B \cap C)}{p(M)} = \frac{p(A) \cdot p(B) \cdot p(C)}{p(M)}$$

$$= \frac{(1/3)(3/4)(1/5)}{(3/20)}$$

$$= 1/20 \times \frac{20}{3}$$

$$p(B/M) = \frac{1}{3}$$

* The probabilities that students A, B, C and D solve a problem are 1/3, 2/5, 1/5 and 1/4 respectively. If all of them try to solve the problem, what is the probability that the problem is solved.

Sol: Let $P(A)$ = Probability that A solves the problem

$P(A) = 1/3$

$P(A') = 1 - P(A) = 1 - \frac{1}{3} = 2/3$

$P(B)$ = Probability that B solves the problem.

$= \cancel{2}/5$

$P(B') = 1 - P(B) = 1 - 2/5 = 1/5$

$P(C)$ = Probability that C solve the problem $= 1/5$

$P(C') = 1 - P(C) = 1 - \frac{1}{5} = 4/5$

$P(D)$ = probability that D solve the problem $= 1/4$

$P(D') = 1 - P(D) = 1 - \cancel{\frac{1}{4}} = 3/4$

The probability that the problem is solved is equal to the probability that the problem is being solved by one of the persons.

$P(A \cup B \cup C \cup D) = 1 - P(A \cup B \cup C \cup D)'$

$= 1 - P(A' \cap B' \cap C' \cap D')$

Since all the events are independent

$= 1 - P(A') \cdot P(B') \cdot P(C') \cdot P(D')$

$= 1 - \frac{2}{5} \cdot \frac{8}{15} \cdot \frac{4}{18} \cdot \frac{3}{4}$

$= 1 - \frac{6}{25}$

$P(A \cup B \cup C \cup D) = \frac{19}{25}$

* A bag contains 5 white, 7 red and 4 black balls. If four balls are drawn one by one with replacement, what is the probability that none is white.

Sol:- Let A denote the event of not getting a white ball in ~~first~~ draw.

Let B denote " " " " " " " ball in second draw

Let C denote " " " " " " " ball in third draw

Let D denote " " " " " " " ball in fourth draw.

Since the balls are drawn with replacement therefore A, B, C, D are independent events.

White	Red	Black	Total
5	7	4	16

Since out of 16 balls, 11 are not white

therefore $P(A) = \frac{11_4}{16_4}$, $P(B) = \frac{11_4}{16_4}$, $P(C) = \frac{11_4}{16_4}$, $P(D) = \frac{11_4}{16_4}$

Required probability $= P(A) \cdot P(B) \cdot P(C) \cdot P(D)$ (A, B, C, D are independent)

$P(A \cap B \cap C \cap D)$

$= \frac{11}{16} \cdot \frac{11}{16} \cdot \frac{11}{16} \cdot \frac{11}{16}$

$= \left(\frac{11}{16}\right)^4$ Ans

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* Multiplication theorem for 3 events:-

If A, B and C are any events then show that

$$P(A\cap B\cap C) = P(A)\cdot P(B/A)\cdot P(C/A\cap B)$$

Proof:- $P(A\cap B\cap C) = P[(A\cap B)\cap C]$

$= P(A\cap B)\cdot P(C/A\cap B)$ from conditional probability

$= P(A)\cdot P(B/A)\cdot P(C/A\cap B)$ from conditional probability.

In general, for any events A_1, A_2, A_3 ---- A_n

$P(A_1\cap A_2\cap A_3\cap$ ---- $\cap A_n) =$

$P(A_1)\cdot P(A_2/A_1)\cdot P(A_3/A_1\cap A_2)\cdot P(A_4/A_1\cap A_2\cap A_3)$ ---- $P(A_n/A_1\cap A_2\cap$ ---- $A_{n-1})$

* Two events A and B are said to be independent

if $P(B/A) = P(B)$

$P(A/B) = P(A)$

i.e the Occurence (or) non-Occurence of event A has no influence on the Occurence or non-Occurence of B.

Otherwise, they are said to be dependent.

$$P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{\cancel{P(A)}\cdot P(B)}{\cancel{P(A)}}$$

$$P(B/A) = P(B)$$

Rule of Elimination (OR) Rule of Total probability:-

Statement: If $E_1, E_2, E_3, \text{------} E_n$ are mutually exclusive events (disjoint) of which one must occur, then

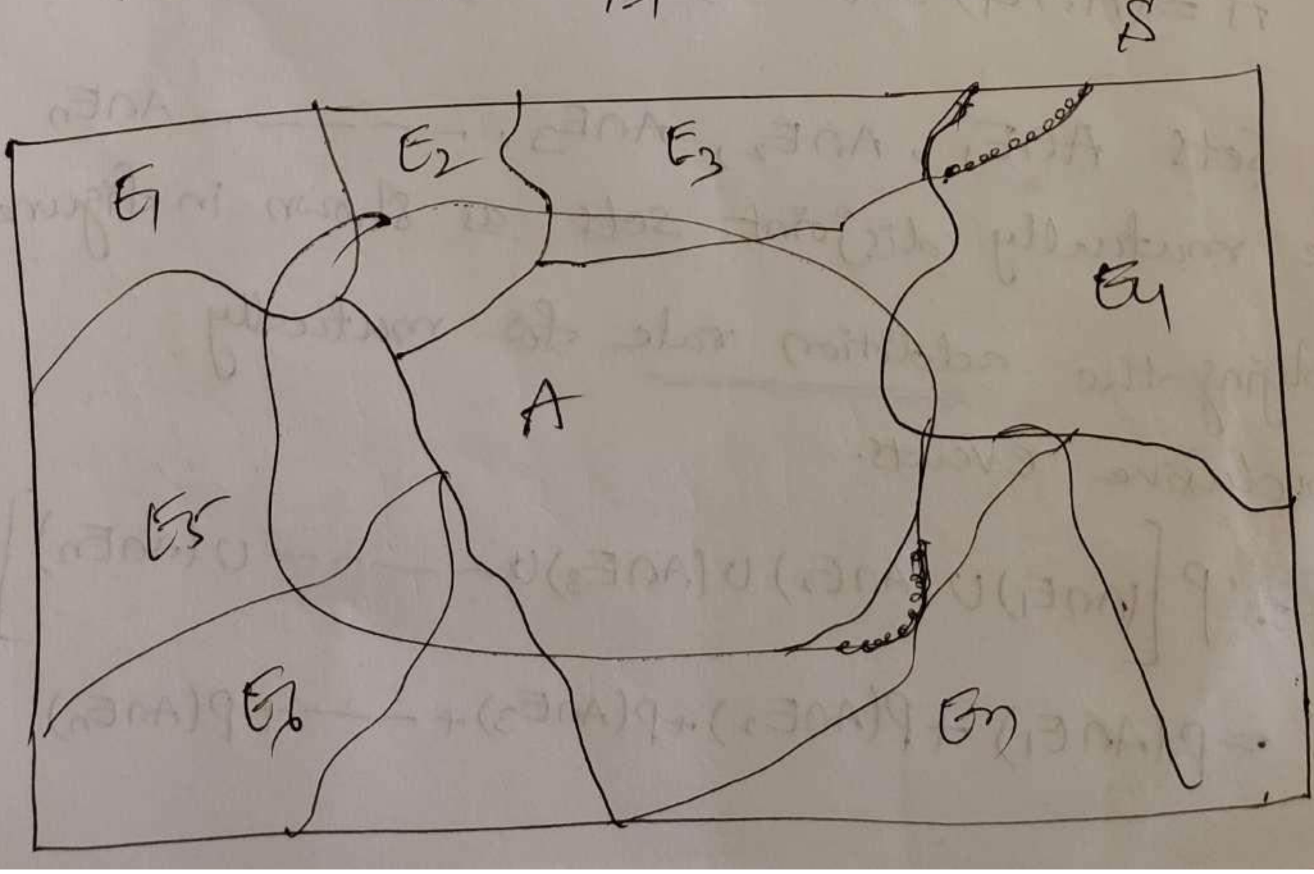
$$p(A) = \Sigma p(E_i)\cdot p(A/E_i).$$

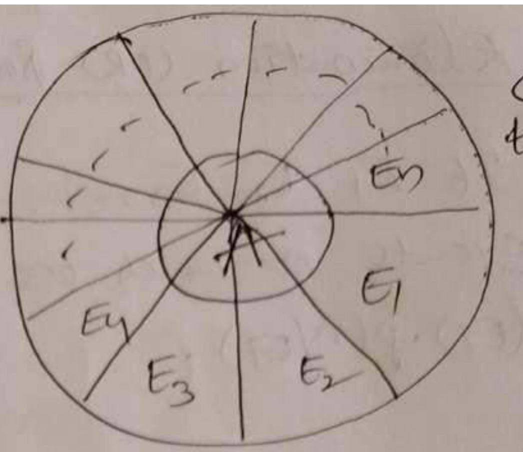
(OR)

Let $E_1, E_2, E_3 \text{--------} E_n$ constitute (3F23,18) a partition of the sample space S with $p(E_i) \neq 0$ for $i = 1, 2, 3, 4 \text{------} n$ then for any event A of S.

$$p(A) = \sum_{i=1}^{n} p(A\cap E_i) = \sum_{i=1}^{n} p(E_i)\cdot p(A/E_i)$$

Proof:- Since $E_1, E_2, E_3 \text{------} E_n$ constitute a partition $S = \bigcup_{i=1}^{n} E_i$





S

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and $E_i \cap E_j = \phi$ for any i and j

i.e their union is S and E_i's are mutually disjoint sets from the figure.

$$A = A \cap S$$

$$A = A \cap \left[\bigcup_{i=1}^{n} E_i\right]$$

$$A = A \cap \left[E_1 \cup E_2 \cup E_3 \cup - - - - - - - \cup E_n\right]$$

$$A = (A \cap E_1) \cup (A \cap E_2) \cup (A \cap E_3) \cup - - - \cup (A \cap E_n)$$

The sets $A \cap E_1$, $A \cap E_2$, $A \cap E_3$, - - - - - - $A \cap E_n$. are mutually disjoint sets as shown in figure

Applying the addition rule for mutually exclusive events.

$$P(A) = P\left[(A \cap E_1) \cup (A \cap E_2) \cup (A \cap E_3) \cup - - - - \cup (A \cap E_n)\right]$$

$$P(A) = P(A \cap E_1) + P(A \cap E_2) + P(A \cap E_3) + - - - + P(A \cap E_n)$$

$$P(A) = \sum_{i=1}^{n} P(A \cap E_i)$$

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Now applying multiplicative rule

$$P(A) = \sum_{i=1}^{n} P(A \cap E_i)$$ (A, Ei are dependent events)

$$P(A) = \sum_{i=1}^{n} P(E_i) \cdot P(A|E_i)$$

Ans

* One bag contains five white and four black balls. Another bag contains seven white and nine black balls. A ball is transferred from the first bag to the second and then a ball is drawn from the second. Find the probability that the ball drawn is white.

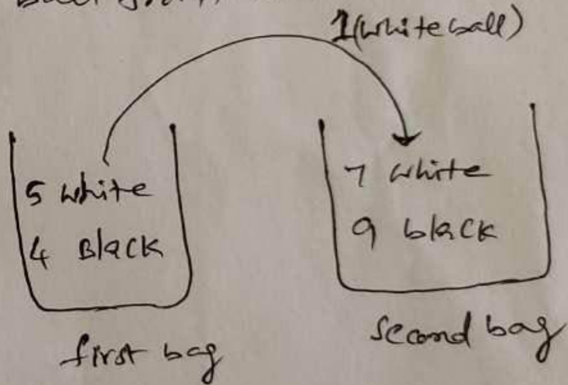
Sol:- The probability of drawing a white ball from box B will depend on whether the transferred ball is black (or) white.

Case I:- When white ball is transferred from the first bag to the second bag.

Its probability $P_1 = \frac{5}{9}$

Now second bag contains 8 white and 9 black balls.

The probability of drawing a white ball from the second bag is $P_2 = \frac{8}{17}$



∴ The probability of both the events occurring together

$$= P_1 P_2 = \frac{5}{9} \times \frac{8}{17} = \frac{40}{153}$$

Case II:- When the black ball is transferred from the first bag to the second bag.

Its probability $P_3 = \frac{4}{9}$

Now second bag contains 7 white and 10 black balls.

The probability of drawing a white ball from the second bag is $P_4 = \frac{7}{17}$

∴ ~~The~~

∴ The probability of both ~~these~~ events occurring together.

$$= P_3 P_4 = \frac{4}{9} \times \frac{7}{17} = \frac{28}{153}$$

∴ The required probability is $= P_1 P_2 + P_3 P_4$

$$= \frac{40}{153} + \frac{28}{153}$$

$$= \frac{68}{153} = \frac{4}{9}$$

Ans

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Multiplication theorem of probability:- (or) theorem of compound probability:-

Statement:- In a random experiment if E_1, E_2 are two events such that $p(E_1) \neq 0$ and $p(E_2) \neq 0$

then $p(E_1 \cap E_2) = p(E_1) \cdot p(E_2/E_1)$

$p(E_2 \cap E_1) = p(E_2) \cdot p(E_1/E_2)$

Proof:- Let S be the sample space associated with the random experiment.

Let E_1, E_2 be two events of S such that $p(E_1) \neq 0$, $p(E_2) \neq 0$.

Since $p(E_1) \neq 0$ by the definition of conditional probability of E_2 given E_1

$$p\left(\frac{E_2}{E_1}\right) = \frac{p(E_1 \cap E_2)}{p(E_1)}$$

$$p(E_1) \cdot p(E_2/E_1) = p(E_1 \cap E_2)$$

$$\boxed{\therefore p(E_1 \cap E_2) = p(E_1) \cdot p(E_2/E_1)}$$

Since $p(E_2) \neq 0$ by the definition of conditional probability of E_1 given E_2.

$$p(E_1/E_2) = \frac{p(E_1 \cap E_2)}{p(E_2)}$$

$$p(E_2) \cdot p(E_1/E_2) = p(E_1 \cap E_2)$$

$$\boxed{\therefore p(E_1 \cap E_2) = p(E_2) \cdot p(E_1/E_2)}$$

Eg: ① If $p(A\cap B) = \frac{1}{6}$, $p(A) = \frac{1}{2}$ then

$$P(B/A) = \frac{p(A\cap B)}{p(A)} = \frac{(1/6)}{(1/2)} = 1/3$$

② If A, B are two events ($\neq \phi$) then

$$P(\bar{A}\cup\bar{B}) = P(A'\cup B') = P(A\cap B)' = 1 - P(A\cap B)$$

$$P(\bar{A}\cup\bar{B}) = 1 - P(A\cap B)$$

$$= 1 - P(A)\cdot P(B/A)$$

$\because (A\cap B)' = A'\cup B'$

$(A\cup B)' = A'\cap B'$

Note:- Multiplication theorem can be extended three events E_1, E_2, E_3 as

$$P(E_1\cap E_2\cap E_3) = P((E_1\cap E_2)\cap E_3)$$

$$= P(E_1\cap E_2)\cdot P(E_3/E_1\cap E_2)$$

$$= P(E_1)\cdot P(E_2/E_1)\cdot P(E_3/E_1\cap E_2)$$

Pairwise independent events:- Three events E_1, E_2, E_3 are mutually independent (or) simply independent if

(i) $P(E_1\cap E_2) = P(E_1)\cdot P(E_2)$

$P(E_2\cap E_3) = P(E_2)\cdot P(E_3)$

$P(E_3\cap E_1) = P(E_3)\cdot P(E_1)$

(ii) $P(E_1\cap E_2\cap E_3) = P(E_1)\cdot P(E_2)\cdot P(E_3)$

Note:- Probability of occurence of atleast one of the independent events E_1, E_2, E_3 $= P(E_1\cup E_2\cup E_3)$

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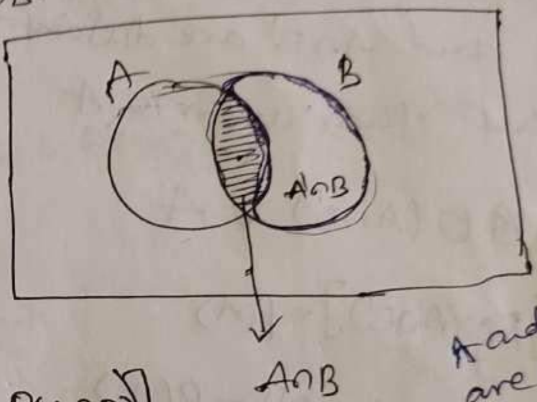
* Addition theorem of probabilities (or) Theorem of Total probability :—

Statement :— If A and B are any two events, then

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

Proof :— A and $A'\cap B$ are disjoint sets and their union is $A\cup B$.



$A\cup B = A\cup(A'\cap B)$

$P(A\cup B) = P[A\cup(A'\cap B)]$

$= P(A) + P(A'\cap B)$ (A and A'∩B are disjoint)

$= P(A) + [P(A'\cap B) + (P(A\cap B) - P(A\cap B))]$

$= P(A) + \{P[(A'\cap B)\cup(A\cap B)] - P(A\cap B)\}$

$= P(A) + \{P(B) - P(A\cap B)\}$ ($\because A'\cap B$ and $A\cap B$ are disjoint)

$= P(A) + P(B) - P(A\cap B)$

$$\boxed{\therefore P(A\cup B) = P(A) + P(B) - P(A\cap B)}$$

Note: ① If A and B are two mutually disjoint events then $A\cap B = \phi$ so that $P(A\cap B) = P(\phi) = 0$

$$\boxed{P(A\cup B) = P(A) + P(B)}$$

② $P(A \cup B)$ is also written as $P(A+B)$; Thus for mutually disjoint events A and B

$$P(A+B) = P(A) + P(B)$$

$P(A \cap B)$ is also written as $P(AB)$

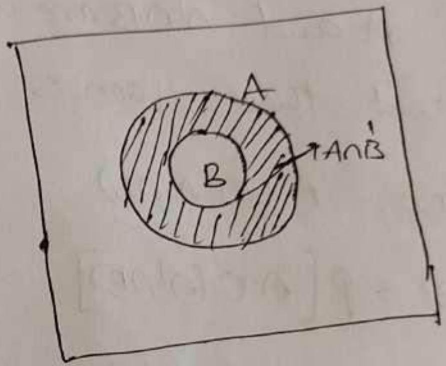
* If $B \subset A$ then

(i) $P(A \cap B') = P(A) - P(B)$ (ii) $P(B) \le P(A)$

So

Proof:- (i) When $B \subset A$,

B and $A \cap B'$ are disjoint and their union is A



$$B \cup (A \cap B') = A$$

$$P[B \cup (A \cap B')] = P(A)$$

$$P(B) + P(A \cap B') = P(A)$$

$$P(A \cap B') = P(A) - P(B)$$

(ii) Now if E is any event, then $0 \le P(E) \le 1$

i.e $P(E) \ge 0$

$\therefore P(A \cap B') \ge 0$

$P(A) - P(B) \ge 0$

$P(A) \ge P(B)$

$\therefore \underline{P(B) \le P(A)}$ Ans

Note:- ① If A be any event in a sample space then $0 \le P(A) \le 1$

② To the entire sample space $P(S) = 1$.

* Two aeroplanes bomb a target in succession. The probability of each correctly scoring a hit is 0.3 and 0.2 respectively. The second will bomb only if the first misses the target. Find the probability that (i) target is hit (ii) both fails to score hits.

Sol:- Let A be the event of first plane hitting the target and B be the event of 2nd plane hitting the target.

The probability of 1st plane hitting the target

$= P(A) = 0.3$

The probability of 2nd plane hitting the target

$= P(B) = 0.2$

$P(A') = 1 - P(A) = 1 - 0.3 = 0.7$

$P(B') = 1 - P(B) = 1 - 0.2 = 0.8$

(i) $P(\text{target is hit}) = P[(A \text{ hits}) \text{ or } (A \text{ fails and } B \text{ hits})]$

$= P[A \cup (A' \cap B)]$ (A, B are independent)

$= P(A) + P(A' \cap B)$

$= P(A) + P(A') \cdot P(B)$ (By multiplication theorem)

$= 0.3 + (0.7)(0.2)$

$= \underline{0.44}$

(ii) $P(\text{both fails}) = P(A \text{ fails and } B \text{ fails})$ (A', B' are independent)

$= P(A' \cap B') = P(A') \cdot P(B')$

$= P(A') \cdot P(B')$

$= (0.7)(0.8)$

$= \underline{0.56}$

~~(OR)~~

$$\cancel{= P[(A\cup B)'] = 1 - P(A\cup B) = 1 - [P(A)+P(B)] = 1 - [0.3+0.2] = 1 - [0.5] = 0.5}$$

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* A, B, C are aiming to shoot a balloon. A will succeed 4 times out of 5 attempts. The chance of B to shoot the balloon is 3 out of 4 and that of C is 2 out of 3. If the three aim the balloon simultaneously, then find the probability that at least two of them hit the balloon.

Sol:- The probability of A hitting the target $= P(A) = \frac{4}{5}$

The " " B " " " $= P(B) = \frac{3}{4}$

The " " C " " " $= P(C) = \frac{2}{3}$

The probabilities of A, B, C not hitting the target respectively are $P(\bar{A}) = 1 - P(A) = 1 - \frac{4}{5} = \frac{1}{5}$

$P(\bar{B}) = 1 - P(B) = 1 - \frac{3}{4} = \frac{1}{4}$

$P(\bar{C}) = 1 - P(C) = 1 - \frac{2}{3} = \frac{1}{3}$

Now the probability that <u>exactly two</u> will hit the balloon $= P(A\cap B\cap C')$ or $P(A\cap B'\cap C)$ or $P(A'\cap B\cap C)$

$= P(A)\cdot P(B)\cdot P(C') + P(A)\cdot P(B')\cdot P(C) + P(A')\cdot P(B)\cdot P(C)$

$= \frac{4}{5}\cdot\frac{3}{4}\cdot\frac{1}{3} + \frac{4}{5}\cdot\frac{1}{4}\cdot\frac{2}{3} + \frac{1}{5}\cdot\frac{3}{4}\cdot\frac{2}{3}$

$= \frac{12+8+6}{60} = \frac{26}{60} = \frac{13}{30}$

The probability that all will hit the balloon

$= P(A\cap B\cap C) = P(A)\cdot P(B)\cdot P(C)$

$= \frac{4}{5}\cdot\frac{3}{4}\cdot\frac{2}{3}$

$= \frac{2}{5}$

∴ The probability that at least two of them will hit the target $= \frac{13}{30} + \frac{2}{5}$

$= \frac{13+12}{30} = \frac{25}{30} = \frac{5}{6}$ Ans

* A class has 12 boys and 8 girls. Three students are selected at random one after another. Find the probability that (i) first two are boys and third is girl. (ii) First and third are of the same sex and the second is of opposite sex.

Sol:- Total number of students in the class $= 12+8 = 20$

Number of ways of selecting 3 students one after another $= n(S) = {}^{20}C_1 \times {}^{19}C_1 \times {}^{18}C_1 =$

(i) E = Event of first two are boys and third is girl.

Then $n(E) = {}^{12}C_1 \times {}^{11}C_1 \times {}^{8}C_1$

$$P(E) = \frac{n(E)}{n(S)} = \frac{12 \times 11 \times 8}{20 \times 19 \times 18} = \frac{44}{285}$$

$\frac{6\frac{1}{2}}{2\frac{1}{2}}$

(ii) E = Event of first and third are of same sex and Second is of Opposite Sex.

This can be that first and third are boys and Second is a girl

(or) First and third are girls and second is boy

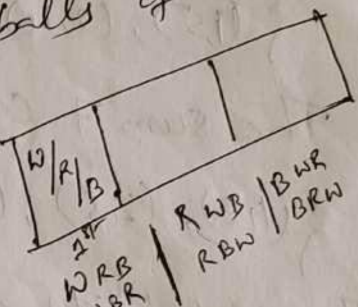
$$n(E) = {}^{12}C_1 \times {}^{8}C_1 \times {}^{11}C_1 + {}^{8}C_1 \times {}^{12}C_1 \times {}^{7}C_1$$

$$P(E) = \frac{nE}{n(S)} = \frac{12 \times 8 \times 11 + 8 \times 12 \times 7}{20 \times 19 \times 18} =$$

1) An urn contains five white, three Red and two black balls. If three balls are drawn in succession, what is the probability that they will be of different colours?

Sol:- Let W, R, B be the events of drawing white, Red and Black balls respectively.

The possible ways of getting the 3 balls of different colours are,



WRB, WBR, BRW, RWB, BWR, RBW

Now the desired probability

P = P(WRB or WBR or BRW or RWB or BWR or RBW)

As the six events (cases) are mutually exclusive P is given by

P = P(WRB) + P(WBR) + P(BRW) + P(RWB) + P(BWR) + P(RBW)

W, R, B are ~~mutually exclusive events~~ independent

$$= \left(\frac{5}{10}\times\frac{3}{9}\times\frac{2}{8}\right)+\left(\frac{5}{10}\times\frac{2}{9}\times\frac{3}{8}\right)+\left(\frac{3}{10}\times\frac{5}{9}\times\frac{2}{8}\right)+\left(\frac{3}{10}\times\frac{2}{9}\times\frac{5}{8}\right)$$

$$+\left(\frac{2}{10}\times\frac{5}{9}\times\frac{3}{8}\right)+\left(\frac{2}{10}\times\frac{3}{9}\times\frac{5}{8}\right)$$

$\frac{1}{4}$ Ans

* If A and B throw alternatively with a pair of ordinary dice. A win if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins find his chances of winning.

Sol:- The sum 6 can be obtained as follows

$\{(1,5),(2,4),(3,3),(4,2),(5,1)\}$

The sum 7 can be obtained as follows

$\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

The probability of A's throwing 6 with 2 dice

$$P(A)=\frac{5}{36}$$

The probability of A's not throwing 6 is

$$P(A')=1-P(A)=1-\frac{5}{36}=\frac{31}{36}$$

The probability of B's throwing 7 is $=\frac{6}{36}=\frac{1}{6}$

The probability of B's not throwing 7 is

$$P(B')=1-P(B)=1-\frac{1}{6}=\frac{5}{6}$$

Now A can win if he throws 6 in the first, third, fifth, seventh, — — — — — — throws

i.e (A wins), (A loses B loses A wins), (A loses B loses A loses B loses A wins), — — — — — — — — —

$= A$ (or) $(A'B'A)$ (or) $(A'B'A'B'A)$ (or) $(A'B'A'B'A'B'A)$ — — — —

$= P(A) + P(A'\cap B'\cap A) + P(A'\cap B'\cap A'\cap B'\cap A) + P(A'\cap B'\cap A'\cap B'\cap A'\cap B'\cap A) + \cdots$

$= P(A) + P(A'B'A) + \cdots$

Since A, B, A', B' are independent

$$= \frac{5}{36} + \left[\frac{31}{36} \times \frac{5}{6} \times \frac{5}{36}\right] + \left[\frac{31}{36} \times \frac{5}{6} \times \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36}\right] + \text{-----}$$

$$= \frac{5}{36}\left[1 + \left(\frac{31}{36} \times \frac{5}{6}\right) + \left(\frac{31}{36} \times \frac{5}{6} \times \frac{31}{36} \times \frac{5}{6}\right) + \text{-----}\right]$$

$$= \frac{5}{36}\left[1 + \left(\frac{31}{36} \times \frac{5}{6}\right) + \left(\frac{31}{36} \times \frac{5}{6}\right)^2 + \left(\frac{31}{36} \times \frac{5}{6}\right)^3 + \text{-----}\right]$$

= The series is in G.P

$$S_\infty = \frac{a}{1-r} \quad \text{if } r<1$$

where $a = 1$, $r = \left(\frac{31}{36} \times \frac{5}{6}\right) < 1$
$= 0.717 < 1$

$$= \frac{5}{36}\left[\frac{1}{1 - \frac{31}{36} \times \frac{5}{6}}\right]$$

$$= \frac{30}{61} \text{ Ans}$$

* "A" can hit a target once in five shots. B can hit two targets in 3 shots. C can hit one target in 4 shots. what is the probability that 2 shots hit the target.

Sol:- Let P(A) be the probability that A hit the target

$= \frac{1}{5}$

Let $P(A') = 1 - P(A) = 1 - \frac{1}{5} = \frac{4}{5}$

Let P(B) be the probability that B hit the target

$P(B) = \frac{2}{3} \Rightarrow P(B') = 1 - \frac{2}{3} = \frac{1}{3}$

Let $P(C)$ be the probability that C hit the target

$$P(C) = \frac{1}{4} \Rightarrow P(C') = 1 - \frac{1}{4} = \frac{3}{4}$$

Probability that two shots hit the target

$$= P\left[(A\cap B\cap C')\cup(A\cap B'\cap C)\cup(A'\cap B\cap C)\right]$$

These two are independent and mutually exclusive also

$$= P\left[A\cap B\cap C'\right] + P\left[A\cap B'\cap C\right] + P\left[A'\cap B\cap C\right]$$

$$= P(A)\cdot P(B)\cdot P(C') + P(A)\cdot P(B')\cdot P(C) + P(A')\cdot P(B)\cdot P(C)$$

$$= \frac{1}{5}\cdot\frac{2}{3}\cdot\frac{3}{4} + \frac{1}{5}\cdot\frac{1}{3}\cdot\frac{1}{4} + \frac{4}{5}\cdot\frac{2}{3}\cdot\frac{1}{4}$$

$$= \frac{15}{60}$$

$$= \frac{1}{4} \text{ Ans}$$

* A, B and C in order toss a coin. The first one to toss a head wins the game. What are their probability of winning? The game finishes if one of them wins the game'

Sol:- probability that A throws a head $P(A) = \frac{1}{2}$

probability that A doesn't throws a head $P(\bar{A}) = \frac{1}{2}$

Probability that B throws a head $P(B) = \frac{1}{2}$

Probability that B doesn't throws a head $P(\bar{B}) = \frac{1}{2}$

Probability that C throws a head $P(C) = \frac{1}{2}$

probability that C doesn't throws a head $P(\bar{C}) = \frac{1}{2}$

A can win the game if he toss a head in the $1^{st}, 4^{th}, 7^{th}$ ______ trial.

i.e (A) or $(A'B'C'A)$ or $(A'B'C'A'B'C'A)$ ______

$$P = P\left[(A) \text{ or } (A' \cap B' \cap C' \cap A) \text{ or } (A' \cap B' \cap C' \cap A' \cap B' \cap C' \cap A) + \text{\_\_\_\_}\right]$$

$$P = P(A) + P(A' \cap B' \cap C' \cap A) + P(A' \cap B' \cap C' \cap A' \cap B' \cap C' \cap A) + \text{\_\_\_\_}$$

$$P = P(A) + P(A') \cdot P(B') \cdot P(C') \cdot P(A) + P(A') \cdot P(B') \cdot P(C') \cdot P(A') \cdot P(B') \cdot P(C') \cdot P(A) + \text{\_\_\_\_}$$

Since the events are independent

$$P = \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$$

$$P = \frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \text{\_\_\_\_}$$

$$P = \frac{1}{2}\left[1 + \frac{1}{2^3} + \frac{1}{2^6} + \text{\_\_\_\_}\right]$$

$$P = \frac{1}{2}\left[1 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2^3}\right)^2 + \text{\_\_\_\_}\right]$$ it is in G.P

where $a = 1$, $r = \left(\frac{1}{2^3}\right) = \frac{1}{8} = 0.125 < 1$

$$S_\infty = \frac{a}{1-r}, \quad r < 1$$

$$P = \frac{1}{2}\left[\frac{1}{1-\frac{1}{8}}\right] = \frac{4}{7}$$

$\therefore P(A \text{ wins the game}) = \frac{4}{7}$

Similarly B can win the game if he throws a head in 2nd, 5th, 8th ———— trial.

$A'B$ or $A'B'C'A'B$ or $A'B'C'A'B'C'AB$ ————

$$P = P\left[(A'\cap B) \text{ or } (A'\cap B'\cap C'\cap A'\cap B) \text{ or } (A'\cap B'\cap C'\cap A'\cap B'\cap C'\cap A\cap B) \text{ or } \text{———}\right]$$

$$P = P(A'\cap B) + P(A'\cap B'\cap C'\cap A'\cap B) + P(A'\cap B'\cap C'\cap A'\cap B'\cap C'\cap A'\cap B) + \text{———}$$

since the events are independent

$$P = P(A)\cdot P(B) + P(A')\cdot P(B')\cdot P(C')\cdot P(A')\cdot P(B) + P(A')P(B')P(C')P(A)P(B')P(C')P(A')P(B) + \text{———}$$

$$P = \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \text{———}$$

$$P = \frac{1}{2^2} + \frac{1}{2^5} + \frac{1}{2^8} + \text{———}$$

$$P = \frac{1}{2^2}\left[1 + \frac{1}{2^3} + \frac{1}{2^6} + \text{———}\right]$$

$$P = \frac{1}{4}\left[1 + \left(\frac{1}{2^3}\right) + \left(\frac{1}{2^3}\right)^2 + \text{———}\right]$$ it is in G.P

where $a = 1$, $r = \frac{1}{2^3} = \frac{1}{8} < 1$, $S_\infty = \frac{a}{1-r}$, $r<1$

$$P = \frac{1}{4}\left[\frac{1}{1-\frac{1}{8}}\right] = \frac{2}{7}$$

$\therefore P(B \text{ wins the game}) = \frac{2}{7}$

Now p (A wins the game) + P(B wins the game)

+ P(C wins the game) = 1

P(C wins the game) = 1 − [P(A wins the game) + P(B wins the game)]

$$= 1 - \left[\frac{4}{7} + \frac{2}{7}\right]$$

$$= 1 - \frac{6}{7}$$

P(C wins the game) $= \frac{1}{7}$ Ans

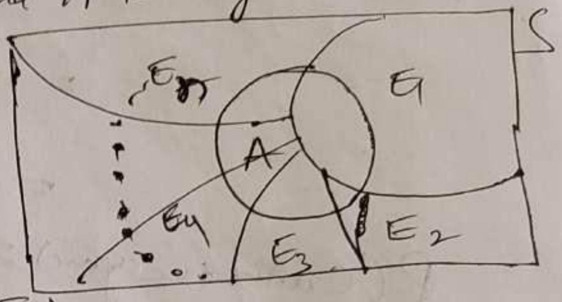
BAYE's Theorem :-

Statement:- E_1, E_2, E_3 ---- E_n are n mutually exclusive and exhaustive events such that $P(E_i) > 0$ $(i = 1, 2, 3, ---- n)$ in a sample space S and A is any other event in S intersecting with every E_i (i.e A can only occur in combination with any one of the events E_1, E_2, E_3, ---- E_n) such that $P(A) > 0$.

If E_i is any of the events of E_1, E_2, E_3 ---- E_n where $P(E_1), P(E_2), P(E_3)$, ------, $P(E_n)$ and $P(A/E_1), P(A/E_2), P(A/E_3)$ ------, $P(A/E_n)$ are known, then

$$P(E_i/A) = \frac{P(E_i)\cdot P(A/E_i)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) + \cdots + P(E_n)\cdot P(A/E_n)}$$

Proof:- $E_1, E_2, E_3 ---- E_n$ are n events of S such that $P(E_i) > 0$ and $E_i \cap E_j = \phi$ for $i \neq j$ where $i, j = 1, 2, 3, ---- n$. Also $E_1, E_2, E_3 ---- E_n$ are enhaustive events of S and A is any other event of S where $P(A) > 0$.



$S = E_1 \cup E_2 \cup E_3 \cup -- \cup E_n = \sum_{i=1}^{n} E_i$

and $A = A \cap S$

$A = A \cap (E_1 \cup E_2 \cup E_3 \cup ---- \cup E_n)$

$A = (A \cap E_1) \cup (A \cap E_2) \cup (A \cap E_3) \cup -- \cup (A \cap E_n)$

Here, $A \cap E_1, A \cap E_2, A \cap E_3 ---- A \cap E_n$ are mutually exclusive events. Then

$$P(E_i/A) = \frac{P(E_i \cap A)}{P(A)}$$

$$= \frac{P(E_i \cap A)}{P[(A \cap E_1) \cup (A \cap E_2) \cup (A \cap E_3) \cup ---- \cup (A \cap E_n)]}$$

$$= \frac{P(E_i \cap A)}{P(E_1 \cap A) + P(E_2 \cap A) + P(E_3 \cap A) + ---- + P(E_n \cap A)}$$

$$P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + ---- + P(E_n) \cdot P(A/E_n)}$$

Note:- Baye's theorem is also known as formula for the probability of "causes" i.e probability of a particular (cause) E_i, given that event A has happened (already).

$P(E_i)$ is "a priori probability" known even before the experiment,

$P(A/E_i)$, $\forall\, i = 1, 2, 3 - - n$ are called Likelihoods and

$P(E_i/A)$, $\forall\, i = 1, 2, 3 - - n$ are called posterior Probabilities determined after the result of the experiment.

~~* Two dice are thrown. Let A be the event that the sum of the points on the faces is 9. Let B be the event that at least one number is 6. Find~~

~~(i) $P(A\cap B)$ (ii) $P(A\cup B)$ (iii) $P(A^c\cup B^c)$~~

~~Sol:- There are 36 sample outcomes when two dice are thrown.~~

~~The event A (= that a sum 9)~~

* A businessman goes to hotels X, Y, Z, 20%, 50%, 30% of the time respectively. It is known that 5%, 4%, 8% of the rooms in X, Y, Z hotels have faulty plumbings. What is the probability that business man's room having faulty plumbing is assigned to hotel Z?

<u>Solution:</u>- Let the probabilities of business man going to hotels X, Y, Z be respectively. $P(X)$, $P(Y)$, $P(Z)$ then

$P(X) = \frac{20}{100} = \frac{2}{10}$

$P(Y) = \frac{50}{100} = \frac{5}{10}$

$P(Z) = \frac{30}{100} = \frac{3}{10}$

Let E be the event that the hotel room has faulty plumbing. Then the probabilities that hotels X, Y, Z have faulty plumbing are

$P(E/X) = \frac{5}{100} = \frac{1}{20}$, $P(E/Y) = \frac{4}{100} = \frac{1}{25}$, $P(E/Z) = \frac{8}{100} = \frac{2}{25}$

The probability that the business man's room having faulty plumbing is assigned to Hotel Z

$$= P(Z/E) = \frac{P(Z)\cdot P(E/Z)}{P(X)\cdot P(E/X) + P(Y)\cdot P(E/Y) + P(Z)\cdot P(E/Z)}$$

$$= \frac{\frac{3}{10}\times\frac{2}{25}}{\frac{2}{10}\cdot\frac{1}{20} + \frac{5}{10}\cdot\frac{1}{25} + \frac{3}{10}\cdot\frac{2}{25}} = \frac{4}{9}$$ Ans

* The probabilities that students A, B, C, D solve a problem are $\frac{1}{3}$, $\frac{2}{5}$, $\frac{1}{5}$ and $\frac{1}{4}$ respectively. If all of them try to solve the problem, what is the probability that the problem is solved.

HW

Sol:- Given the probability of A, B, C, D solving the problem is $P(A)=\frac{1}{3}$, $P(B)=\frac{2}{5}$, $P(C)=\frac{1}{5}$, $P(D)=\frac{1}{4}$

The probability that the problem is not solved by A, B, C, D are $P(A')=\frac{2}{3}$, $P(B')=\frac{3}{5}$, $P(C')=\frac{4}{5}$, $P(D')=\frac{3}{4}$

The probability that the problem is not solved when A, B, C, D try together (independently)

$$= P(A'\cap B'\cap C'\cap D')$$

$$= P(A')\cdot P(B')\cdot P(C')\cdot P(D')$$

$$= \frac{2}{3}\cdot\frac{3}{5}\cdot\frac{4}{5}\cdot\frac{3}{4}$$

$$= \frac{6}{25}$$

$\therefore$ The probability that the problem is solved $= 1-\frac{6}{25}$

$$= \frac{19}{25} \text{ Ans}$$

**Box A contains 5 red and 3 white marbles and box B contains 2 red and 6 white marbles. If a marble is drawn from each box, what is the probability that they are both of same colour.

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Sol:- Let A and B denote the events of selecting box A & B respectively. then $P(A)=1/2$, $P(B)=1/2$

Suppose E_1 = The event that the marble is drawn from box A and is red.

$$P(E_1)=\frac{1}{2}\left(\frac{5}{8}\right)=\frac{5}{16}$$

and E_2 = The event that the marble is drawn from box B and is red.

$$P(E_2)=\frac{1}{\cancel{2}}\cdot\left(\frac{\cancel{2}}{8}\right)=\frac{1}{8}$$

The probability that both the marbles are red is

(E_1, E_2 are independent)

$$P(E_1\cap E_2)=P(E_1)\cdot P(E_2)$$

$$=\frac{5}{16}\cdot\frac{1}{8}=\frac{5}{128}$$

Let E_3 = The event that the marble is drawn from box A and is white

$$P(E_3)=\frac{1}{2}\cdot\left(\frac{3}{8}\right)=\frac{3}{16}$$

Let E_4 = The event that the marble is drawn from box B is white

$$P(E_4)=\frac{1}{\cancel{2}}\cdot\left(\frac{\cancel{6}^{3}}{8}\right)=\frac{3}{8}$$

and $P(E_3 \cap E_4) = \frac{3}{16} \cdot \frac{3}{8} = \frac{9}{128}$

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The Probability that the marbles are of same colour

$= P(E_1 \cap E_2) \text{ or } P(E_3 \cap E_4)$

$= P(E_1 \cap E_2) + P(E_3 \cap E_4)$

$= \frac{5}{128} + \frac{9}{128} = \frac{7}{64} = 0.109$

Two dice are thrown. Let A be the event that the sum of the points on the faces is 9. Let B be the event that at least one number is 6. Find

(i) $P(A \cap B)$ (ii) $P(A \cup B)$ (iii) $P(A^C \cup B^C)$

Sol:- There are 36 simple outcomes when two dice are thrown. i.e $n(S) = 36$

The event A that a sum 9 occurs in the following way:

$A = \{(3,6), (4,5), (5,4), (6,3)\}$, $n(A) = 4$

$P(A) = \frac{4}{36}$

The event B that at least one number is 6 occurs in the following way:

$B = \{(1,6), (2,6), (3,6), (4,6), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\} = 11$

$P(B) = \frac{11}{36}$

$A \cap B = \{(3,6), (6,3)\}$ (i) $P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

(ii) $P(A\cup B) = P(A) + P(B) - P(A\cap B)$ 78

$$= \frac{4}{36} + \frac{11}{36} - \frac{2}{36} = \frac{13}{36}$$

(iii) $P(A^c \cup B^c) = P\left[(A\cap B)^c\right] = 1 - P(A\cap B)$

$$= 1 - \frac{1}{18} = \frac{17}{18}$$ Ans

8 A problem in Statistics is given to the 3 students A, B, C whose chances of solving it are $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that the problem is solved?

Sol:- Given $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{4}$, $P(C) = \frac{1}{4}$

$P(A') = 1 - P(A) = \frac{1}{2}$, $P(B') = \frac{1}{4}$, $P(C') = \frac{3}{4}$

The probability that the problem is not solved is given by $P(A'\cap B'\cap C') = P(A')\cdot P(B')\cdot P(C')$

$$= \frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4} = \frac{3}{32}$$

The probability that the problem is solved

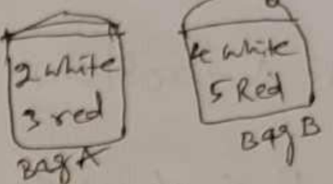
$$= P(A\cup B\cup C)$$

$$= 1 - P(A'\cap B'\cap C')$$

$$= 1 - \frac{3}{32} = \frac{29}{32} = 0.90625$$

*** A bag A contains 2 white and 3 red balls and a bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that the red ball drawn is from bag B.

Sol:- Let A and B denote the events of selecting bag A and bag B respectively.



Then $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$

Let R denote the event of drawing a red ball.

Having selected bag A, the probability to draw a red ball from A $= P\left(\frac{R}{A}\right) = \frac{3}{5} = \frac{3}{5}$

Similarly $P(R/B) = \frac{5}{9} = \frac{5}{9}$

one of the bags is selected at random and from it a ball is drawn at random.

It is found to be red. Then the probability that the selected bag is B.

$$P(B/R) = \frac{P(B)\cdot P(R/B)}{P(A)\cdot P(R/A) + P(B)\cdot P(R/B)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{5}{9}\right)}{\frac{1}{2}\cdot\frac{3}{5} + \frac{1}{2}\cdot\frac{5}{9}} = \frac{25}{52}$$

Ans

*** Suppose 5 men out of 100 and 25 women out of 10,000 are colour blind. A colour blind person is choosen at random. what is the probability of the person being a male (Assume male and female to be in equal numbers)? (80)

Sol:- Given that 5 men out of 100 and 25 women out of 10,000 are colour blind

A colour blind person is choosen at random.

The probability that the choosen person is male

$= P(M) = \frac{1}{2}$

Similarly the probability that the choosen person is female

$= P(W) = \frac{1}{2}$.

Let B represent a blind person. Then

$$P(B/M) = \frac{5}{100} = 0.05$$

$$P(B/W) = \frac{25}{10,000} = 0.0025$$

The probability that the choosen person is male is given by

$$P(M/B) = \frac{P(M)\cdot P(B/M)}{P(M)\cdot P(B/M) + P(W)\cdot P(B/W)}$$

$$= \frac{0.05 \times 0.5}{(0.05 \times 0.5) + (0.5 \times 0.0025)}$$

$$= \underline{0.95} \text{ Ans}$$

** In a bolt factory machines A, B, C manufacture 20%, 30%, 50% of the total of their output and 6%, 3% and 2% are defective. A bolt is drawn at random and found to be defective. Find the probabilities that it is manufactured from (i) Machine A (8)
(ii) Machine B (iii) Machine C.

Sol:- Let $P(A), P(B), P(C)$ be the probabilities of the ~~probabilities of the~~ events that the bolts are manufactured by the machines A, B, C respectively.

Then $P(A) = \frac{20}{100} = \frac{1}{5}$, $P(B) = \frac{30}{100} = \frac{3}{10}$, $P(C) = \frac{50}{100} = \frac{1}{2}$

Let D denote that the bolt is defective. Then

$$P(D/A) = \frac{6}{100}, \quad P(D/B) = \frac{3}{100}, \quad P(D/C) = \frac{2}{100}$$

(i) If bolt is defective, then the probability that it is from machine $A = P(A/D)$

$$P(A/D) = \frac{P(D/A)\cdot P(A)}{P(D/A)\cdot P(A) + P(D/B)\cdot P(B) + P(D/C)\cdot P(C)} = \frac{12}{31}$$

(ii) Similarly, $P(B/D) = \frac{9}{31}$

(iii) $P(C/D) = 10/31$

*** A bag contains 5 Red 7 Black balls and a second bag contains 4 blue and 3 Green balls. A ball is taken out from each bag. Find the probability that (82)

(i) one ball is Red and the other is Blue.

(ii) one ball is Black and the other Green.

Sol: (i) A = The event of getting Red ball from the 1^{st} bag.

B = The event of getting blue ball from the 2^{nd} bag.

Then $n(A) = {}^5C_1 = 5$

$n(B) = {}^4C_1 = 4$

and for A, $n(S) = 5+7 = 12$

for B, $n(S) = 4+3 = 7$

$P(A) = \frac{n(A)}{n(S)} = \frac{5}{12}$

$P(B) = \frac{n(B)}{n(S)} = \frac{4}{7}$

Since A and B are independent.

$P(A \cap B) = P(A) \cdot P(B)$

$= \frac{5}{\cancel{12}_3} \cdot \frac{\cancel{4}}{7} = \frac{5}{21}$

(ii) C = The event of getting Black ball from 1^{st} bag

D = The event of getting Green ball from 2^{nd} bag.

Since C and D are independent events

$P(C \cap D) = P(C) \cdot P(D) = \frac{\cancel{7}}{\cancel{12}_4} \cdot \frac{\cancel{3}}{\cancel{7}} = \frac{1}{4}$

*** The contents of urns I, II and III are as follows (8)
1 white, 2 Black and 3 Red balls; 2 white, 1 black and 1 Red balls; 4 White, 5 blacks and 3 Red balls one urn is choosen at random and two balls are drawn. They happen to be white and Red, what is the probability that they come from urns I, II & III?

Sol:- Let E_1, E_2 and E_3 denote the event that urn I, II, III are selected respectively. and let A be the event that the two balls taken from the selected urn are white and Red. Then

I	II	III
1W, 2B, 3R	2w, 1b, 1R	4w, 5b, 3R

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}.$$

$$P(A/E_1) = \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2} = \frac{1}{5}$$ (since there are 1 white, 2 black 3 Red balls in a total of 6 balls in urn I)

$$P(A/E_2) = \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2} = \frac{1}{3}$$ (since there are 2 white, 1 black, 1 Red balls in a total of 4 balls in urn II)

$$P(A/E_3) = \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2} = \frac{2}{11}$$ (since there are 4 white, 5 black, 3 Red balls in a total of 12 balls in urn III)

Hence,

$$P(E_2/A) = \frac{P(E_2)\cdot P(A/E_2)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2) + P(E_3)\cdot P(A/E_3)}$$

$$= \frac{\frac{1}{3}\cdot\frac{1}{3}}{\frac{1}{3}\cdot\frac{1}{5}+\frac{1}{3}\cdot\frac{1}{3}+\frac{1}{3}\cdot\frac{2}{11}}$$

$$= \frac{55}{118}$$

Similarly

$$P(E_3/A) = \frac{\frac{1}{3}\cdot\frac{2}{11}}{\frac{1}{3}\cdot\frac{1}{5}+\frac{1}{3}\cdot\frac{1}{3}+\frac{1}{3}\cdot\frac{1}{11}} = \frac{30}{118}$$

$$P(E_1/A) = 1 - P(E_2/A) - P(E_3/A) = \frac{33}{118}$$

Random Variables

8

Introduction:— The events described on a random experiment may be numerical (or) non-numerical (descriptive). For example, the out comes that we obtain, when we throw a die are numerical (we get the out comes as 1, 2, 3, 4, 5, 6). Hence the sample space is numerical. But the outcomes we obtain, when we toss a coin are non-numerical. we get the out comes as head (or) tail. Here the sample space is non-numerical/descriptive.

It is inconvenient to deal with these descriptive outcomes mathematically. Hence for easy of manipulation, we may assign a real number to each of the out comes using a fixed rule or mapping

For example when we toss a coin we get two outcomes. namely head & tail. We can assign numerical values say 1 to head and 0 to tail.

This rule or mapping from the original sample space (numerical or non-numerical) to a numerical (real) sample space, subjected to certain constraints is called a random variable. This random variable is real valued function which maps the numerical or non-numerical sample space (domain) of the random experiment to real values (co-domain or range).

Random Variable :- A real variable X whose value is determined by the outcome of a random experiment is called a random variable. (80)

Eg:- The sample space corresponding to tossing of two coins.

When we toss two coins, its outcomes (or) sample points can be $S = \{HH, HT, TH, TT\}$

we count the number of tails and denoted it by X

The first outcome HH has 0 tail so $X=0$.

The second outcome HT has 1 tail so $X=1$

The ~~third~~ outcome TH has 1 tail so $X=1$

The Fourth outcome TT has 2 Tails so $X=2$

Thus X takes the values 0, 1, 1, 2.

i.e $X = 0, 1, 2$.

A random variable is also called a stochastic variable (or) simply a variable or a chance variable.

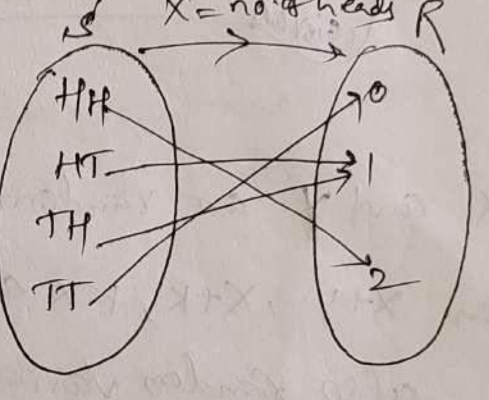
Random variables are usually denoted by capital letters of English alphabets and particular values which the random variable takes are denoted by the corresponding small letters.

Eg:- Consider a random experiment consisting of tossing a coin twice. The sample space (87)

$S = \{HH, HT, TH, TT\}$ consists of four elements (sample points). Define a function $X : S \longrightarrow R$ by $X(s)$ = number of heads.

Then $X(HH) = 2$, $X(HT)=1$, $X(TH)=1$, $X(TT)=0$

Range of $X = \{0, 1, 2\}$



Eg. Consider the random experiment of throwing a pair of dice and noting the sum. The sample space consists of points

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)$

$(2,1)\ (2,2)\ (2,3), (2,4), (2,5), (2,6)$

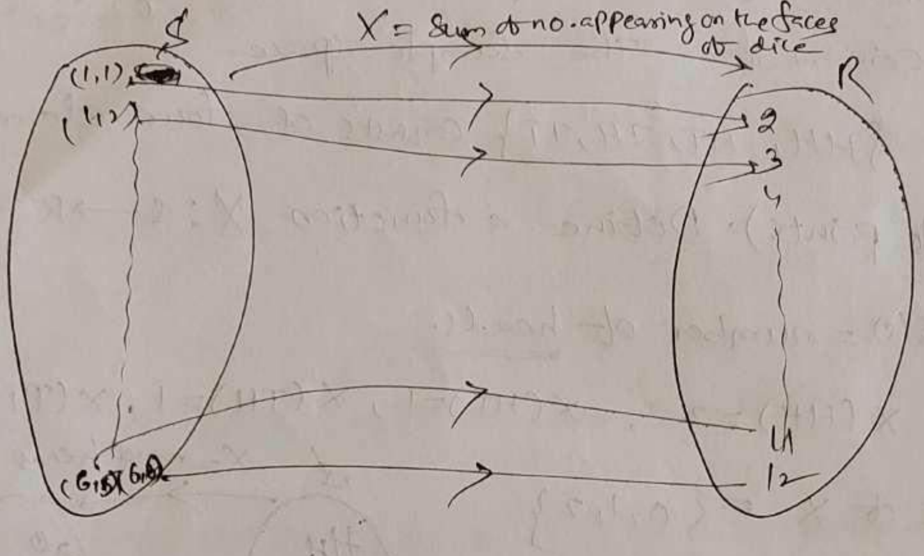
$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

If X is the "sum of numbers appearing on the faces of dice", X is a random variable.

a random

Here $X: S \to R$ is defined as

(88)



* If X and Y are random variables on the sample space S, then X+Y, X+K, KX and KY (K is a constant) are also random variables on S.

Types of Random Variables:—

Random Variable is of two types:

(i) Discrete Random Variable.

(ii) Continuous Random Variable.

Discrete Random Variable:— A random variable X which can take only a finite number of discrete values in an interval of domain is called a discrete random variable.

In other words, if the random variable takes the values only on the set $\{0, 1, 2, 3 \text{——} n\}$ is called a discrete random variables

(or)
A discrete variable takes only certain values in a range.

Tossing of a coin, throwing a dice, number of children in a family, number on a dice, the number of detectives in a sample of electric bulbs, the number of printing mistakes in each page of a book, the number of telephone calls received by the telephone operator, etc. Can take only integral values.

Thus to each out come s of a random experiment there corresponds a real number X(s) which is defined for each point of the sample space S.

<u>Continuous random variable</u>:- A random variable X which can take values continuously i.e which takes all possible values in a given interval is called a continuous random variable.

For example, the height, age and weight of individuals, temparature, time are continuous random variables.

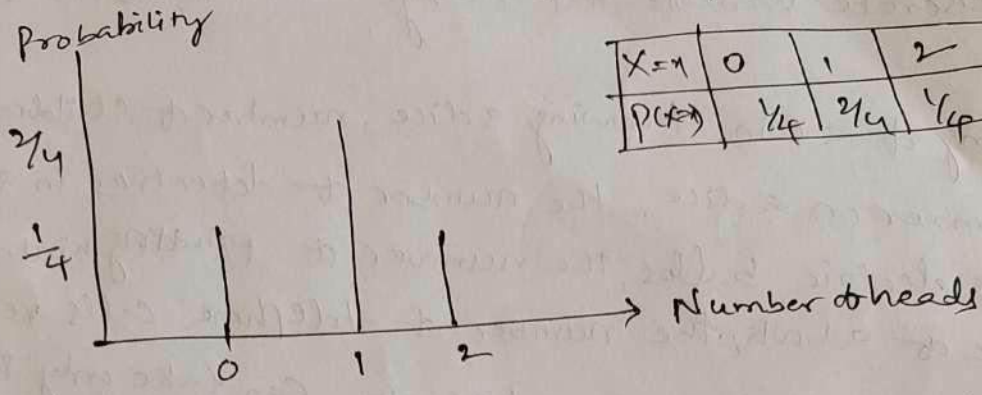
<u>Probability Function</u>:-

<u>Eg:- Distribution of probability over number of Heads</u>

when two coins are tossed, the number of heads is a random variable which can take the values 0, 1, 2.

The associated probabilities are $\frac{1}{4}, \frac{2}{4}, \frac{1}{4}$ and is shown below

$X=x$	0	1	2
$P(x)$	$1/4$	$2/4$	$1/4$



It may be noted that total of all the three probabilities $\frac{1}{4}, \frac{2}{4}$ and $\frac{1}{4}$ is 1.

The above diagram shows how the total probability $(=1)$ is distributed over the range of the variable. these Graphs are discribed by a mathematical function which is refferred to as a probability Function (or) Statistical distribution.

Probability Function OF A Discrete random Variable:-

If for a discrete random Variable X, the real valued function $p(x)$ is such that $P(X=x) = p(x)$ then $P(x)$ is called probability function (or) probability density function of a discrete random variable X.

probability function $P(x)$ gives the measure of probability for different values of X.

Properties of a probability function:—

If $P(x)$ is a probability function of a random Variable X, then is Possesses the following properties

(i) $P(x) \geq 0$ for all x

(ii) $\Sigma P(x) = 1$ Summation is taken over for all values of x

(iii) $P(x)$ Cannot be negative for any value of x.

Probability Distribution Function:—

Let X be a random Variable. Then the probability distribution function associated with X is defined as the probability that the outcome of an experiment will be one the outcomes for which $X(s) \leq x$, $x \in R$, That is the function $F(x)$ ie $F_x(x)$ defined by $F_x(x) = P(X \leq x) = P\{S : X(s) \leq x\}$, $-\alpha < x < \alpha$

Is called the distribution function of X.

Probability mass function (or) Discrete Probability distribution:—

Suppose a discrete Variate X is the outcome of Some experiment. If the probability that X takes the values x_i is P_i then

$$P(X = x_i) = P_i \quad (\text{or}) \quad P(x_i) \text{ for } i = 1, 2, 3 \text{ ———}$$

If the numbers $P(x_i)$, $i = 1, 2, 3$, ——— Satisfy the two conditions

(i) $P(x_i) \geq 0$ for all values of i

(ii) $\Sigma P(x_i) = 1$

then the function $p(x)$ is called the probability mass function of the random Variable X and the set $\{p(x_i)\}$, $i = 1, 2, 3$ —— is called the discrete Probability distribution of the discrete random Variable X. (2)

The Probability distribution of the random Variable X is given by means of the following table.

X	x_1	x_2	x_3	x_4	----	x_i	---- x_n
$p(x)$	p_1	p_2	p_3	p_4	----	p_i	---- p_n

Further $P(X < x_i) = P(x_1) + P(x_2) + P(x_3) + \text{——} + P(x_{i-1})$

$$P(X \leq x_i) = P(x_1) + P(x_2) + P(x_3) + \text{——} + P(x_i)$$

$$P(X > x_i) = 1 - P(X \leq x_i)$$

Eg:- In tossing a coin two times $S = \{TT, TH, HT, HH\}$

X = getting number of heads

$P(X=0)$ = Probability of getting two tails (no heads)

$$P(\{T,T\}) = \frac{1}{4}$$

$P(X=1)$ = probability of getting one head

$$P(\{H,T\},\{T,H\}) = \frac{2}{4}$$

$P(X=2)$ = Probability of getting two heads

$$P(\{H,H\}) = \frac{1}{4}$$

Thus the total Probability 1 is distributed into three parts $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$ according to whether $X=0,1,2$ (93)

This probability distribution is given in the following table

$X=x_i$	0	1	2
$P(X=x_i)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Cumulative Distribution Function of a discrete random Variable:-

Suppose that X is a discrete random variable. Then the Discrete distribution function (or) Cumulative distribution function $F(x)$ is defined by

$$F(x) = P(X \leq x) = \sum_{i=1}^{x} P(x_i) \text{ where } x \text{ is any integer}$$

Note:- If $P(x)$ i.e $f(x)$ is the probability function (or) probability distribution then the value of $\sum_{x=0}^{x} P(x)$ i.e $\sum_{x=0}^{x} f(x)$ denoted by $F(x)$ is called the Cumulative distribution function (or) simply distribution Function.

Suppose if X takes only a finite number of values $x_1, x_2, x_3 \text{------} x_n$ then the cumulative distribution function is given by

$$F(x) = \begin{cases} 0 & , \; -\infty < x < x_1 \\ p(x_1) & , \; x_1 \leq x < x_2 \\ p(x_1) + p(x_2) & , \; x_2 \leq x < x_3 \\ \text{------} & \\ \text{------} & \\ \text{------} & \\ p(x_1) + p(x_2) + \text{---} + p(x_n) & , \; x_n \leq x < \infty \end{cases}$$

Probability density function :- The probability density function $f_x(x)$ is defined as the derivative of the probability distribution function, $F_x(x)$, of the random variable X.

Thus $f_x(x) = \frac{d}{dx}\left[F_x(x)\right]$

Continuous probability distribution :- When a variate X takes every values in an interval, it gives rise to continuous distribution of X.

The probability distribution of a continuous variate x is defined by a function $f(x)$ such that the probability of the variate x falling in the small interval $x-\frac{1}{2}dx$ to $x+\frac{1}{2}dx$ is $f(x)dx$. Symbolically it can be expressed as $P(x-\frac{1}{2}dx \le x \le x+\frac{1}{2}dx) = f(x)dx$.

Then $f(x)$ is called the <u>probability density function</u> and the continuous curve $y=f(x)$ is called the probability curve.

The density function $f(x)$ is always positive and $\int_{-\infty}^{\infty} f(x)dx = 1$ (i.e the total area under the probability curve and the X-axis is unity which corresponds to the requirements that the total probability of happening of an event is unity)

<u>Cumulative distribution function of a Continuous random variable :-</u> If $F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)dx$

then $F(x)$ is defined as the cumulative distribution function or simply the distribution function of the continuous variate X.

The distribution function $F(x)$ has the following properties

(i) $F'(x) = f(x) \ge 0$ so that $F(x)$ is a non-decreasing function

(ii) $F(-\infty)=0$ (iii) $F(\infty)=1$

(iv) $P(a \le x \le b) = \int_a^b f(x)dx = \int_{-\infty}^{b} f(x)dx - \int_{-\infty}^{a} f(x)dx = F(b)-F(a)$.