

UNIT-II

SET THEORY

2.1:- Introduction

~~Algebraic Systems~~

2.2:- Basic Concepts of Set theory.

2.3:- Representation of Discrete Structures.

2.4:- Relations and Ordering.

2.5:- Functions.

07/03/24

* Set Theory :-

Set is a well defined collection of objects ~~which say~~.

→ Set is denoted by Capital letter $A, B, C \dots Z$.

→ Elements or object will be denoted by lower cases $a, b, c \dots x, y, z$.

→ the phrase is belongs to will be denoted by the symbol, ' $\in$ '.

Ex:- the states in India.

the students who have joined CSE branch in the college.

* Element or number :-

The object which makes a set are called element or number of the set.

Ex:- ① $x \in A$

x is belongs to A .

② $S = \{ \text{All even number} \}$

$S = \{ 2, 4, 6, 8, \dots \}$.

$2 \in S, 4 \in S$.

Set can be represented by using braces.

2.2:- Basic Concepts of Set Theory.

Set Notation:-

In generally, a set is represent in 2 ways. they are

① Roaster Notation.

② Set Builder Notation.

1) Roaster Notation:-

In roaster notation all the elements are listed & separated by comma & enclosed by braces.

Ex:- ① Set of all vowels in alphabet

$$V = \{a, e, i, o, u\}$$

② Set of even +ve integers $x < 10$

$$E = \{2, 4, 6, 8, 10\}$$

Let A be a set of letters in the word book.

$$A = \{B, O, O, K\}$$

2) Set Builder notation:-

A set builder notation we define the elements of the set by specifying the characteristic property with all the elements in the set process.

Ex: $V = \{x: x \text{ is a vowel}\}$
 $E = \{x: x \text{ is a true integer } \leq 10\}$

* Types of Sets:-

1) Null Set:-

A set is said to be null if doesn't consist any elements.

Ex: $\{3, \emptyset\}$

2) Singleton Set:-

A Singleton Set consists only one element.

Ex: $\{1\}, \{2\}$

3) Subset:-

Let A & B are 2 sets, then A is said to be subset of B . If every element of A is an element of B .

$B = \{2, 3, 4, 5, 6, 7\}$

$A = \{3, 4\}$

$A \subseteq B$

4) Universal Set:-

If all possible sets are subset of a fixed set then a fixed set is called as Universal Set & denoted by U .

5) Complement Set:-

Set contains all elements of universal set except set A is called complement.

Ex:- $U = \{1, 2, 3, 4, 5, 6\}$.

$A = \{4, 5, 6\}$

$A^c = \{1, 2, 3\}$

$$A^c = U - A$$

6) Finite Set:- If the no. of elements in a set are finite & countable then we say a set is a finite set.

Ex:-

7) Infinite Set:- If the no. of elements in a set are infinite or non countable then we say that a set is infinite set.

Ex:- Set of stars in sky.

8) Proper Subset:-

If A is subset of B & there is at least one element of B which is not in A. The symbol subset ~~is~~ $\subset$ stands

for a proper subset of A & B denoted by

$$A \subset B.$$

Ex:- $A = \{2, 3, 4, 5, 8, 9\}$

$B = \{2, 3, 4, 5, 8, 9, 10, 11\}$

$A \subset B = \{10, 11\}$

Equal set:- Two sets A & B are said to be equal, if they have precisely B elements.

Ex:- $A = \{1, 2, 3, 4\}$

$B = \{x/x \text{ is a positive integer}$

with $x^2 < 20\}$

$\therefore A = B$

power set:- Given a set A, a collection of all subset of A.

• power of set A is denoted by $P(A)$

• Suppose set A has n elements. the $P(A)$ contains 2^n .

Ex:- let $A = \{1, 2, 3\} = 2^3 = 8$

$$P(A) = \{\{1\}, \{2\}, \{3\}, \{1, 2, 3\}$$

$$\{1, 2\}, \{2, 3\}, \{1, 3\}, \emptyset\}$$

cartesian product.

let A & B are any two sets of all ordered pairs such that the first number is element of A & second number is element of B is called cartesian product of A & B . can be written as $A \times B$.

$$A \times B = \{(x, y) / (x \in A) \wedge (y \in B)\}$$

Ex:- let $A = \{1, 3, 5\}$ $C = \{4, 6\}$

$$B = \{2, 3\}$$

1.] find $A \times B, B \times A, (A \cup B) \times C, (A \times B) \cap C$
 $B \times C, A \cup B, (A \times B) \cap (B \times C)$

$$A \times B = \{(1, 2), (1, 3), (3, 2), (3, 3), (5, 3), (5, 2)\}$$

$$B \times A = \{(2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5)\}$$

$$(A \cup B) = \{1, 2, 3, 5\}$$

$$(A \cup B) \times C = \{(1, 4), (1, 6), (2, 4), (4, 6), (3, 4), (3, 6), (5, 4), (5, 6)\}$$

$$B \times C = \{(2, 4), (2, 6), (3, 4), (3, 6)\}$$

$$(A \times B) \cap (B \times C) = \emptyset$$

2.] Find x & y in each of the following cases :-

$$(2x, x+y) = (6, 1) \quad A \times B \text{ both } [1]$$

$$2x = 6$$

$$x+y=1 \quad A \times B$$

$$x = 3$$

$$x+y=1$$

$$y = 1 - 3$$

$$y = -2$$

Relations :-

The relationship between elements of two sets A & B we can use ordered pairs.

An ordered pair commonly known as a point.

If R is a set of ordered pairs
 given (a, b)

where $a \in A$ & $b \in B$ then R is a relation from A to B .

- The relation of set A is R which is defined as a subset of $A \times A$.

$\therefore$ The relation between the set is

$$R \subseteq A \times A.$$

- The relation of set A & B is R which is defined as subset of $A \times B$.

$$R \subseteq A \times B.$$

- The relation between a & b is denoted as $(a, b) \in R$.

Ex: Find out the relation between

Set A and B on relation R .

$$A = \{1, 2, 3\}, B = \{1, 2, 3\}, R = \{(1, 1), (1, 2), (1, 3), (2, 3)\}$$

what is $A \times B$.

$$A \times B = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2), (3, 3)\}$$

$$R \subseteq A \times B.$$

properties of relations (or) types of relations :-

A relation has mainly $\in$ properties.

1. Reflexive property.
2. Irreflexive property.
3. Symmetric.
4. Anti-symmetric.
5. Asymmetric.
6. Transitive property.

1.] Reflexive property :-

A relation R on a set A is said to be reflexive if $(a, a) \in R$, for every element of $a \in A$.

If $a \in A$, & $(a, a) \in R$.

then R is Reflexive.

Ex: let $A = \{1, 2, 3\}$

$$R = \{(1, 1), (2, 2), (3, 3)\}$$

find out reflexive relation of set

A on relation R

Sol: $1 \in A, (1, 1) \in R$

$2 \in A, (2, 2) \in R$

$3 \in A, (3, 3) \in R$

$\therefore R$ is reflexive.

2.] let $A = \{1, 2, 3, 4\}$

$R = \{(1, 1), (2, 2), (3, 3), (3, 4)\}$

Sol: $1 \in A, (1, 1) \in R$

$2 \in A, (2, 2) \in R$

$3 \in A, (3, 3) \in R$

$4 \in A, (3, 4) \in R$

$\therefore R$ is not reflexive.

2.] Irrreflexive property :-

A relation R on set A is called irreflexive if $(a, a) \notin R$ for every element of $a \in A$.

If $a \in A$, & $(a, a) \notin R$
then R is irreflexive

Ex: let $A = \{1, 2, 3\}$

$$R = \{(1, 1), (1, 2), (3, 2), (3, 3)\}$$

Sol: $1 \in A, (1, 1) \in R$

$\therefore R$ is not Reflexive.

2. let $A = \{1, 2, 3\}$ $R = \{(1, 1), (3, 2), (2, 3)\}$

Sol: $1 \in A, (1, 1) \notin R$

$2 \in A, (2, 2) \notin R$

$3 \in A, (3, 3) \notin R$

$(1, 2) \in R, (3, 2) \in R, (2, 3) \in R$

$\therefore$ It is irreflexive.

$A \times B$

3.] Symmetric :-

A relation R on a set A is called symmetric, if whenever $(a, b) \in R$ then $(b, a) \in R$ for all $(a, b) \in A$.

If $(a, b) \in A$ and $(a, b) \in R$
then $(b, a) \in R$ then
 R is symmetric.

4.] Antisymmetric :-

A relation R on a set A is
called antisymmetric if whenever

$(a, b) \in R$ & $(b, a) \in R$ then $a = b$

A relation R on set A is called
not antisymmetric if whenever
 $(a, b) \in R$ & $(b, a) \in R$ then $a \neq b$.

Ex: $A = \{1, 2, 3, 4\}$

$R = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

Sol: $(1, 1) \in R$, $(1, 1) \in R$ then $1 = 1$.

$(2, 2) \in R$

, then

$2 = 2$

$(3, 3) \in R$

, then

$3 = 3$

$(4, 4) \in R$

, then

$4 = 4$

$\therefore R$ is antisymmetric.

5.7 Asymmetric :-

A relation R on a set A is called asymmetric

if $(a, b) \in R$ implies that $(b, a) \notin R$,
for all $(a, b) \in A$.

Ex: $A = \{1, 2, 3, 4\}$

$R = \{(1, 2), (3, 1), (3, 4), (3, 2)\}$

Sol:- $(1, 2) \in R$ then $(2, 1) \notin R$

$(3, 1) \in R$ then $(1, 3) \notin R$

$(3, 4) \in R$ then $(4, 3) \notin R$

$(3, 2) \in R$ then $(2, 3) \notin R \dots$

$\therefore R$ is asymmetric.

6.] Transitive property:

A relation R on a set A is called transitive if whenever

$(a, b) \in R$ & $(b, c) \in R$ then $(a, c) \in R$
for all $(a, b, c) \in A$

Ex: $A = \{1, 2, 3, 4\}$

$R = \{(1, 2), (2, 4), (1, 4), (3, 2), (2, 1), (3, 1)\}$

Sol: $[(1, 2) \in R, (2, 4) \in R, (1, 4) \in R]$

$[(3, 2) \in R, (2, 1) \in R, (3, 1) \in R]$

$\therefore R$ is transitive.

⇒ Representation of relation:-

There are 2 methods used for representing the relations.

1. Using matrix.
2. Directed graphs or digraphs.

Relation matrix:-

Let $A = \{a_1, a_2, a_3, \dots, a_m\}$

$B = \{b_1, b_2, b_3, \dots, b_n\}$ are finite sets containing m & n elements respectively and

R is a relation from A to B .

- Then we can represent a relation R by $m \times n$ matrix called relation matrix.

Representation of relations

2 methods

① Using matrix

② Directed graph or digraph.

① Relation matrix:-

$$\text{Let } A = \{a_1, a_2, a_3, \dots, a_m\}$$

$$B = \{b_1, b_2, b_3, \dots, b_n\}$$

are finite sets containing m & n elements respectively. R is a relation from

A to B .

→ then we can represent a relation R by $m \times n$ matrix called relation matrix.

It is denoted by $M_R = [M_{ij}]$.

where $M_{ij} = \begin{cases} 1, & \text{if } (a_i, b_j) \in R \\ 0, & \text{if } (a_i, b_j) \notin R \end{cases}$

Ex:- Let $A = \{1, 2, 3\}$

$$B = \{1, 2, 3\}$$

$$R = \{(1,1), (2,2), (3,3)\}$$

$$\text{① } A \times B = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$$

2) Matrix form $M(A \times B) = \begin{bmatrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

3) If $(a, b) \in A \times B$ & $(a, b) \in R$ then result is '1'. Otherwise result is '0'.

$(1, 1) \in (A \times B)$, $(1, 1) \in R \Rightarrow 1$

$(1, 2) \in (A \times B)$, $(1, 2) \notin R \Rightarrow 0$

$(1, 3) \in (A \times B)$, $(1, 3) \notin R \Rightarrow 0$

$(2, 1) \in (A \times B)$, $(2, 1) \notin R \Rightarrow 0$

$(2, 2) \in (A \times B)$, $(2, 2) \in R \Rightarrow 1$

$(2, 3) \in (A \times B)$, $(2, 3) \notin R \Rightarrow 0$

$(3, 1) \in (A \times B)$, $(3, 1) \notin R \Rightarrow 0$

$(3, 2) \in (A \times B)$, $(3, 2) \notin R \Rightarrow 0$

$(3, 3) \in (A \times B)$, $(3, 3) \in R \Rightarrow 1$

4) Represent the relation in matrix form

$$M(R) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3}$$

5) the complement of relation R is R^c .

$$R^c = U - R$$

$$R^c = \{ (A \times B) - R \}$$

$$R^c = \{ (1,2) (1,3) (2,1) (2,3) (3,1) (3,2) \}$$

6) R^c in matrix form.

$$M[R^c] = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

② Relation representation using digraph/
directed graph.

A relation can be represented
pictorially by drawing its graph.

① A small circle is drawn from each
element of A & marked with correspondingly
element this circle is called vertices.

② An arrow is drawn from the
vertices a_i to a_j if & only if $(a_i, a_j) \in R$
is called as edge.

③ If (a, a) belongs to R then arrow
is drawn from a to a such edge
is called as loop.

④ this pictorial representation of R is called
as directed graph.

Sol Let $A = \{1, 2, 3, 4, 6\}$ & R be a relation on A defined by $a R b$ if and only if a is a multiple of b . Represent the relation R as a matrix and draw its digraph.

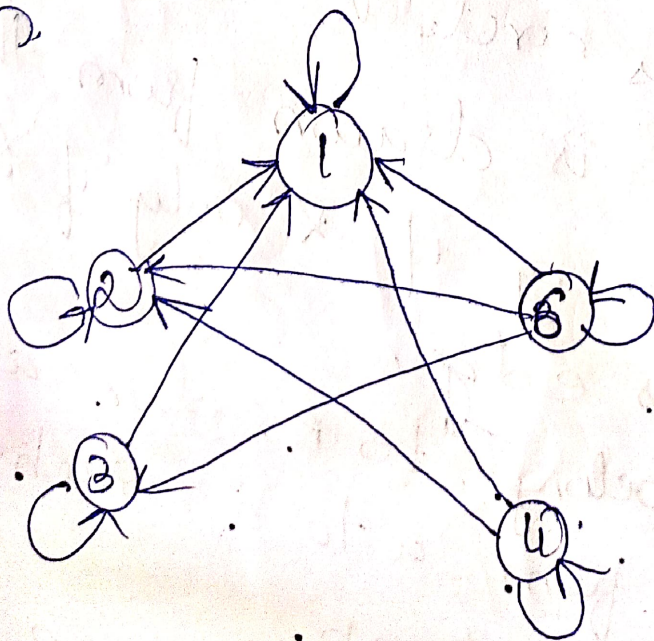
Sol Given,

$$R = \{(1, 1), (2, 1), (2, 2), (3, 1), (3, 3), (4, 1), (4, 2), (4, 4), (6, 1), (6, 2), (6, 3), (6, 6)\}$$

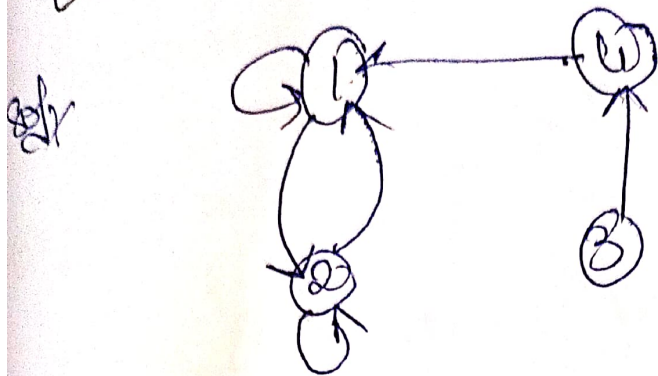
By examining the elements of R , we find that the matrix of R is

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Digraph



1) let $A = \{1, 2, 3, 4\}$ & $R = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,1)\}$ be a relation on A . Write down the digraph of R .



2) let $A = \{1, 2, 3, 4\}$ & let R be the relation on A defined by $x R y$ if & only if " x divides y ", written $x|y$.

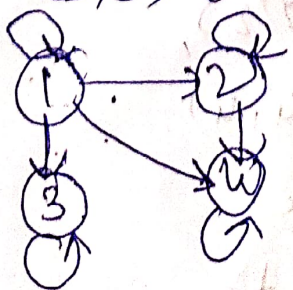
- Write down R as a set of ordered pairs.
- Draw the digraph of R .
- Determine in-degree & out-degree of the each vertex.

Ex Given,

i) $A = \{1, 2, 3, 4\}$

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$.

ii)



iii) In degree

$V_1 \rightarrow 1$

$V_2 \rightarrow 2$

$V_3 \rightarrow 2$

$V_4 \rightarrow 3$

Out

$V_1 \rightarrow 4$

$V_2 \rightarrow 1$

$V_3 \rightarrow 1$

$V_4 \rightarrow 1$

Equivalence
A relation R is said to be equivalence relation if it is reflexive, symmetric & transitive.

② Find out whether R is equivalent or not.

$$A = \{1, 2, 3\}$$

$$R = \{(1, 1), (2, 2), (3, 3), (2, 3), (3, 1)\}.$$

Reflexive

Sol^x $1 \in R, (1, 1) \in R$

$$2 \in R, (2, 2) \in R$$

$$3 \in R, (3, 3) \in R.$$

$\therefore R$ is reflexive.

Symmetric

$$(1, 1) \in R$$

$$(2, 2) \in R$$

$$(3, 3) \in R$$

$$(2, 3) \in R, (3, 2) \notin R$$

$$(3, 1) \in R, (1, 3) \notin R.$$

$\therefore R$ is not symmetric

$\therefore R$ is not equivalent.

→ Find out R is equivalent or not
 $A = \{1, 2, 3, 4\}$
 $R = \{(1,1) (2,2) (3,3) (4,4) (1,2) (2,1) (2,3) (3,2) (4,3) (3,4)\}$

Reflexive

$$1 \in A, (1,1) \in R$$

$$2 \in A, (2,2) \in R$$

$$3 \in A, (3,3) \in R$$

$$4 \in A, (4,4) \in R$$

$\therefore R$ is reflexive

Symmetric

$$(1,1) (2,2) (3,3) (4,4) \in R$$

$$(1,2) \in R, (2,1) \in R$$

$$(2,3) \in R, (3,2) \in R$$

$$(4,3) \in R, (3,4) \in R$$

$\therefore R$ is symmetric

transitive

$$(1,1), (1,2) \in R \text{ then } (1,2) \in R$$

$$(1,2), (2,1) \in R \text{ then } (1,1) \in R$$

$$(2,3), (3,2) \in R \text{ then } (2,2) \in R$$

$$(4,3), (3,4) \in R \text{ then } (4,4) \in R$$

$\therefore R$ is transitive

$\therefore R$ is equivalent

⇒ Compatibility Relation

In compatibility A relation R is said to be CR if it is reflexive &

Symmetric.

⇒ All equivalence relations are compatibility relation.

⇒ Composition of relations

Let R be a relation from x to y & S be a relation from y to z then the composition relation of R & S is the relation consisting of ordered

pairs (x, z) where $x \in X, z \in Z$ & for which there exist a element $y \in Y$ such that $(x, y) \in R$ & $(y, z) \in S$.

the elements of CR is

$R: X \rightarrow Y \quad x \in X$

$S: Y \rightarrow Z \quad y \in Y$

$ROS: X \rightarrow Z \quad z \in Z$

Let $A = \{1, 2, 3, 4\}$ & R & S are 2 relations on set A defined by.

$$R = \{(1, 2) (1, 3) (2, 4) (4, 4)\}.$$

$$S = \{(1, 1) (1, 2) (1, 3) (1, 4) (2, 3) (2, 4)\}.$$

find out ROS , SOR , $(ROS) \circ R$, $(ROR) \circ R$, SOS , $RO(SOR)$.

~~Sol~~ $ROS = \{ (1, 2) \in R, (2, 3) \in S \text{ then } (1, 3) \}$
 $(1, 2) \in R, (2, 4) \in S \text{ then } (1, 4)$

$$ROS = \{(1, 3) (1, 4)\}.$$

$SOR \Rightarrow (1, 1) \in S, (1, 2) \in R \text{ then } (1, 2)$

$(1, 1) \in S, (1, 3) \in R \text{ then } (1, 3)$

$(1, 4) \in S, (1, 4) \in R \text{ then } (1, 4)$

$(2, 4) \in S, (4, 4) \in R \text{ then } (2, 4)$

$$SOR = \{(1, 2) (1, 3) (1, 4) (2, 4)\}.$$

$(ROS) \circ R = (1, 4) \in ROS, (4, 4) \in R \text{ then } (1, 4)$

$$(ROS) \circ R = \{(1, 4)\}.$$

$(ROR) = (1, 2) \in R, (2, 4) \in R \text{ then } (1, 4)$

$(2, 4) \in R, (4, 4) \in R \text{ then } (2, 4)$

$(4, 4) \in R, (4, 4) \in R \text{ then } (4, 4)$

$$(ROR) = \{(1, 4) (2, 4) (4, 4)\}.$$

$(R \cup R) \cup R = \{(1,4) \in R \cup R, (4,4) \in R \text{ then } (1,4) \in R \cup R, (4,4) \in R \text{ then } (2,4) \in R \cup R, (4,4) \in R \text{ then } (2,4) \in R \cup R\}$

$$(R \cup R) \cup R = \{(1,4), (2,4)\}$$

$S \cup S = \{(1,1) \in S, (1,1) \in S \text{ then } (1,1) \in S\}$

$\{(1,1) \in S, (1,2) \in S \text{ then } (1,2) \in S\}$

$\{(1,1) \in S, (1,3) \in S \text{ then } (1,3) \in S\}$

$\{(1,1) \in S, (1,4) \in S \text{ then } (1,4) \in S\}$

$\{(1,2) \in S, (2,3) \in S \text{ then } (1,3) \in S\}$

$\{(1,2) \in S, (2,4) \in S \text{ then } (1,4) \in S\}$

$$S \cup S = \{(1,1), (1,2), (1,3), (1,4)\}$$

$R \cup (S \cup R) = \{(1,2) \in R, (2,4) \in S \cup R \text{ then } (1,4) \in R \cup (S \cup R)\}$

$$R \cup (S \cup R) = \{(1,4)\}$$

* Partially ordered set (POSET):-

A relation R on a set P is said to be partially ordering relation if & only if R is reflexive, antisymmetric & transitive.

→ Partially ordering is denoted by " $\leq$ "
 → If $\leq$ a PO on a set P then the order pair $(P, \leq)$ is called POSet.

$\rightarrow$ Let $(P, \leq)$ is a POset if for every 2 elements i.e. $(a, b) \in P$ we have either $a \leq b$, (or) $b \leq a$ from 's' is called a simple ordering or linear ordering on P & $(P, \leq)$ is called a totally ordered set.

Note: ① $(P, \leq)$ is dual of $(P, \geq)$
 ② $(P, \geq)$ is dual of $(P, \leq)$.

Ex 1. Let $A = \{1, 2, 3, 4\}$ then
 $R = \{(1, 2), (2, 3), (1, 1), (1, 3), (2, 2), (3, 3), (4, 4)\}$
 find whether R is a poset or not.

Sol: $1 \in A, (1, 1) \in R$
 $2 \in A, (2, 2) \in R$
 $3 \in A, (3, 3) \in R$
 $4 \in A, (4, 4) \in R$

$\therefore R$ is reflexive.

$(1, 2) \in R, (2, 3) \in R$ then $(1, 3) \in R$
 $(2, 3) \in R, (3, 3) \in R$ then $(2, 3) \in R$.

$\therefore R$ is transitive.

$(1, 1) \in R \quad 1 = 1$
 $(2, 2) \in R \quad 2 = 2$
 $(3, 3) \in R \quad 3 = 3$
 $(4, 4) \in R \quad 4 = 4$

$\therefore R$ is antisymmetric.
 $\therefore R$ is poset.

Hasse diagram

The digraph of poset is called Hasse diagram (or) poset diagram.

Rules for Hasse diagram

- 1) No loops for Reflexive.
- 2) No Edge for transitive Relation.
- 3) If $(a, b) \in R$ there must be an upward edge from a to b .
- 4) down to up.

Example:-

1) If R is a relation on the set $A = \{1, 2, 3, 4\}$ defined by xRy if x divides y . Prove that (A, R) is a poset. Draw its Hasse diagram.

$$R = \{(x, y) \mid x, y \in A \text{ and } x \text{ divides } y\}$$

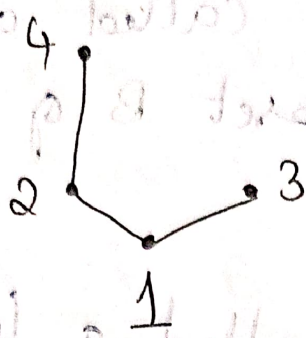
$$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$$

$$\Rightarrow \{(1,1), (2,2), (3,3), (4,4)\} \in R \rightarrow \text{Reflexive}$$

$$\Rightarrow \{(1,2) \in R \text{ but } (2,1) \notin R, (1,3) \in R \text{ but } (3,1) \notin R, (2,4) \in R \text{ but } (4,2) \notin R\} \rightarrow \text{Antisymmetric}$$

$$\Rightarrow \{(1,2), (2,4) \Rightarrow (1,4) \in R\} \rightarrow \text{Transitive}$$

So (A, R) is a poset.

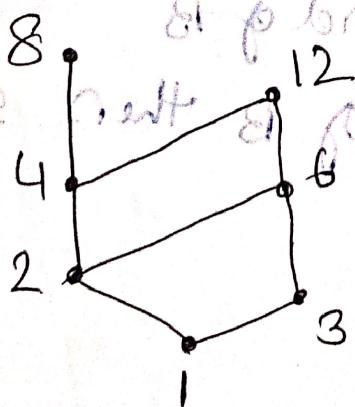


a) Let $A = \{1, 2, 3, 4, 6, 8, 12\}$ on A , define the partial ordering relation R by $a R b$ if and only if $a|b$. Draw the Hasse diagram for R .

b) Write down the relation matrix for R .

Ans. $R = \{ (1,1) (1,2) (1,3) (1,4) (1,6) (1,8) (1,12)$
 $(2,2) (2,4) (2,6) (2,8) (2,12)$
 $(3,3) (3,6) (3,12) (4,4) (4,8) (4,12)$
 $(6,6) (6,12) (8,8) (12,12) \}$

The Hasse diagram for this R is as shown below.



$(1,2) (2,4) \Rightarrow (1,4)$
 $(1,3) (3,6) \Rightarrow (1,6)$
 $(1,6) (6,12) \Rightarrow (1,12)$
 $(1,2) (2,6) \Rightarrow (1,6)$
 $(1,4) (4,8) \Rightarrow (1,8)$

$$R = \{ (1, 2) (1, 3) (1, 4) (2, 4) \}$$

$$(1, 1) (2, 2) (3, 3) (4, 4)$$

1) Draw the Hasse diagram for the divisibility relation defined on set ~~A~~

$$A = \{ 2, 3, 6, 12, 24, 36 \}$$

Solⁿ $R = \{ (2, 2) (2, 6) (2, 12) (2, 24) (2, 36) (3, 3) (3, 6) (3, 12) (3, 24) (3, 36) (6, 6) (6, 12) (6, 24) (6, 36) (12, 12) (12, 24) (12, 36) (24, 24) (36, 36) \}$

Step-2: Remove Self loops.

Step-3: Remove Transitive edges.

$$(2, 6) \& (6, 12) \rightarrow (2, 12)$$

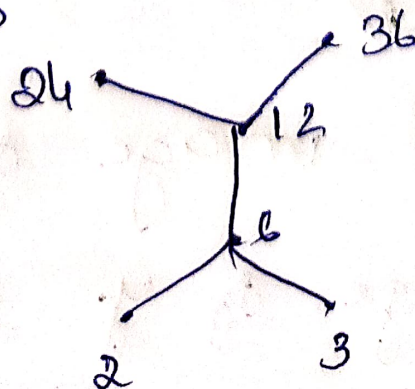
$$(2, 6) \& (6, 24) \rightarrow (2, 24)$$

$$(2, 12) \& (12, 36) \rightarrow (2, 36)$$

$$(6, 12) \& (12, 24) \rightarrow (6, 24)$$

$$(6, 12) \& (12, 36) \rightarrow (6, 36)$$

Step 4: Hasse diagram

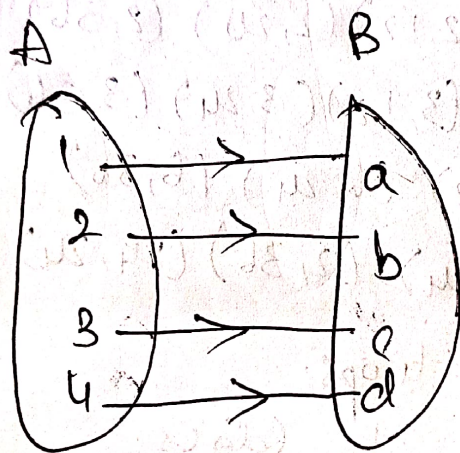


* Functions

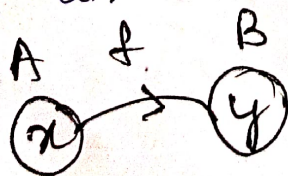
A mapping f i.e. $A \rightarrow B$ is said to be function if every element of A having unique image in B . If A, B are two sets then A is called as domain & B is range for that domain.

Representation of function

$$f: A \rightarrow B$$



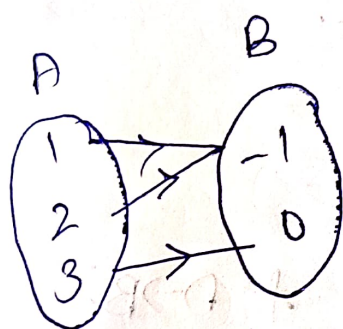
Let A & B are two sets A function f i.e. $A \rightarrow B$ is a relation from A to B such that for every element in set A there is an element in range in set B .



Here ' y ' is called image of x & x is called the preimage of y under function ' f '.

1) Let $A = \{1, 2, 3\}$ & $B = \{-1, 0\}$, S is a relation from A to B defined by $S = \{(1, -1), (2, -1), (3, 0)\}$. If S is a function or not. and find the domain & co domain of S .

Given sets are $A = \{1, 2, 3\}$
 $B = \{-1, 0\}$



$\therefore S$ is a function

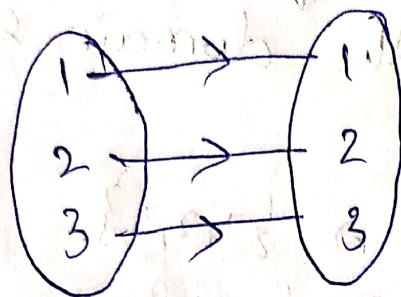
Domain $A = \{1, 2, 3\}$
 Co-Domain $B = \{-1, 0\}$

Types of function:-

- 1) Identity.
- 2) Constant
- 3) One-One function
- 4) Onto function.
- 5) Bijective.

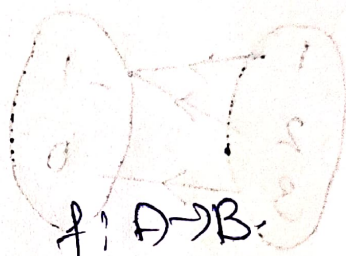
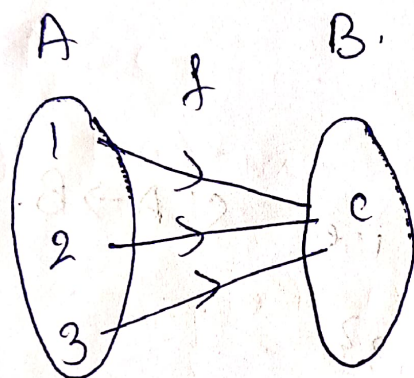
1) Identity: If a function $f(x)$ is said to be identity function if $f(x) = x$.

Ex:-



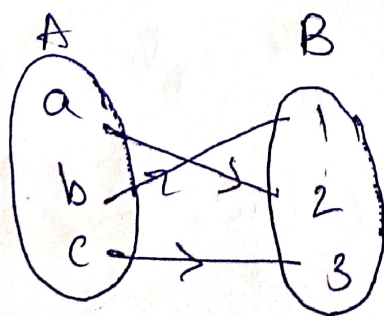
2) Constant: If a function $f(x)$ is said to be constant if $f(x) = c$.

Ex:-



3) One-One function:

A function $f(x)$ is said to be one-one function if every element of A must have distinct image in B.



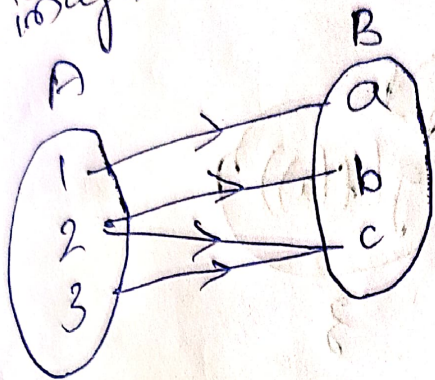
$$f(a) = 2$$

$$f(b) = 1$$

$$f(c) = 3$$

ii) Onto function :-

A fun $f(x)$ is said to be onto if every element of B must have pre-image in A



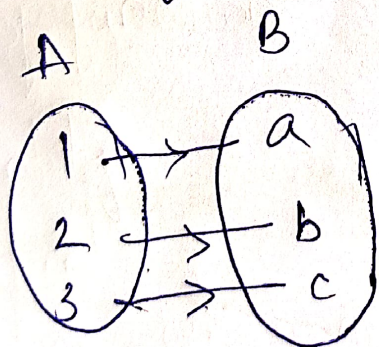
1' is the pre image of a

2' is the pre-image of b, c.

3' is the pre image of c.

ii) Bijective :-

A function $f(x)$ is said to be bijective. If the function is one-one & onto



$f: A \rightarrow B$

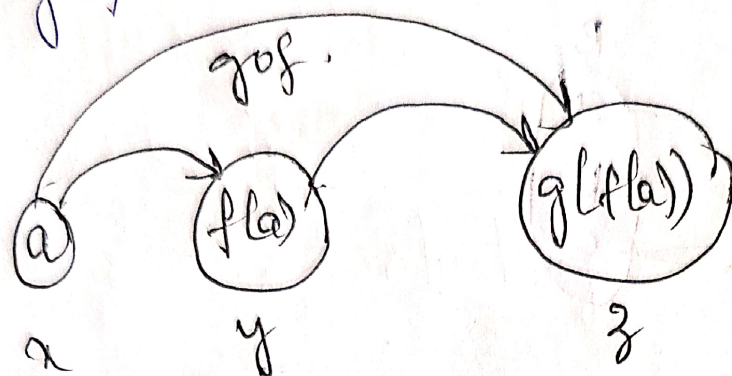
1 is the pre-image of a

2 is the pre-image of b

3 is the pre-image of c.

Composite functions:

let us consider $f: A \rightarrow B$, $g: B \rightarrow C$
then the composite function is defined
as $g \circ f: A \rightarrow C$.



$$g \circ f: A \rightarrow C$$

$$g(f(a)) = f(a) = a$$

1) let $f(x) = x^2$, $g(x) = x+1$, & $h(x) = x$, then
find $f \circ g \circ h$, $g \circ f \circ h$, $g \circ h \circ f$ & $f \circ h \circ g$.

$$\begin{aligned} \text{Sol. } f \circ g \circ h &= f \circ g \circ h(x) \\ &= f(g(h(x))) \\ &= f(x+1) \\ &= (x+1)^2. \end{aligned}$$

$$\begin{aligned} g \circ f \circ h &= g \circ f \circ h(x) \\ &= g(f(x)) \\ &= g(x^2) \\ &= x^2 + 1 \end{aligned}$$

$$\begin{aligned}
 g \circ h \circ f &= g \circ h \circ f(x) \\
 &= g \circ h(x^2) \\
 &= g(x^2) \\
 &= x^2 + 1
 \end{aligned}$$

$$\begin{aligned}
 f \circ h \circ g &= f \circ h \circ g(x) \\
 &= f \circ h(x+1) \\
 &= f(x+1) \\
 &= (x+1)^2
 \end{aligned}$$

Inverse function:-

A function $f: A \rightarrow B$ is said to be invertible if the selection is $f^{-1}: B \rightarrow A$ is also function then f^{-1} is called Inverse of f .

→ A function is said to be invertible if it must be one-one & onto function.

Steps to follow the finding inverse of a function

- 1) Replace $f(x)$ with y .
- 2) Interchange x 's with y 's.
- 3) Solve y .
- 4) Replace y with $f^{-1}(x)$.

i) let $f(x) = x+5$ & $g(x) = x^2+1$ then find:

$$f^{-1} \circ g^{-1} \text{ \& \> } g^{-1} \circ f^{-1}$$

Sol: $f(x) = x+5$, $g(x) = x^2+1$

$$\text{let } f(x) = y$$

$$x+5 = y$$

$$y+5 = x \text{ (Interchange)}$$

$$y = x-5$$

$$f^{-1}(x) = x-5$$

$$g(x) = x^2+1$$

$$\text{let } g(x) = y'$$

$$y' = x^2+1$$

$$x = (y')^2+1$$

$$x-1 = (y')^2$$

$$\sqrt{x-1} = y'$$

$$\Rightarrow g^{-1}(x) = \sqrt{x-1}$$

$$f^{-1} \circ g^{-1} = f^{-1} \circ g^{-1}(x)$$

$$= f^{-1}(\sqrt{x-1})$$

$$= \sqrt{x-1} - 5$$

$$\begin{aligned}
 g^{-1} \circ f^{-1} &= g^{-1} \circ f^{-1}(x) \\
 &= g^{-1}(x-5) \\
 &= \sqrt{x-5-1} \\
 &= \sqrt{x-6}.
 \end{aligned}$$

Recursive functions

A function f on a argument x is generally specified by indicating the value of $f(x)$ for every value of x in the domain of f in the explicit form.

→ Sometimes it is possible to define a function in terms of itself. This process is called Recursive function.

→ Apply a rule to its result is $f(x) = n!$ here n is every natural number.

$f(0) = 1$ & $f(n) = 1, 2, 3, \dots, n$ for $n \in \mathbb{Z}^+$

→ The recursive method of describing the function is $f(0) = 1$ & $f(n) = n f(n-1)$ for $n \in \mathbb{Z}^+$.

→ the recursive function for square of n is

$$f(n) = 0$$

$$f(n) = f(n-1) + 2n-1$$

$$f(1) = f(1-1) + 2(1) - 1 = 1 = 1^2$$

$$f(2) = f(2-1) + 2(2) - 1 = f(1) + 4 - 1 = 1 + 4 - 1 = 4 = 2^2$$

$$f(3) = f(3-1) + 2(3) - 1 = f(2) + 6 - 1 = 4 + 6 - 1 = 9 = 3^2$$

→ for factorial $f(n)$, n is defined recursively

by: $f(0) = 1$

$$f(n) = n \cdot f(n-1)$$

$$f(1) = 1 \cdot f(1-1)$$

$$= 1 \cdot f(0) = 1 \cdot 1 = 1$$

$$f(2) = 2 \cdot f(2-1)$$

$$= 2 \cdot f(1) = 2 \cdot 1 = 2$$

$$f(3) = 3 \cdot f(3-1)$$

$$= 3 \cdot f(2) = 3 \cdot 2 = 6$$

$$f(4) = 4 \cdot f(4-1)$$

$$= 4 \cdot f(3) = 4 \cdot 6 = 24$$